Finite Blocklength Performance of a Multi-Relay Network with Best Single Relay Selection

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***Abstract*—We consider a multi-relay system where the best single relay is selected to assist transmissions. We study the performance bound of the system in the finite blocklength regime. On the one hand, we extend Polyanskiy's finite blocklength model of a single-hop scenario to the considered relaying system and derive the corresponding achievable throughput. On the other hand, by employing a practical coding scheme, namely polar codes, a finite blocklength performance bound attainable by a low-complexity coding scheme is provided. Through simulation and numerical investigations, we show the appropriateness of the proposed bounds. Moreover, we evaluate both the achievable performance with finite blocklengths and the performance of polar codes in the best single relay scenario in comparison to direct transmission.**

***Index Terms*—best single relay, decode-and-forward, finite blocklength regime, relaying, polar codes.**

I. Introduction

Low-latency is one of the major concerns in the design of future wireless networks. Increasingly both researchers and designers of future wireless networks are interested in allowing for wireless links to carry latency-critical traffic as relevant for, *e.g.*, haptic feedback in virtual and augmented reality, e-health, autonomous driving, industrial control applications and cyber-physical systems.

Common scenarios in these networks feature multiple nodes densely deployed. Cooperative relaying significantly promotes the transmission performance and greatly capitalizes on dense node packings [1], [2], [3]. Consequently, the performance in multi-node networks may be leveraged by applying relaying, as each node may potentially act as a relay, *e.g.*, via best single relay (BSR) selection, assisting transmissions for peer nodes [4], [5], [6]. However, the above studies of relaying and the application of relaying in multi-node scenarios are under the ideal assumption of communicating arbitrarily reliable at rates close to Shannon's channel capacity. They thus implicitly assume an infinite blocklength (IBL) regime, which does not allow for accurately assessing the performance in low-latency, short blocklength scenarios.

In the finite blocklength (FBL) regime, the error probability of the communication is not negligible. Early in 1962, the normal approximation of coding rate is discussed [7]. More recently, an achievable upper bound on the coding rate is identified in [8] for a single-hop transmission system, taking the error probability into account. The result of [8] has been extended to Gilbert-Elliott Channels [9], as well as to quasi-static fading channels [10], [11]. Recently, an achievable FBL performance for relaying under a single relay scenario was addressed analytically in [12], [13], [14], [15]. Furthermore, bounds when using single relays with practical codes, *e.g.*, polar codes (PCs), are discussed in [16]. In fact, PCs have been considered in several relaying scenarios. In [17], PCs are employed to devise coding schemes for decode-and-forward (DF), as well as compress-and-forward relaying scenarios assuming relay channels with orthogonal receivers. A DF relaying scenario without the assumption of orthogonal receivers is addressed in [18], by implementing a Markov block coding scheme proposed in [19] using PCs. Inspired by [17], PCs are presented in an opportunistic cooperative DF relaying scenario [20]. While these works exploit structural properties of PCs, *e.g.*, the subset property of the information bit index sets in case of degraded channels as in [18], the relay scenarios considered are, to the best of our knowledge, either two-hop situations assuming a single-relay, or, as in [20] or [21], a main link assisted by a two-hop side channel based on a single relay network.

To the best of our knowledge, for low-latency, multi-node scenarios with BSR, both the achievable performance bound as well as practical bounds, *i.e.*, performance achieved by specific coding systems, have not been studied. We address these problems by providing an analytical attempt to solve the problem, and validate our results with simulations, employing PCs as an example of a low-complexity coding system. On the one hand, we derive the achievable performance bound based on Polyanskiy's FBL model. On the other hand, to support our theoretical findings, a practical FBL performance bound based on PCs is provided. The rest of the paper is organized as follows. Section II describes the system model and briefly introduces the theoretical background of the FBL regime, as well as the relevant aspects of polar codes. In Section III, we first derive the FBL performance of the system assuming static channels, and subsequently extend the performance model to a quasi-static channel fading scenario. Section IV presents our numerical results. Finally, we conclude our work in Section V.

