Simultaneous Wireless Information and Power Transfer in Relay Networks With Finite Blocklength Codes

(Invited Paper)

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Abstract

This paper considers simultaneous wireless information and power transfer (SWIPT) mechanisms in a relaying network with finite blocklength (FBL) codes. The reliability of the network is optimized under both a power splitting (PS) protocol and a proposed joint PS and time switching (TS) protocol. Under both protocols, we first determine the overall error probability formulation of the SWIPT-enabled two-hop transmission in the FBL regime. Subsequently, we state and solve optimization problems minimizing the overall error probability. Via numerical analysis, we show the appropriateness of our analytical model and demonstrate the performance advantage of the proposed protocol in comparison to TS and PS protocols. In addition, we provide interesting insights on the system behavior by characterizing the impact of the blocklength, transmit power and packet size on the reliability performance.

Keywords

Energy harvesting, finite blocklength, relay, SWIPT.

I. Introduction

Future wireless networks are expected to provide ultra reliable and low-latency communication services while reducing the energy consumption. On the one hand, there has recently been significant interest in having wireless links to support latency-critical traffic as relevant in several applications [1], [2] involving, e.g., haptic feedback in virtual/augmented reality and E-health. The common characteristic of these applications is that the coding blocklengths for a wireless transmission are quite short due to the low latency constraint.

On the other hand, integrating energy harvesting (EH) technologies into communication networks has been shown to be a promising way to reduce energy consumption [3], [4] and improve the energy efficiency [5]–[7], while the energy can be replenished from various sources, e.g., solar, wind, vibrations, and radio frequency (RF) radiation. Among numerous energy sources, the RF radiation is of particular interest as the source sends signals carrying both energy and information at the same time [8], which is so-called simultaneous wireless information and power transfer (SWIPT). In particular, it has shown that relay-assisted SWIPT significantly improves the overall transmission performances [9].

Typically, two protocols have proposed for SWIPT networks [10], namely, time switching (TS) and power splitting (PS). In [11], TS and PS protocols have been studied in relaying networks, while the outage probability is determined under the protocols. The reliability performance has been studied in a multi-user cooperative decode-and-forward (DF) relaying network in [12]. In [13], the error probability of SWIPT protocols has been investigated in a large-scale relay network. In an interference relay network, a distributed PS framework using game theory is developed to improve the network-wide performance [14]. However, all the above studies on the SWIPT-enabled relaying network are performed under the ideal assumption of communicating arbitrarily reliably at Shannon's channel capacity, i.e., codewords are assumed to be infinitely long.

In the finite blocklength (FBL) regime, the data transmission is no longer arbitrarily reliable. Especially when the blocklength is short (due to the low-latency requirement), the error probability becomes significant even if the coding rate is below the Shannon limit. Taking this into account, an accurate approximation of the achievable coding rate under the finite blocklength assumption for an additive white Gaussian noise (AWGN) channel was derived in [15] for a single-hop transmission system. Subsequently, the initial work for AWGN channels was extended to Gilbert-Elliott channels [16] and quasi-static fading channels [17], [18]. In our previous work [19]–[22], the FBL performance model was generally developed for a relaying network without SWIPT protocols. More recently, the FBL throughput of a SWIPT-enabled relaying network under the TS protocol is evaluated in [23]. However, the optimal transmission design, especially the performance and optimal scheduling of PS protocols, in SWIPT-enabled relaying networks has not been addressed in the FBL regime.

In this work, we consider a SWIPT-enabled relaying network, where the source sends wireless information and energy simultaneously to a relay, and the relay subsequently forwards the received data to the destination powered by harvested energy. The relay is assumed to be able to work with both the TS and PS models. Our aim is to minimize overall error probability in the FBL regime. The contributions of this paper can be further detailed as follows:

- We study a the SWIPT-enabled relaying network with a PS protocol in the FBL regime and determine the overall error probability. We prove that the error probability under the PS protocol is convex in the PS ratio, i.e., the optimization problem minimizing the error probability can be solved efficiently.
- We propose a joint TS and PS protocol for the relaying network and determine the overall error probability in the FBL regime. An algorithm is proposed to minimize the overall error probability under the protocol.
- Via numerical analysis, we show the performance advantages of the proposed joint TS and PS protocol. In addition, it is shown that the proposed algorithm achieves the same performance as the executive search. In addition, the impact of blocklength, transmit power and packet size on the reliability performance is characterized.

The remainder of the paper is organized as follows: In Section II, we describe the system model and briefly provide the background on the FBL regime. In Section III, we first study the PS protocol and minimize the error probability in the FBL regime. Subsequently, we propose a joint TS-PS protocol and minimize the overall error probability under the proposal. We provide our numerical results in Section IV and finally conclude the paper in Section V.

