Power Allocation for Orthogonally-Observed Multi-Target Sensor Networks

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***Abstract*—In this paper we study the performance bound of a wireless sensor network which is capable of estimating the true values of several active targets. We assume that sensors can observe each target separately, assuming either targets are orthogonal in frequency or physically distinguishable. The aforementioned assumption is of practical relevance for targets with different physical nature, e.g., heat, humidity, pressure etc. Another interesting example of separable observation is power grids where the voltage of each node can be exclusively estimated at any of its neighboring nodes by measuring the flowing current and applying the Ohm's law. Even though such a sensor network is not wireless, the current work provides a suitable framework for state estimation and planning of the smart meters in smart grids. We then propose a novel unbiased estimator. Moreover, as an additional design variable, we perform power allocation and optimal fusion of data to further improve the performance of the proposed estimator. The optimal fusion rules are provided in closed-form, while power allocation is optimally done by means of convex optimization.**

***Index Terms*— optimal fusion, target detection, classification, state estimation, separable targets**

I. Introduction

The rise of certain applications in the 5[th] generation wireless systems (5G) drastically increases the importance of sensor networks for sensing and monitoring purposes. However, an optimal resource utilization is necessary for an accurately performed sensing and monitoring task, since the estimation performance increases with the energy and power consumption of the network. Hence, the optimization of power and energy resources for a required performance is of high interest and studied in many publications, especially for scenarios with a single target. In [1] we have solved the power allocation problem in closed-form s.t. individual power limitations of the sensors as well as a given sum-power constraint for a single target network. In multi-target scenarios the main topics for investigations address the tracking and coverage problem. For example, the focus in [2] is to maximize the lifetime s.t. power constraints and coverage regions. In the present work we minimize the estimation error instead of maximizing the lifetime. The authors in [3] use the GaussMarkov mobility model to formulate the tracking problem as a hierarchical Markov decision process and solve it with the aid of neurodynamic programming. Due to difficulties of a centralized processing to handle multi-target problems, the authors in [4] have studied the tracking problem by a distributed data processing approach. In [5] a special scenario is considered in which sensor nodes can be put into a sleep mode with a timer, that determines the sleep duration. By optimizing the sleep duration they show an improvement of the tracking performance in sensor networks. In contrast to [4] and [5], we investigate the centralized scenario and determine the least reliable sensor nodes to keep them asleep for a uniform time duration, respectively. It is to mention, that our approach is more general and it can be used not only for tracking but also for detection and classification of targets, cf. [6]. In this paper we extend our previously published work [7], where we consider a wireless sensor network for observing L active targets by deploying K wireless sensors. The current work expands that setup by assuming that sensing channels are orthogonal which means each sensor can observe each target, separately. For instance, targets are either orthogonal in frequency or of different physical nature such that they can be observed by sensors without inter-target interference. This demands in turn, sophisticated sensor nodes which are capable of observing targets over different frequency bands or targets with different physical nature such as humidity, temperature, etc. In such a condition, sensors can relay their observations over orthogonal communication resources using standard techniques of division multiple access such that there is no inter-sensor and intra-sensor interference.

An interesting application for such a network is voltage estimation in power grids, as the sensing channels are cables and, thus, orthogonal. Even though the cable sensing channels are not wireless, this work enables to provide a suitable framework for power grid state estimation.

The sensors in our sensor network consume no power for observation, but only for relaying their observation to the fusion center. The targets, on the other hand, actively transmit signals in their environment. The relaying communication from sensor nodes towards the fusion center is subject to both per-sensor node output power constraints and a sum-power constraint on all sensors. Then, at the fusion center a linear fusion rule, i.e., a matrix multiplication, is applied to estimate the true values of the target signals from its received observations. We propose an unbiased estimator for our sensor network. The total estimation error of the estimator is further minimized by optimal linear fusion of observations and also power allocation. For the fusion coefficients, the optimal solution is provided in closed-form, while the global optimum of the power allocation is achieved by means of convex optimization.