II. Preliminaries

A. System Model

We consider a source S, a destination D and J DF relays R_j, where $j \in \mathcal{J} = \{1, \ldots, J\}$. This scenario is schematically

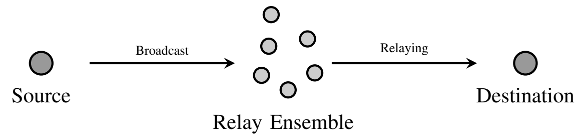

Fig. 1. Example of the considered best single relay (BSR) network.

depicted in Figure 1. In general, the distances among relays are significantly shorter than the distances between the source, the relay group and the destination. The entire system operates in a slotted fashion, where time is divided into transmission periods of length $2m$ symbols. Thus, each transmission period accommodates two phases of length m each, which are referred to as broadcasting phase and relaying phase. In the broadcasting phase, the source sends a data block to the relays. After reception, if any relay decodes the block successfully, such relay will be able to forward the data to the destination. We refer to such relays as *active relays*. In the subsequent relaying phase, one of the active relays, namely that with the best channel to the destination, will be selected to forward the data to the destination. We refer to this scenario as the BSR network.

Channels are assumed to suffer from Rayleigh block-fading. Hence, channels are constant during the duration of each transmission period but vary from one period to the next. We denote the instantaneous channel gains from the source to relay R_j and from R_j to the destination by $z_{\mathrm{S},j}$ and $z_{j,\mathrm{D}}$, $j \in \mathcal{J}$. Due to the random Rayleigh fading, tha channel gains follow exponential distribution with unit mean, i.e., $\mathbb{E}(z_{\mathrm{S},j}) = \mathbb{E}(z_{j,\mathrm{D}}) = 1$. The instantaneous signal-to-noise ratio (SNR) from the source to R_j and from R_j to the destination are denoted by $\gamma_{\mathrm{S},j}$ and $\gamma_{j,\mathrm{D}}$. We consider a homogeneous scenario where the average received SNRs of all the links from the source to relays are the same, denoted by $\bar{\gamma}_{\mathrm{S,R}}$, while the same applies to the average SNR of the links from relays to the destination, denoted by $\bar{\gamma}_{\mathrm{R,D}}$. Both the source as well as each of the relays is assumed to have access to perfect channel state information (CSI).

These assumptions allows us to study the fundamental performance of the considered multi-relay network, *i.e.*, not for a specific network topology. Therefore, we have $\gamma_{\mathrm{S},j} = z_{\mathrm{S},j}\bar{\gamma}_{\mathrm{S,R}}$ and $\gamma_{j,\mathrm{D}} = z_{j,\mathrm{D}}\bar{\gamma}_{\mathrm{R,D}}$, $j \in \mathcal{J}$. It is possible that multiple relays decode the data packet from the source correctly and become active relays, *i.e.*, multiple candidates may be selected to forward the data block in the upcoming relaying phase. Then, by selecting the best active relay to forward the data, the destination obtains a selective channel gain with the SNR given by $\max\limits_{j\in\mathcal{J}_\mathrm{a}}\{\gamma_{j,\mathrm{D}}\}$, where $\mathcal{J}_\mathrm{a}$ is the *active relay set.*

B. Blocklength-Limited Performance of Single-Hop Transmission Scenario with Perfect CSI

For additive white Gaussian noise (AWGN) channels, reference [8] derives a tight achievable bound for the coding rate of a single-hop transmission. With blocklength m, block error probability ϵ and signal-to-noise ratio (SNR) γ, the coding rate in bits per channel use is given by

$$\frac{1}{2}\log(1+\gamma) - \frac{\log e}{\gamma+1}\sqrt{\gamma\frac{\gamma+2}{2m}}Q^{-1}(\epsilon) + \frac{O(\log m)}{m}\,, \quad (1)$$

where $Q^{-1}(\cdot)$ is the inverse of the Q-function given by

$$Q(w) = \frac{1}{\sqrt{2\pi}}\int_w^\infty e^{-t^2/2}dt. \quad (2)$$

In [22], [23], the above result has been extended to a complex channel model with received SNR γ, where the coding rate in bits per channel use is

$$r = \mathcal{R}(\gamma,\epsilon,m) \approx \mathcal{C}(\gamma)\sqrt{\frac{V}{m}}Q^{-1}(\epsilon), \quad (3)$$

where $\mathcal{C}(\gamma)$ is the Shannon capacity of the channel used. For a known SNR, it is given by $\mathcal{C}(\gamma) = \log_2(1+\gamma)$. Moreover,

$$V = \left(1 - \frac{1}{1+\gamma^2}\right)(\log_2 e)^2 \quad (4)$$

is the channel dispersion [8, Def.1]. Hence, for a single hop transmission with blocklength n and coding rate r, the block error probability at the receiver is given by

$$\epsilon = \mathcal{P}(\gamma,r,m) \approx Q\left(\sqrt{\frac{m}{V}}\left(\mathcal{C}(\gamma)-r\right)\right). \quad (5)$$