II. Preliminaries

A. System description

We consider a dual-hop relay system with a source S, a DF relay R and a destination D, as shown in Fig. 1. The system operates in a time-slotted fashion, where time is divided into frames. The length of each frame is of M symbols, corresponding to $T = MT_{\mathrm{c}}$ seconds, where T_{c} is a symbol length in time. In a frame, the source is required to transmit a data packet of a fixed size k bits to the destination with the help of the relay. In the frame, the relay first harvests energy and receive information from the source. If the relay decodes the data packet successfully, it forwards the data packet to destination in the subsequent hop using the harvested energy.

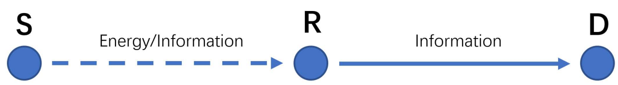

Fig. 1. An example of the considered network.

We denote by φ_1 and φ_2 the path-losses of the S-R link and the R-D link. In addition, denote by P_s the transmission power at the source while the noise power of the two links are denoted by σ_i, $i = 1, 2$. Channels of the two links are assumed to be independent and experience the

Nakagami-m quasi-static block-fading, i.e. the states of channels are constant during one block, and vary independently to the next. Denote the gain of channels from S to R and from R to D by z_1 and z_2, respectively. Then, the probability density function (PDF) of $z_i, i = 1, 2$, is given by $f_z(z_i|m) = \frac{m^m}{\Gamma(m)} z_i^{m-1} e^{-mz_i}$, where m is the shape factor.

B. Finite blocklength performance model of a single-hop transmission

For AWGN channels, [15] derives a tight bound for the coding rate of a single-hop transmission system. With blocklength n, block error probability ε and SNR γ, the coding rate (in bits per channel use) is given by: $r = \frac{1}{2}\log_2(1+\gamma) - \sqrt{(1-\frac{1}{(1+\gamma)/2n^2})}Q^{-1}(\varepsilon)\log_2 e + \frac{O(\log_2 n)}{n}$, where $Q^{-1}(\cdot)$ is the inverse of the Q-function given by $Q(w) = \int_w^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$. In [18], the above result has been extended to a complex channel model with received SNR γ, where the coding rate (in bits per channel use) is

$$r = \mathcal{R}(\gamma, \varepsilon, n) \approx \mathcal{C}(\gamma) - \sqrt{\frac{V}{n}} \log_2 e \cdot Q^{-1}(\varepsilon), \tag{1}$$

where $\mathcal{C}(\gamma)$ is the Shannon capacity. For a known SNR of the channel, it is given by $\mathcal{C}(\gamma) = \log_2(1+\gamma)$. Moreover, V is the channel dispersion [9, Def.1]. Under a complex AWGN channel, $V = 1 - \frac{1}{(1+\gamma)^2}$. Hence, for a single hop transmission with blocklength n and coding rate r, the decoding (block) error probability at the receiver is given by

$$\varepsilon = \mathcal{P}(\gamma, r, n) = Q\left(\sqrt{\frac{n}{V}}\,(\mathcal{C}(\gamma) - r)\log_e 2\right). \tag{2}$$

So far, we have introduced the system model and the performance model of a single-hop transmission with FBL codes. In the following, we further study the FBL performance of SWIPT-enabled two-hop relaying networks.

III. FBL PERFORMANCE OF SWIPT PROTOCOLS IN RELAYING NETWORKS

In this section, we study the reliability performance of an energy efficient, low-latency (FBL) SWIPT-enabled relaying network. We are in particular interested in scenarios supporting reliable transmissions, where the error probability of each link is required to be lower than a threshold $\varepsilon_{\mathrm{th}}$, i.e., $\varepsilon_{\mathrm{th}} \leq 10^{-1}$. On the other hand, as channel gains are randomly distributed, it is possible that channels are extremely poor, which results in that received SNRs at the relay and the destination based on the channels will be extremely low, i.e., being not able to satisfy the reliability requirement (as the packet size is fixed). Note that the energy efficiency needs also to be considered in the future network design, we therefore assume that the source does not transmit signal to the relay when channels are extremely bad i.e., $\gamma_i, i = 1, 2$, are lower than a threshold γ_{th}, where γ_{th} is corresponding to $\varepsilon_{\mathrm{th}}$ and it holds for $\gamma_{\mathrm{th}} \gg 0$ dB. In the following, we study and optimize the