The organization of this paper is as follows: the system model is described in Section II. We propose an unbiased estimator in Section III whose variance of error can be further minimized by optimizing the power allocation among the sensor nodes as well as optimizing a set of fusion rules (per sensor node) at the fusion center. The resulting optimization problems are solved in Section IV and Section V. While the simulation results are presented in Section VI, Section VII concludes this paper.

Notations: The notation used throughout the paper is as follows: x denotes a scalar x while $\mathbf{x}$ is a vector $\mathbf{x}$ with entries x_i. $\mathbf{X}$ represents a matrix $\mathbf{X}$ with entries x_{ij}. x^*, $\mathbf{x}^*$ and $\mathbf{X}^*$ stand for the complex conjugate of scalar x and complex conjugate transpose of vector $\mathbf{x}$ and matrix $\mathbf{X}$, respectively. Also, $\mathbf{x}'$ and $\mathbf{X}'$ are the transpose of vector $\mathbf{x}$ and matrix $\mathbf{X}$, respectively. A diagonal matrix with diagonal entries $\mathbf{x}$ is written as $\mathbf{\Lambda}_{\mathbf{x}}$. $\mathcal{E}(\cdot)$ refers to the statistical expectation. The sets $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{C}$ denote the set of all integer positive and non-zero numbers, the set of real numbers and the set of all complex numbers, respectively, while $\mathbb{C}^{m\times n}$ the set of all complex matrices of the size of $m\times n$. Finally, the Kronecker delta function is denoted as δ_{lm}.

II. System Model

The wireless sensor network of interest in the current work is consisted of $K\in\mathbb{N}$ passive sensor nodes. Such a sensor network is illustrated in Fig. 1 and can estimate the true values of $L\in\mathbb{N}$ unknown, complex-valued and active targets, i.e., $r_1,\ldots,r_L$. The index sets $\mathbb{F}_K=\{1,\ldots,K\}$ and $\mathbb{F}_L=\{1,\ldots,L\}$ correspond to the set of all sensors and targets, respectively.

We assume that each target has the known power, i.e., $R_l:=\mathcal{E}(|r_l|^2), l\in\mathbb{F}_L$. Nonetheless, their true unknown values change slowly such that they can be assumed constant over each round that we perform estimation.

The target signal r_l propagates towards the sensor node k over the so-called sensing channel $g_{lk}\in\mathbb{C}$ and then is influenced by additive measurement noise $m_{lk}\in\mathbb{C}$. We assume the sensing channel to be nearly constant over each estimation interval. Hence, it can be treated as a time-invariant deterministic channel coefficient. The noise is zero-mean, identically and independently distributed (iid) with variance of M_{lk}, which is also independent from the target signals. So, it is correct to state

$$\mathcal{E}(m_{lk}m^*_{l\prime k\prime})=\delta_{kk\prime}\delta_{ll\prime}M_{lk},\ \forall k,k\prime,l,l\prime, \tag{1a}$$
$$\mathcal{E}(m_{lk}r^*_{l\prime})=\mathcal{E}(m_{lk})\,\mathcal{E}(r^*_{l\prime})=0,\ \forall k,l,l\prime. \tag{1b}$$

Each sensor accordingly amplifies its received signal form target $l\in\mathbb{F}_L$ by the complex-valued coefficient $u_{lk},\ k\in\mathbb{F}_K$ and transmits it towards the fusion center. The output of sensor k corresponding to target l, i.e., x_{lk} is represented by

$$x_{lk}=u_{lk}(m_{lk}+r_l\,g_{lk}),\ k\in\mathbb{F}_K. \tag{2}$$

Thus, the output power of sensor k for target l is derived below

$$X_{lk}:=\mathcal{E}\big(|x_{lk}|^2)\big)=|u_{lk}|^2\big(M_{lk}+R_l|g_{lk}|^2\big). \tag{3}$$

The overall output power X_k of each sensor is limited by the individual power budget for sensor $k\in\mathbb{F}_K$, denoted by P_k, which results in the individual power constraint

$$X_k:=\sum_{l\in\mathbb{F}_L}X_{lk}=\sum_{l\in\mathbb{F}_L}|u_{lk}|^2\big(M_{lk}+R_l|g_{lk}|^2\big)\le P_k\,. \tag{4}$$