C. Polar Codes

Recently introduced by Arıkan [24], PCs answer a long-standing, open question in information theory by providing a practical coding scheme which provably achieves the symmetric capacity of any binary-input discrete memoryless channel (B-DMC). Relying on channel polarization, PCs are constructed explicitly targeting a specific design channel. To do so, we may select an index set $\mathcal{A} \subseteq \mathbb{N}$ for a PC based on the probabilities of decision error under successive cancellation (SC) decoding

$$P_\mathrm{e}(\mathcal{W}_{m,i}) = \sum_{\mathbf{y}'\in\mathcal{Y}^m\times\mathcal{X}^{i-1}} \frac{1}{2}\min\left\{W_{m,i}(\mathbf{y}'\,|\,0), W_{m,i}(\mathbf{y}'\,|\,1)\right\}, \quad (6)$$

which estimates

$$\hat{u}_i = \underset{u_i\in\{0,1\}}{\arg\max}\left\{W_{m,i}\big(\mathbf{y},\hat{\mathbf{u}}_1^{i-1}\,|\,u_i\big)\right\} \quad (7)$$

based on a received block $\mathbf{y}$ and assuming $i-1$ correctly estimated information bits $\hat{\mathbf{u}}_1^{i-1}$. In the decisions, $W_{m,i}$ denotes the channel statistic of the (virtual) channels $\mathcal{W}_{m,i}$,

$$\mathcal{W}_{m,i} : \{0,1\} \to \mathcal{Y}^m \times \{0,1\}^{i-1},\ 1 \le i \le m\,, \quad (8)$$

modelling these decisions, where $\mathcal{Y}$ denotes the output alphabet of the basic channel $\mathcal{W}$. As $m \to \infty$, these polarize in the sense that the fraction of decisions vanishes for which $P_\mathrm{e}(\mathcal{W}_{m,i})$ does not approach either 0 or $\frac{1}{2}$.

Encoding and decoding using SC [24] is possible in quasi-linear complexity of $O(m\log m)$, where m is the blocklength. To increase the performance at short lengths, SC decoding has been extended to SC list (SCL) decoding [25], [26], which runs in $O(L\cdot m\log m)$ with list size L, *i.e.*, the number of candidate prefixes maintained in each step of the decoding process.

III. Achievable Throughput of BSR at Finite Blocklengths

A. Achievable FBL Throughput on Static Channels

In the FBL regime, the transmission is not error-free even with perfect CSI. According to (5), with coding rate r the achievable error probability of the link from the source to R_j is given by $\varepsilon_{\mathrm{S},j} = \mathcal{P}(\gamma_{\mathrm{S},j}, r, m)$. In other words, with probability $1 - \varepsilon_{\mathrm{S},j}$ R_j will decode the data block correctly. Hence, regardless of r, there is a positive probability that some relays decode the data block successfully and can be active in the relaying phase. Denote by n_{a} the size of the active relay set $\mathcal{J}_{\mathrm{a}}$, *i.e.*, $\mathcal{J}_{\mathrm{a}}$ contains n_{a} active relays which decode the data block correctly. Then, we have $n_{\mathrm{a}} = \sum_{j=1}^{J} x_j$, where x_j is a Bernoulli variable, *i.e.*, $x_j \sim \mathrm{Ber}(1 - \varepsilon_{\mathrm{S},j})$, indicating if R_j is in $\mathcal{J}_{\mathrm{a}}$. Hence, the expected value of n_{a} is given by $\mathbb{E}[n_{\mathrm{a}}] = \sum_{j=1}^{J} (1 - \varepsilon_{\mathrm{S},j})$.