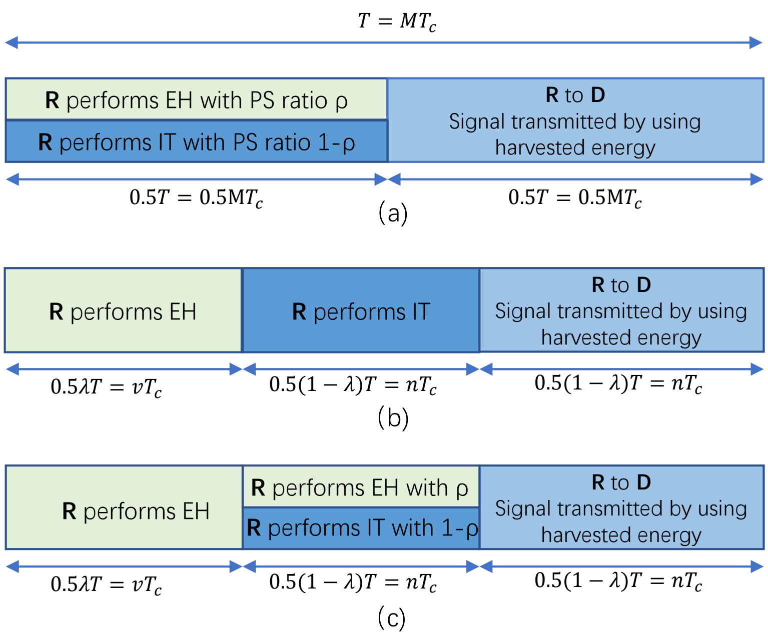

Fig. 2. Frameworks for information/data transmitting (IT) and EH under SWIPT-enabled transmission protocols: (a) PS protocol, (b) TS protocol, (c) The proposed PT-S protocol.

reliability of the network. We first study the PS protocol in the FBL regime. Subsequently, a joint PS-TS protocol is proposed and investigated[1].

A. PS protocol

As shown in Fig. 2-a, under the PS protocol a frame contents two phases while each phase has a time duration of $T_c M/2$. The received signal at the relay in the first phase is split such that an $\rho \in [0, 1]$ portion of the signal power is used for data packet decoding and the remaining $1 - \rho$ portion of the power for EH.

Therefore, the signal split for decoding packet at the relay is with SNR:

$$\gamma_1 = \beta_1(1 - \rho), \tag{3}$$

where $\beta_1 = \frac{P_s z_1}{\varphi_1 \sigma_1^2}$. In addition, the harvested energy is

$$E_{\mathrm{H}} = nT_c \frac{\eta \rho P_s z_1}{\varphi_1}, \tag{4}$$

where η is the energy conversion efficiency, i.e., $0 < \eta < 1$. Based on the harvested energy, the relay forwards the packet to the destination. The SNR of the signal received at the destination in

[1]Please refer to [23] for the general FBL performance of a SWIPT-enabled relaying network with the TS protocol.

the second phase from the relay is given by

$$\gamma_2 = \frac{E_{\mathrm{H}} z_2}{n T_c \varphi_2 \sigma_2^2} = \beta_2 \rho, \tag{5}$$

where $\beta_2 = \frac{\eta P_s z_1 z_2}{\varphi_1 \varphi_2 \sigma_2^2}$.

Recall that the packet size is fixed as k while the blocklength of each phase/hop is M/2. Hence, the coding rate (bit/symbol) of the two phase are $r_1 = r_2 = 2k/M$. According to (2), the error probabilities of the first and second hop are $\varepsilon_1 = \mathcal{P}(\gamma, 2k/M, M/2)$ and $\varepsilon_2 = \mathcal{P}(\gamma, 2k/M, M/2)$. As a result, the overall error probability of the two-hop transmission under the PS protocol is given by

$$\varepsilon_{\mathrm{O}} = \varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2. \tag{6}$$

Our aim is to minimize this overall probability. Hence, the optimization problem under the PS protocol is stated by

$$\begin{aligned} &\min_{\rho} \ \varepsilon_{\mathrm{O}}(\rho) \\ &s.t.: \gamma_i \geq 1, i = 1, 2; \\ &\qquad \rho \in (0, 1). \end{aligned} \tag{7}$$

We then have a lemma for solving the problem.

Lemma 1. *With the constraint $\gamma_i \geq 1, i = 1, 2$, the overall error probability ϵ_{O} under the PS protocol is convex in ρ.*

Proof: See Appendix.A. ■

According to the above Lemma, problem (7) can be efficiently solved by applying optimization techniques.

B. A joint PT-S protocol

In the FBL regime, the key differences between PS and TS protocols are as follows: A PS protocol splits the received power at the relay and therefore has a lower SNR for decoding data packet. In comparison to the PS protocol, a TS protocol provides a relatively higher SNR but shorter blocklengths for the data transmission, as shown in Fig. 2-b. According to (2), the error probability is influenced by both the blocklength and the SNR. This motivate us to propose a joint PS-TS protocol, namely PT-S, to improve the reliability performance by introducing a trade-off between reducing the SNR and shortening the blocklength.

In the proposed PT-S protocol, the whole transmission frame with length M is divided into three phases, with lengths v, n and n, as shown in Fig. 2-c. In the first phase, the relay performs a pure EH receiver and harvests energy from the source. In second phase, the relay works under a PS protocol with splitting ratio ρ. In the last phase, the relay forwards the packet to the destination based on the energy harvested in the previous two phases.