The transmitted signal from each sensor propagates through the communication channel and arrives at the fusion center. We denote this signal by y_{lk} which can be derived by:

$$y_{lk}:=n_{lk}+h_{lk}x_{lk}=n_{lk}+h_{lk}u_{lk}(m_{lk}+r_l\,g_{lk}), \tag{5}$$

where h_{lk} is the communication channel coefficient between the sensor node k and the fusion center. Similarly, the communication channel h_{lk} is almost constant during the interval of estimation, and thus deterministic and time-invariant. Also, n_{lk} represents the additive noise at the fusion center antenna, which is assumed to be zero-mean and iid with variance N_{lk}. Therefore, we can write

$$\mathcal{E}(n_{lk}n^*_{l\prime k\prime})=\delta_{kk\prime}\delta_{ll\prime}N_{lk},\ \forall k,k\prime,l,l\prime, \tag{6a}$$
$$\mathcal{E}(m_{lk}n^*_{l\prime k\prime})=\mathcal{E}(m_{lk})\,\mathcal{E}(n^*_{l\prime k\prime})=0,\ \forall k,k\prime,l,l\prime. \tag{6b}$$

For a compact representation of our system, we introduce the following vector notation. The active targets are written as the vector $\mathbf{r}=\big[r_1,\ldots,r_L\big]'$, $\mathbf{r}\in\mathbb{C}^{L\times1}$, while $\mathbf{x}_l=[x_{l1},\ldots,x_{lK}]'$, $\mathbf{x}_l\in\mathbb{C}^{K\times1}$ is the vector containing the outputs of all sensors corresponding to one specific target $l\in\mathbb{F}_L$. The other system variables $\mathbf{y}_l,\mathbf{u}_l,\mathbf{g}_l,\mathbf{h}_l,\mathbf{m}_l$ and $\mathbf{n}_l\in\mathbb{C}^{K\times1}$ are defined, accordingly. Then, (2) can be recast into the vector form

$$\mathbf{x}_l=\mathbf{\Lambda}_{\mathbf{u}_l}(\mathbf{m}_l+r_l\,\mathbf{g}_l), \tag{7}$$

where we define $\mathbf{\Lambda}_{\mathbf{u}_l}:=\mathrm{diag}(\mathbf{u}_l)$. Also, by using (7) and defining $\mathbf{\Lambda}_{\mathbf{h}_l}=\mathrm{diag}(\mathbf{h}_l)$, we rewrite (5) into the vector form

$$\mathbf{y}_l=\mathbf{n}_l+\mathbf{\Lambda}_{\mathbf{h}_l}\mathbf{x}_l=\mathbf{n}_l+\mathbf{\Lambda}_{\mathbf{h}_l}\mathbf{\Lambda}_{\mathbf{u}_l}(\mathbf{m}_l+r_l\,\mathbf{g}_l). \tag{8}$$

Since the system can make observations of each target separately, the fusion center fuses them separately by multiplying its input corresponding to target l with the so-called fusion vector $\mathbf{v}_l\in\mathbb{C}^{K\times1}$ which results into the observation values

$$\tilde{r}_l=\mathbf{v}_l'\mathbf{y}_l=h_lr_l+w_l\,, \tag{9}$$

where $h_l:=\mathbf{v}_l'\mathbf{c}_l$ is the effective observation channel and

$$[\mathbf{c}_l]_k:=h_{lk}u_{lk}g_{lk}\,. \tag{10}$$

Also $w_l:=\mathbf{v}_l'\mathbf{n}_l+\mathbf{v}_l'\mathbf{\Lambda}_{\mathbf{h}_l}\mathbf{\Lambda}_{\mathbf{u}_l}\mathbf{m}_l$ is the effective noise.

To examine the performance bound of such a sensor network, we assume that the channel coefficients are perfectly estimated and known at the fusion center. Finally, the overall power that is consumed in the network, i.e., the sum of the output powers of all sensors from (4), is limited by the available sum-power P_{tot}, which leads to the sum power constraint

$$\sum_{k\in\mathbb{F}_K}X_k=\sum_{k\in\mathbb{F}_K}\sum_{l\in\mathbb{F}_L}|u_{lk}|^2\big(M_{lk}+R_l|g_{lk}|^2\big)\le P_{\text{tot}}. \tag{11}$$

This allows to increase the life time of our network, since in each round of estimation the power is allocated optimally.