From $\mathcal{J}_{\mathrm{a}}$, the relay with the highest SNR of the link to the destination is selected to achieve the highest reliability for the second hop. Hence, the received SNR at the destination is given by $\max_{j\in\mathcal{J}_{\mathrm{a}}}\{\gamma_{j,\mathrm{D}}\}$. The error probability of the second hop is given by $\mathcal{P}\big(\max_{j\in\mathcal{J}_{\mathrm{a}}}\{\gamma_{j,\mathrm{D}}\}, r, m\big)$. Therefore, the expected overall error probability of the considered network, *i.e.*, transmission assisted by BSR, is given by

$$\varepsilon_{\mathrm{BSR}} = \sum_{\mathcal{J}_{\mathrm{a}}\in\mathbb{P}(\mathcal{J})} \Big\{ \prod_{j\notin\mathcal{J}_{\mathrm{a}}} \varepsilon_{\mathrm{S},j} \prod_{n\in\mathcal{J}_{\mathrm{a}}} (1-\varepsilon_{\mathrm{S},n})\, \mathcal{P}\Big(\max_{j\in\mathcal{J}_{\mathrm{a}}}\{\gamma_{j,\mathrm{D}}\}, r, m\Big)\Big\}, \tag{9}$$

where $\mathbb{P}(\mathcal{J})$ is the powerset of the relay set $\mathcal{J}$. Thus, with coding rate r the expected FBL-throughput facing static channels is given by

$$\mu_{\mathrm{FBL}} = (1 - \varepsilon_{\mathrm{BSR}})\frac{r}{2}. \tag{10}$$

Note that $\varepsilon_{\mathrm{BSR}}$ is a function of r as described in (5), hence the maximal FBL-throughput is given by

$$\mu_{\mathrm{FBL}}^{\max} = \max_{r} \{\mu_{\mathrm{FBL}}\}. \tag{11}$$

B. Achievable Average FBL Throughput on Fading Channels

In this section, we extend the above analysis to quasi-static Rayleigh channels. Recall that the size of the forwarding relay set, given by n_{a}, is a sum of Bernoulli variables, *i.e.*, $x_j \sim \mathrm{Ber}(\varepsilon_{\mathrm{S},j})$, $j \in \mathcal{J}$. To the best of our knowledge, as of now there is no accurate closed-form expression characterizing the distribution of the sum variable n_a. In the following, we propose an approximation for modeling the achievable FBL performance. For all these homogeneous links from the source to the relays, the expected error probability using the fading channel are the same, which are given by $\bar{\varepsilon}_{\mathrm{S,R}} = \mathbb{E}_{\gamma_{\mathrm{S},j}}[\varepsilon_{\mathrm{S},j}]$, $j \in \mathcal{J}$. We write $\bar{n}_{\mathrm{a}}$ as the number of active relays based on this expected error probability, where $\bar{n}_{\mathrm{a}}$ is binomially distributed, *i.e.*, $\bar{n}_{\mathrm{a}} \sim \mathrm{Bin}(J, \bar{\varepsilon}_{\mathrm{S,R}})$. According to [27], we have

$$\mathcal{B}(\bar{n}_{\mathrm{a}}, J, \bar{\varepsilon}_{\mathrm{S,R}}) = \binom{J}{\bar{n}_{\mathrm{a}}} (1 - \bar{\varepsilon}_{\mathrm{S,R}})^{\bar{n}_{\mathrm{a}}} \bar{\varepsilon}_{\mathrm{S,R}}^{J-\bar{n}_{\mathrm{a}}}. \tag{12}$$

We approximate the average FBL throughput by assuming $\bar{n}_{\mathrm{a}}$ as the true number of active relays for each transmission period, hence use an approximation based on treating the expected error probability $\bar{\varepsilon}_{\mathrm{S,R}}$ as the instantaneous error probability for each link of the first hop.