Under the proposed PT-S protocol, the energy harvested by the relay in the first phase is given by $E_{\mathrm{TS}} = (M-2n)T_c\frac{\eta P_s z_1}{\varphi_1}$. In addition, the energy harvested in the second phase by the relay with splitting ratio ρ is given by $E_{\mathrm{PS}} = nT_c\frac{\eta\rho P_s z_1}{\varphi_1}$. At the same time, the SNR for decoding packet at the relay actually share the same expression as γ_1 under the PS protocol providing in (3), given by $\gamma_1' = \gamma_1 = \beta_1(1-\rho)$, Note that under the PT-S protocol, each hop of data transmission needs to send k bits via a block of length n. Hence, the coding rate is given by $r_1' = r_2' = k/n$. Then, we have the decoding error probability at the relay $\varepsilon_1' = \mathcal{P}(\gamma_1', k/n, n)$.

Note that the energy in total collected by the relay under the PT-S protocol is $E_{\mathrm{H}}' = E_{\mathrm{TS}} + E_{\mathrm{PS}}$, which is used at the relay to forward the data packet (with size k) to the destination via a block of length n. Hence, the SNR of the received signal at the destination is given by

$$\begin{aligned}\gamma_2' = \frac{E_{\mathrm{H}}' z_2}{nT_c\varphi_2\sigma_2^2} &= \frac{\eta P_s z_1 z_2}{\varphi_1\varphi_2\sigma_2^2}\frac{M-2n}{n} + \frac{\rho\eta P_s z_1 z_2}{\varphi_1\varphi_2\sigma_2^2}\\ &= \beta_2\rho + \theta,\end{aligned} \tag{8}$$

where θ is given by

$$\theta = \beta_2\frac{M-2n}{n}. \tag{9}$$

Based on γ_2', the decoding error probability at the destination is given by $\varepsilon_2' = \mathcal{P}(\gamma_2', k/n, n)$. Hence, the overall error probability $\varepsilon_{\mathrm{O}}'$ under the PT-S protocol can be determined by inserting ε_1' and ε_2' into (6). Note that n is the blocklength which should be a positive integer, i.e., $n \in \mathbb{Z}^+$. The optimization problem under the PT-S protocol is

$$\begin{aligned}\min_{\rho,n}\ & \varepsilon_{\mathrm{O}}'(\rho, n)\\ s.t. :\ & \gamma_i' \geq 1, i = 1,2;\\ & n \in \mathbb{Z}^+; n \leq M/2;\\ & \rho \in (0,1).\end{aligned} \tag{10}$$

We first consider the relaxation (on n) of the above problem, which is given by

$$\begin{aligned}\min_{\rho,n}\ & \varepsilon_{\mathrm{O}}'(\rho, n)\\ s.t. :\ & \gamma_i' \geq 1, i = 1,2;\\ & n \in (0, M/2);\\ & \rho \in (0,1).\end{aligned} \tag{11}$$

It is clear that the overall error probability $\varepsilon'_{\mathrm{O}}$ under the PT-S protocol is subject to decisions of both ρ and n. In comparison to γ_2 provided in (5), γ'_2 in (8) has an addition term θ, which makes γ'_2 be a function of n. In particular, considering the function $\varepsilon'_2 = \mathcal{P}(\gamma'_2, k/n, n) = Q\left(\sqrt{\frac{n}{V(\gamma'_2)}}\left(\mathcal{C}(\gamma'_2) - k/n\right)\log_e 2\right)$, all inputs γ'_2, k/n and n are either n or subject to n. As a result, it is unlikely to show the convexity of Problem (11).

However, if we let θ be a constant with respect to n, γ'_2 becomes a constant in n. Then, we have a convexity characteristic provided in the following lemma

Lemma 2. *If $\gamma_i \geq 1, i = 1, 2$, and θ is fixed, the overall error probability ϵ'_{O} under the proposed PT-S protocol is convex in (ρ, n).*

Proof: See Appendix.B. ■

According to Lemme 2, we propose the following algorithm to solve the optimization problem. We first interactively solve Problem (11), while each local problem is an optimization problem guaranteed by Lemma 2. Then, the solution of Problem (10) is determined subsequently.

Algorithm 1 : Optimal Scheduling Algorithm under the PT-S protocol.

Step 1: Solving the relaxed problem

a) Initialize $n = M/2$.
b) Determine θ according to (9).
c) Let θ be fixed. Hence, γ'_2 is constant in n. Then, determine the optimal ρ° and n°_{rlx} minimizing ϵ'_{O} according to Lemma 2.
d) According to (9), update θ based on the updated n°_{rlx} in Step c.
e) Check if θ converges to a constant:
f) **if** the gap between the updated θ and the previous one become relatively constant and small enough,
g) **then** θ converges. The optimal solution of the relaxed Problem (11) is determined: $(\rho^*, n^*_{\mathrm{rlx}}) = (\rho^\circ, n^\circ)$.
h) **else** return to Step c.

Step 2: Solving the original problem

i) **if** n^*_{rlx} is an integer,
j) **then** the optimal solution of the original Problem (10) is the same as the relaxed one, i.e., $(\rho^*, n^*) = (\rho^*, n^*_{\mathrm{rlx}})$. Jump to the end.
k) **else** Let $n_{\mathrm{ceil}} = \lceil n^*_{\mathrm{rlx}} \rceil$ and $n_{\mathrm{floor}} = \lfloor n^*_{\mathrm{rlx}} \rfloor$, where $\lceil\cdot\rceil$ and $\lfloor\cdot\rfloor$ are ceil and floor functions, respectively. For each of the two possible choices of n, $n \in \{n_{\mathrm{ceil}}, n_{\mathrm{floor}}\}$, update θ accordingly and determine the optimal ϵ'_{O} as well as the corresponding optimal solution of ρ according to Lemma 2. Then, the optimal value of Problem (10) is $\min\limits_{n \in \{n_{\mathrm{ceil}}, n_{\mathrm{floor}}\}} \epsilon'_{\mathrm{O}}$.

end

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, we provide our numerical results to show the appropriateness of our analytical model and evaluate the system performance. The results of the proposed PT-S protocol are com-

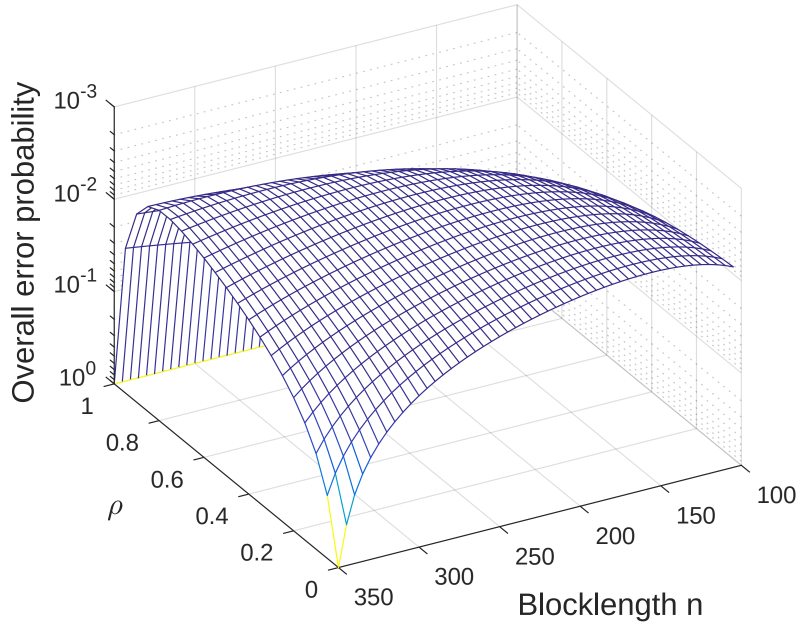

Fig. 3. Error probability ε against n and ρ, while packet size is set to $k = 250$ bits.

pared with the PS protocol and the TS protocol (following the model provided in [23]). In the numerical analysis, we consider the following parameterization adopted from [23], [24]: Firstly, the transmission power at the source is set to $P_s = 1$ Joule/sec. Secondly, noise powers of the two links are set to $\sigma_1^2 = \sigma_2^2 = 0.01$. In addition, the energy conversion efficiency is set to $\eta = 0.5$. Finally, the path-losses are obtained based on the $\varphi_i = d_i^{-2.7}$, $i = 1, 2$, where d_1 and d_2 are the distance of the two links and the distances are normalized to unit value $d_1 = d_2 = 1$ m.

We start with the investigation on the impact of ρ and n on the overall error probability under the proposed PT-S protocol. The results are provided in Fig. 3. First, the results match well with Lemma 2 that the overall error probability in convex in (ρ, n). Secondly, in the figure the frame length is set to $M = 700$ (symbols). Hence, the performance of the PS protocol can also be observed from the figure on the curve which varys ρ at point $n = M/2 = 350$, i.e., the left-side edge of the surface. It can seen that when $n = 350$, the overall error probability is convex in ρ, which matches with our Lemma 1.

Next, we study the impact of blocklength on the reliability. We evaluate the overall error probability under the PS protocol by applying optimization techniques (backtracking line search) in comparison to the exhaustive search. In addition, the performance of the proposed Algorithm 1 for the proposed PT-S protocol is also compared with the results of exhaustive search. The comparison results are shown in Fig. 4, where the results of the TS protocol is also provided as a contrast. We observe that the proposed algorithm achieves the same performance of the exhaustive search under