In the presence of Rayleigh fading, the probability density function (PDF) of the fading gain z is given by $f(z) = e^{-z}$. Hence, the PDF of the received SNR of a second hop link with average SNR $\bar{\gamma}_{\mathrm{R,D}}$ is given by

$$f(\gamma) = \frac{1}{\bar{\gamma}_{\mathrm{R,D}}} e^{-\frac{\gamma}{\bar{\gamma}_{\mathrm{R,D}}}}, \tag{13}$$

whereas the cumulative density function (CDF) is

$$F(\gamma) = 1 - e^{-\frac{\gamma}{\bar{\gamma}_{\mathrm{R,D}}}}. \tag{14}$$

Then, the CDF of the received SNR at the second hop conditioned on $\bar{n}_{\mathrm{a}}$, *i.e.*, the optimal SNR among links from $\bar{n}_{\mathrm{a}}$ active relays to the destination, is given by

$$F_{\max|\bar{n}_{\mathrm{a}}}(\gamma) = \Big(1 - e^{-\frac{\gamma}{\bar{\gamma}_{\mathrm{R,D}}}}\Big)^{\bar{n}_{\mathrm{a}}}, \tag{15}$$

with PDF

$$f_{\max|\bar{n}_{\mathrm{a}}}(\gamma) = \frac{\bar{n}_{\mathrm{a}}}{\bar{\gamma}_{\mathrm{R,D}}}\Big(1 - e^{-\frac{\gamma}{\bar{\gamma}_{\mathrm{R,D}}}}\Big)^{\bar{n}_{\mathrm{a}}-1} e^{-\frac{\gamma}{\bar{\gamma}_{\mathrm{R,D}}}}. \tag{16}$$

Hence, the overall error probability conditioned on $\bar{n}_{\mathrm{a}}$ is

$$\mathbb{E}\big[\varepsilon_{\mathrm{BSR}|\bar{n}_{\mathrm{a}}}\big] = \int_0^{+\infty} f_{\max|\bar{n}_{\mathrm{a}}}(\gamma)\mathcal{P}(\gamma, r, m)\, d\gamma. \tag{17}$$

Consequently, the average error probability over $\bar{n}_{\mathrm{a}}$ is given in (18).

By construction, $\mathbb{E}[\varepsilon_{\mathrm{BSR}}]$ is a function of the coding rate r. For a given r, the average FBL-throughput then calculates to

$$\mu_{\mathrm{FBL}} = (1 - \mathbb{E}[\varepsilon_{\mathrm{BSR}}])\frac{r}{2}. \tag{19}$$

In particular, the achievable FBL-throughput is obtained at the optimal choice of r. Then, the average of the achievable FBL-throughput over fading is

$$\mu_{\mathrm{FBL}}^{\max} = \frac{1}{2}\mathop{\mathbb{E}}_{\gamma}\Big[\max_{r}\{(1 - \mathbb{E}[\varepsilon_{\mathrm{BSR}}])r\}\Big]. \tag{20}$$

C. Throughput Bound of Polar Codes

While there are plenty of well-established choices for a coding system, we opt for PCs for several reasons. By design, PCs allow for a wide range of rates via fixing the number of channel uses employed to convey information to an arbitrary integer. Combined with an explicit upper bound on the probability of block error under successive cancellation (SC) decoding, but also to that of the same code under more powerful, but still low-complexity decoders, this allows for great flexibility for numerically evaluating our findings.

To construct the PCs used throughout this work, we employ an optimized variant [28] of the construction heuristic [29]. Assuming an AWGN channel of SNR γ using binary phase-shift keying and hard-decision at the receiver, we target the equivalent binary symmetric channel during construction. We then select index sets based on the approximated channel

$$
\begin{aligned}
\mathbb{E}[\varepsilon_{\mathrm{BSR}}] &= \mathop{\mathbb{E}}_{\bar{n}_{\mathrm{a}}}\left[\mathbb{E}[\varepsilon_{\mathrm{BSR}|\bar{n}_{\mathrm{a}}}]\right] = \sum_{\bar{n}_{\mathrm{a}}=1}^{N}\int_{0}^{+\infty} f_{\max|\bar{n}_{\mathrm{a}}}(\gamma)\mathcal{P}\left(\gamma,r,m\right)\mathcal{B}\left(\bar{n}_{\mathrm{a}},J,\bar{\varepsilon}_{\mathrm{S,R}}\right)d\gamma \\
&= \sum_{\bar{n}_{\mathrm{a}}=1}^{N}\int_{0}^{+\infty}\mathcal{P}\left(\gamma,r,m\right)\binom{J}{\bar{n}_{\mathrm{a}}}\left(1-\bar{\varepsilon}_{\mathrm{S,R}}\right)^{\bar{n}_{\mathrm{a}}}\bar{\varepsilon}_{\mathrm{S,R}}^{J-\bar{n}_{\mathrm{a}}}\frac{\bar{n}_{\mathrm{a}}}{\bar{\gamma}_{\mathrm{R,D}}}\left(1-e^{-\frac{\gamma}{\bar{\gamma}_{\mathrm{R,D}}}}\right)^{\bar{n}_{\mathrm{a}}-1}e^{-\frac{\gamma}{\bar{\gamma}_{\mathrm{R,D}}}}d\gamma.
\end{aligned}
\tag{18}
$$