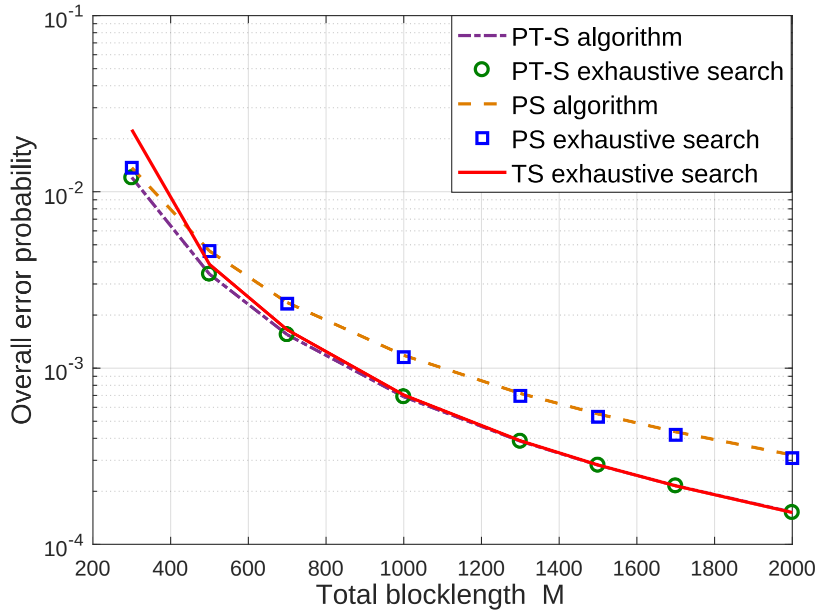

Fig. 4. Performance comparison.

the PT-S protocols. This is also true for the PS protocol, under which the optimization algorithm (backtracking line search) performs the same as the exhaustive search. These results show again the appropriateness of our analytical mode. More importantly, it can be found that the proposed PT-S protocol outperforms both the PS and the TS protocols, which confirms that the PT-S protocol achieves a better trade-off the between the SNR and the blocklength in minimizing the overall error probability. Moreover, in comparison to the PS protocol, the TS protocol is more reliable for long blocklength scenarios, while the PS protocol is preferred than the TS one when the blocklength is quite short.

Then, we move on to investigate the impact of transmit power at the source on the overall error probability. The results are provided in Fig. 5. It is shown that all the PS, TS and PT-S protocols are more reliable when the source power increases, while the PT-S protocol provides the best reliability performance. Moreover, the relationship (performance order) between these three protocols is not changed by increasing the transmit power.

Finally, we vary the packet size k and show the results in Fig. 6. As expected, a bigger packet size results in a higher error probability for all protocols. In addition, the PT-S protocol shows the reliability advantage in compassion to the TS protocol and the PS protocol. Moreover, when comparing the PS protocol with the TS one, we find that the TS protocol is preferred when the packet is small, which matches the results in Fig. 5. On the other hand, the PS protocol has a better reliability performance than the TS one in the relative big packet size region.

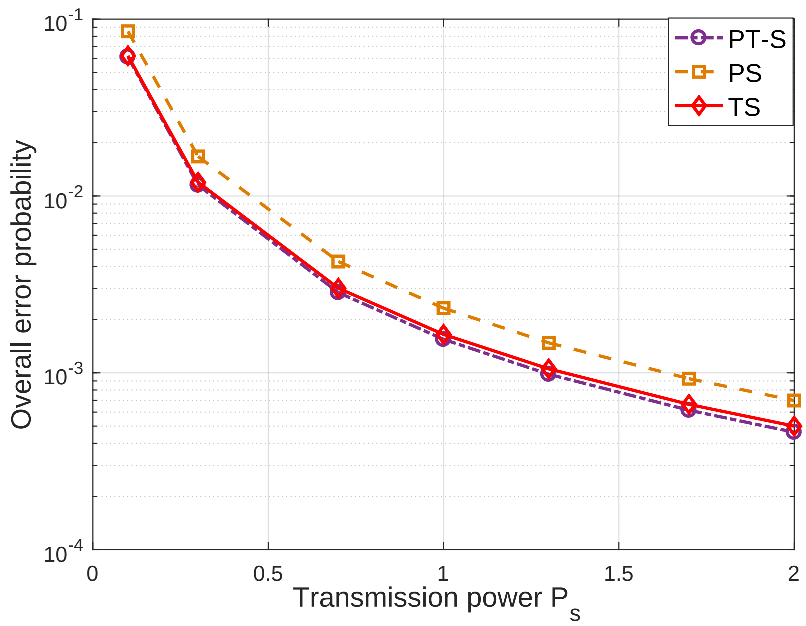

Fig. 5. The impact of the source transmit power P_s (in Joule/sec) on the overall error probability. In the figure, we set $k = 150$ bits.