parameters $P_{\mathrm{e}}(\mathcal{W}_{m,i})$ as given in (6) such that the union bound of block error

$$
\mathcal{P}_{\mathrm{PC}}\left(\gamma,r,m\right) \leq \sum_{i\in\mathcal{A}} P_{\mathrm{e}}(\mathcal{W}_{m,i}) \tag{21}
$$

is minimized. During our experiments, $r = \frac{|\mathcal{A}|}{m}$. Hence, defining $|\mathcal{A}|$ will allow for m different rates $r\in\left\{\frac{1}{m},\ldots,1\right\}$. Note that by this approach, in addition to the construction of a specific code instance used in simulations, we obtain a theoretically justified, explicit upper error bound on the performance of such code under SC decoding, which translates to a lower bound on throughput under SC decoding. To obtain a lower bound on the throughput of PCs in the BSR network, we then employ the upper bound on $\mathcal{P}_{\mathrm{PC}}\left(\gamma,r,m\right)$ as in (21) as $\mathcal{P}\left(\gamma,r,m\right)$, and obtain the achievable FBL throughput as in the previous sections.

IV. Numerical Results and Discussion

In this section, we present simulative and numerical results to support our findings. We first validate our performance models by simulations. In addition to that, we evaluate the BSR performance in comparison to direct transmission in the FBL regime. In the simulation and numerical analysis, we consider an urban scenario, where distances of broadcasting, relaying and direct links are set to 200m, 200m and 275m. We set the transmit and noise power to 25 dBm and -85 dBm, respectively. In addition, we utilize the well-known COST [30] model for calculating the pathloss. Our system operates in a very low SNR regime, thus motivating the application of relays to enhance transmission reliability.

The results are shown in Figure 2, where simulative results for PCs are based on a PC of length $m = 512$ and $r = \frac{3}{4}$ under SC decoding as in [24], as well as SCL decoding with a list size of $L = 16$ as in [25], [26]. The FBL simulative results are obtained by applying Polyanskiy's achievable error probability at each single link and each randomly generated channel realization.

We observe a very close match between the FBL simulations and the proposed FBL analytical model, which indicates that the employed approximation for the active relay number in (12) is appropriate. In addition to that, the analytical bound of PCs based on (21) is slightly lower and bounds the simulative results, tightly for SC decoding. This is in line with our model, as (21) provides an upper bound on the probability of block error under SC decoding which in turn is as good as SCL decoding with $L = 1$, and weaker for greater L.

Hence (21) provides a basis for an analytical lower bound of PC throughput in a BSR network setting under each of the three decoders employed. Moreover, as SCL decoding helps to overcome the performance limitations of PCs at short

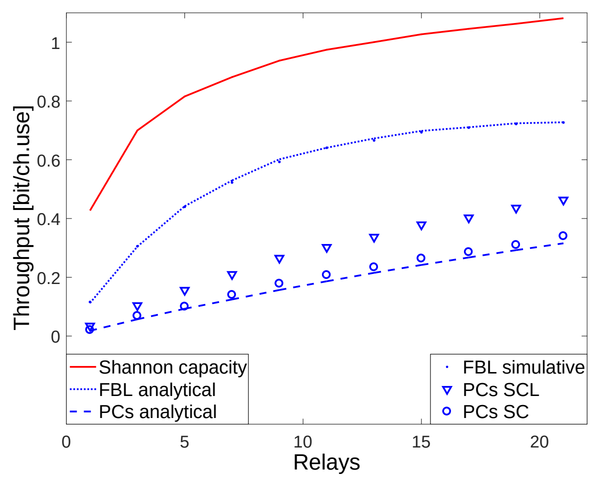

Fig. 2. Simulations compared to analytical bounds (m = 512, r = 0.75).

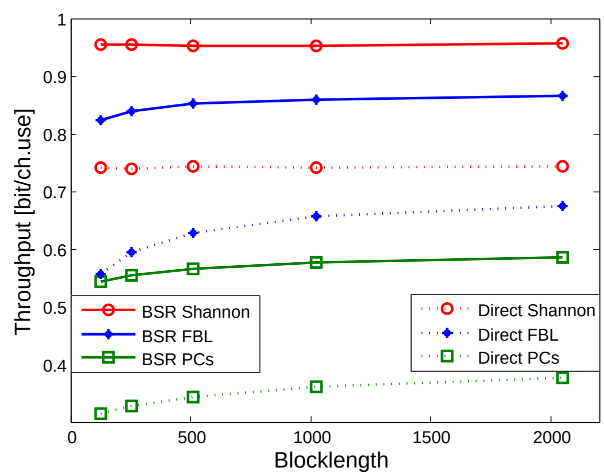

Fig. 3. Throughputs in the best-single relay scenario with J=10 relays.

blocklengths and makes them comparable to other codes as LDPC codes, *cf. e.g.*, [26], we expect these results to be good surrogates for the performance of other codes at equal lengths.

However, as we restrict modulation to binary signalling in this work, we expect performance gains to be reaped by considering higher-order modulations or SNR-adaptive modulation schemes.

The average throughput of the considered BSR network with Rayleigh fading is shown in Figure 3, where the blocklength is varied. In the figure, the throughput at each channel realization is obtained based on the optimal choice of coding rate and the respective upper bound on the block error probability under SC decoding given by (21). In addition, we investigate BSR performance in comparison to the direct scheme, using

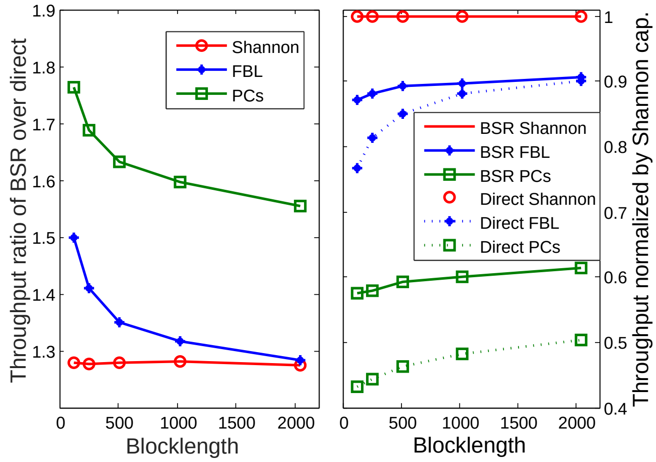

Fig. 4. Throughput ratios corresponding to Figure 3.

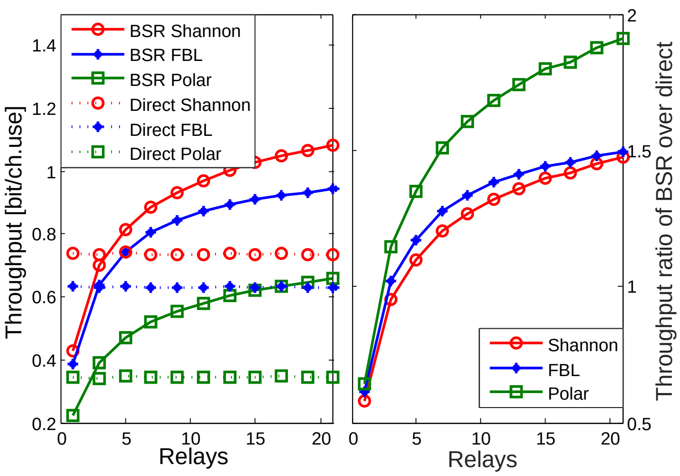

Fig. 5. Throughputs of BSR vs. relay number. m = 512.

a doubled blocklength for the latter. Consequently, to have a fair comparison, due to the two-hop setting the successfully received bits are reduced by a factor of $\frac{1}{2}$ to obtain the overall throughput of BSR, while this does not apply to the single-hop direct transmission.