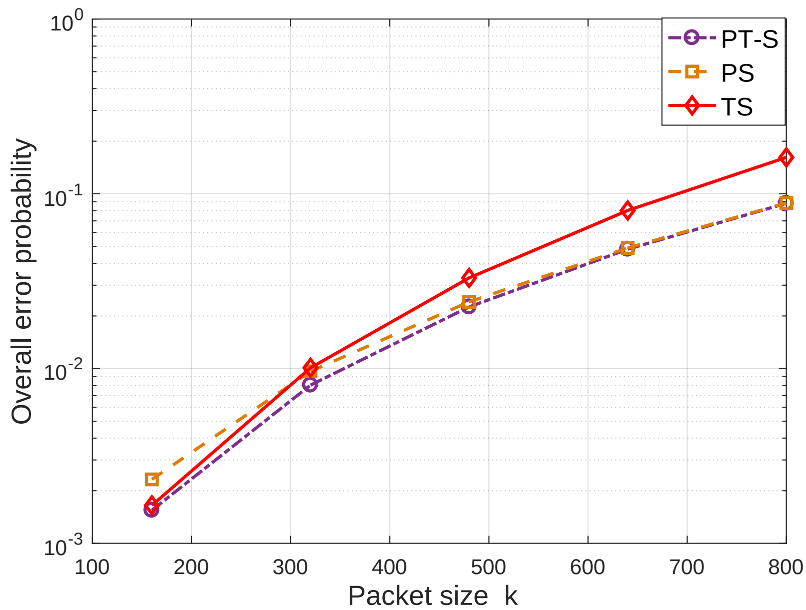

Fig. 6. The impact of packet size k on the overall error probability.

V. CONCLUSION

In this paper, we studied the reliability performance of a SWIPT-enabled relaying network in the FBL regime. We first considered the PS protocol and determined the overall error probability. We proved that the overall error probability under the PS protocol is convex in the PS ratio, i.e., the optimization problem minimizing the error probability can be solved efficiently by applying optimization techniques. In addition, we introduced a PT-S protocol aim at a better tradeoff between the SNR and the blocklength than both the TS and the PS protocols. We determined the FBL performance of the PT-S protocol and proposed an algorithm to minimize the overall error probability under the protocol. Via numerical analysis, we showed the appropriateness of our analytical model. In addition, it was observed that the performance of the proposed algorithm achieves the same performance as the executive search. More importantly, we demonstrated the performance advantage of the proposed PT-S protocol in comparison to the TS and the PS protocols. Moreover, we found that in comparison to the PS protocol, the TS protocol is more reliable only under long blocklength or/and small packet size scenarios. To extend our findings, future work will focus on the theoretical underpinnings of the performance comparisons between TS, PS and PT-S protocols for FBL scenarios presented here, as well as on optimization problems maximizing the throughput under given target error probability.

APPENDIX A
PROOF OF LEMMA 1

Note that the source transmits signals only when the channels are not extremely poor, which could support a relatively reliable transmission, i.e., $\varepsilon_i \leq 10^{-1}$, $i=1,2$. Hence, $\varepsilon_1+\varepsilon_2 \gg \varepsilon_1 * \varepsilon_2$. Then, we have $\varepsilon_{\mathrm{O}} \approx \varepsilon_1+\varepsilon_2$. In the following, we show the appropriateness of Lemma 1 based on this approximation. Note that two channels are independent to each other, the second derivative of ε_{O} with respect to ρ is

$$\frac{\partial^2 \varepsilon_{\mathrm{O}}}{\partial \rho^2} \approx \frac{\partial^2 \varepsilon_1}{\partial \rho^2} + \frac{\partial^2 \varepsilon_2}{\partial \rho^2}. \tag{12}$$

According to (2), for link i, $i=1,2$, we have

$$\frac{\partial^2 \varepsilon_i}{\partial \rho^2} = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{Z^2(\gamma_i)}{2}\right)\left(Z(\gamma_i)\left(\frac{\partial Z(\gamma_i)}{\partial \rho}\right)^2 - \frac{\partial^2 Z(\gamma_i)}{\partial \rho^2}\right), \tag{13}$$

where $Z(\gamma_i)=\frac{\mathcal{C}(\gamma_i)-r}{\sqrt{V(\gamma_i)/n}}$. Hence $\frac{\partial^2 \varepsilon_i}{\partial \rho^2} \geq 0$ if $\frac{\partial^2 Z(\gamma_i)}{\partial \rho^2} \leq 0$. Then, we have

$$\begin{aligned}\frac{\partial^2 Z(\gamma_i)}{\partial \rho^2} = \frac{\beta_i^2}{\sqrt{\frac{V(\gamma_i)}{n}}(\gamma_i^2-r+1)^2}\Bigg(\Bigg(&\frac{3\log_e 2}{((\gamma_i+1)^2-1)^2} \\ &+\frac{3\log_e 2}{((\gamma_i+1)^2-1)}\Bigg)(\mathcal{C}(\gamma_i)-r)\ -1-\frac{2}{(\gamma_i+1)^2-1}\Bigg).\end{aligned} \tag{14}$$

Hence, $\frac{\partial^2 Z(\gamma_i)}{\partial \rho^2} \leq 0$, if the following condition holds

$$3\log_e 2(\mathcal{C}(\gamma_i)-r) \geq (\gamma_i+1)^2-1. \tag{15}$$

It is easy to show $(\gamma_i+1)^2-1 \geq 3\log_e 2\mathcal{C}(\gamma_i) \geq 3\log_e 2(\mathcal{C}(\gamma_i)-r)$, $\forall\ \gamma_i \geq 1$. Hence, $\frac{\partial^2 \varepsilon_{\mathrm{O}}}{\partial \rho^2} \geq 0$

holds under the same condition.