Inspecting Figure 3, we note a performance loss (in comparison to the Shannon capacity) when using either BSR or direct transmission, which is due to the significantly limited blocklength. Furthermore, we note that even though the average SNR of the direct link is comparable to the broadcasting and relaying links (recall that they have comparable transmission distance), with 10 relays the BSR scheme significantly outperforms the direct transmission scheme. On the other hand, the limited power of PCs at such short lengths is exposed.

To further compare these schemes, we consider the following two perspectives on the results shown in Figure 3. *Perspective I*: We consider throughput ratios of BSR over direct transmission with respect to the Shannon capacity, FBL throughput and PC throughput, respectively, given in Figure 4-left. From the figure, the performance improvements by applying BSR are far more prominent in the FBL regime than in the IBL regime (BSR Shannon and Direct Shannon curves), as both the ratios of FBL and PCs are higher than the ratio of the Shannon capacity. In particular, for a system operating with PCs applying BSR is more beneficial. Furthermore, BSR helps most for short blocklengths. As a result, BSR poses a valid strategy to overcome the performance impairments of short codes in general, exemplified by PCs at short lengths in this work.

Then, in Figure 4-right we consider *Perspective II*, for which we normalize the BSR throughputs by the BSR Shannon capacity and normalize the direct transmission throughputs by that of the direct transmission, respectively. We observe gains reaped by both BSR and direct transmission in the FBL regime as well as in the IBL regime. Figure 4-right shows that the performance gap between the Shannon curve and the FBL/PC curve due to short blocklengths is more significant for direct transmissions. This result is consistent with Figure 4-left. BSR is more beneficial than direct transmission in the FBL regime, especially for the PCs employed and at such short blocklengths. The advantages of applying BSR in the FBL regime is in accord with our previous study [12], [13], [14] of a single relay network. We further investigate this performance advantage of relaying in the BSR network with a variable number of relays J, which also covers the single relay scenario, *i.e.*, $J = 1$. Figure 5 shows maximum throughputs using the optimal coding rate and corresponding ratios obtained based on *Perspective I*. We also investigated *Perspective II* for Figure 5 and observe well-matching results, omitted here due to space considerations.

Inspecting Figure 5-left we note that deploying more relays improves the performance of BSR in both the FBL regime and the IBL regime. In addition, Figure 5-right also shows that the gain of deploying more relays in the FBL regime (both FBL achievable bound and the PC bound) is more significant than in the IBL regime (Shannon bound). These results indicate that the selective diversity gain of BSR is more beneficial for short blocklength systems, an effect even more pronounced for PCs. For the design of low-latency short blocklength systems, this result suggests a large cluster size of nodes acting as relay candidates. Moreover, for a network with a relatively large number of relays, the diversity gains of BSR with respect to the Shannon capacity, as well as that with respect to the achievable bound in the FBL regime, increase slowly in the relay number and converge. At the same time, PCs again significantly profit from an increasing number of relays available.

We conclude the discussion by reiterating that the results presented for PCs in Figures 3 to 5 are based on the upper bound of block error under SC decoding, as it allows for analytically bounding the performance, while for the SCL decoder no sharper bounds exist. However, we expect the gaps observed between the achievable FBL bounds and the PC bounds to be less pronounced for more powerful decoders, as well as when using codes with better short blocklength performance.

V. Conclusion

We characterized the achievable FBL performance bound for a BSR network and illustrated it employing PCs as an example of a low-complexity coding system which greatly profits from BSR-induced diversity gains. The appropriateness of the bounds is validated by simulations. Our evaluation suggests great benefits of BSR compared to direct transmission in the FBL regime, especially for PCs in short blocklengths scenarios. In comparison to the single relay network, having more relay candidates is beneficial in both IBL and FBL regimes. Moreover, the achievable performance in both regimes is improved to a similar extent, while the impact on the performance of short PCs is significant. To extend our findings, future work will focus on the theoretical underpinnings of the performance analysis model for FBL scenarios presented here, as well as on different coding schemes in multi-node scenarios with BSR and other multi-relay selection strategies. In addition to that, the impact of imperfect or outdated CSI on the system performance will be addressed.

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