Appendix B
Proof of Lemma 2

Similar as the proof of Lemma 1, here we prove Lemma 2 by showing the convexity of ε_i' in (ρ, n). The Hessian matrix of ε_i' with respect to (ρ, n) is

$$H_i = \begin{bmatrix} \frac{\partial^2 \varepsilon_i'}{\partial \rho^2} & \frac{\partial \varepsilon_i'}{\partial \rho \partial n} \\ \frac{\partial \varepsilon_i'}{\partial n \partial \rho} & \frac{\partial^2 \varepsilon_i'}{\partial n^2} \end{bmatrix}. \tag{16}$$

According to Lemma 1, $\frac{\partial^2 \varepsilon_i'}{\varepsilon' \rho^2} > 0$. Hence, H_i is positive semi-definite if $\det H_i \geq 0$. In fact, $\det H_i$ is given by:

$$\det H_i = \frac{\partial^2 \varepsilon_i'}{\partial n^2} \frac{\partial^2 \varepsilon_i'}{\partial \rho^2} - \left(\frac{\partial \varepsilon_i'}{\partial \rho \partial n} \right)^2 = \frac{1}{\sqrt{2\pi}} e^{-Z(\gamma_i')} \left(A + B \right), \tag{17}$$

where

$A = -Z(\gamma_i') \left(\frac{\partial Z}{\partial n}\right)^2 \frac{\partial^2 Z(\gamma_i')}{\partial \rho^2} + 2Z(\gamma_i') \frac{\partial Z(\gamma_i')}{\partial n} \frac{\partial Z(\gamma_i')}{\partial \rho} \frac{\partial^2 Z(\gamma_i')}{\partial n \partial \rho}$ and $B = -Z(\gamma_i') \left(\frac{\partial Z(\gamma_i')}{\partial \rho} \right)^2 \frac{\partial^2 Z(\gamma_i')}{\partial \rho^2} + \frac{\partial^2 Z(\gamma_i')}{\partial n^2} \frac{\partial^2 Z(\gamma_i')}{\partial \rho^2} - \left(\frac{\partial^2 Z(\gamma_i')}{\partial n \partial \rho} \right)^2$.

Actually, it holds that

$$\begin{aligned} A > {} & K_1[(\gamma_i' + 1)^2 - 2(\gamma_i' + 1)^2 - 3\log_e 2((\gamma_i' + 1)^2 - 1)(\mathcal{C}(\gamma_i') - r)] \\ & + K_2[(kn - 2k\log_e 2) + n(\gamma_i' + 1)^2 - 1) - n\log_e 2(\mathcal{C}(\gamma_i') - r] \end{aligned} \tag{18}$$

and

$$B > K_3 \left(\frac{2\log_e^3(2)}{9} k(\mathcal{C}(\gamma_i') - r)(\gamma_i' + 1)^2 - (\gamma_i' + 1)^2 + 1 \right), \tag{19}$$

where K_1, K_2 and K_3 are positive terms: $K_1 = \frac{k^2 \log_e 2\beta_i^2}{n((\gamma_i'+1)^2-1)^3\sqrt{V(\gamma_i')n}}$, $K_2 = \frac{k\log_e 2\beta_i^2}{n^2((\gamma_i'+1)^2-1)^3} \left(1 + \frac{n(\mathcal{C}(\gamma_i')-r)}{2}\right)$ and $K_3 = \frac{\beta_i^2}{n^2 \frac{V(\gamma_i')}{n}((\gamma_i'+1)^2-1)(\gamma_i'+1)^2}$.

Note that the packet size k is positive. For a blocklength with a pratical interest, i.e., $n \gg 1$, it holds that

$$kn - 2k\log_e 2 > 0. \tag{20}$$

On the other hand, recall that $\gamma_i' \geq 1$, the following inequalities therefore hold

$$n((\gamma_i' + 1)^2 - 1) - n\log_e 2(\mathcal{C}(\gamma_i') - r) > 0, \tag{21}$$

$$(\gamma_i' + 1)^4 - 2(\gamma_i' + 1)^2 - 3\log_e 2((\gamma_i' + 1)^2 - 1)(\mathcal{C}(\gamma_i') - r) > 0, \tag{22}$$

$$\frac{2\log_e^3(2)}{9}k(\mathcal{C}(\gamma_i') - r)(\gamma_i'+1)^2 - (\gamma_i'+1)^2 + 1 > 0. \tag{23}$$

Then, we have $A > 0$ based on (18) and (20)-(22) and $B > 0$ according to (19) and (23), respectively. As a result, $\det H_i > 0$ holds and H_i is positive semi-definite.

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