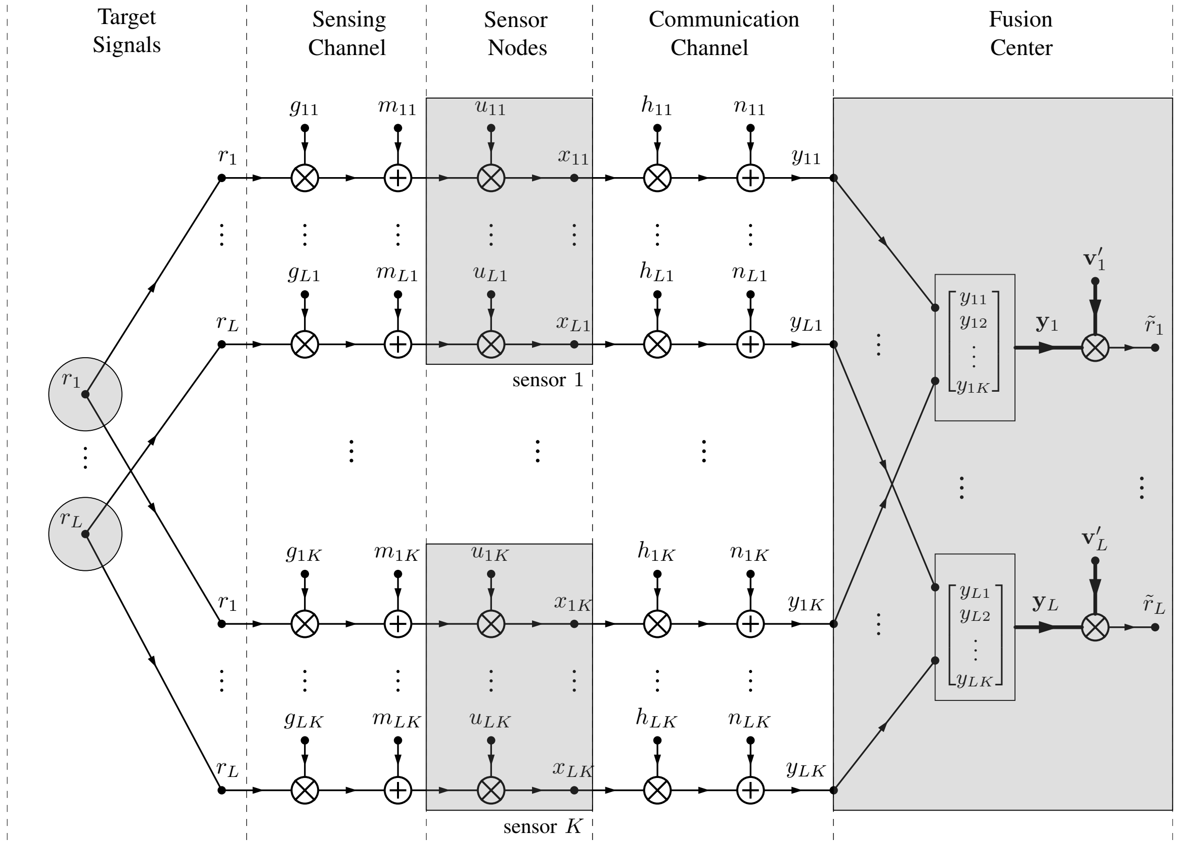


Fig. 1: Block diagram of the multi-target wireless sensor network. Targets are observed separately at each sensor node and fused separately at the fusion center.

III. Proposed Estimator

An efficient estimator, which is unbiased and further attains the Cramer-Rao lower bound, is possible to find only when the probability distribution function of the observation data is known [8]. Unfortunately, in our case this requirement is not met, as we know only mean and variance of the signals. Consequently, it is also not viable to come up with a minimum variance unbiased estimator (MVUE). Given these facts, we are ideally interested in the best linear unbiased estimator (BLUE), $\hat{\mathbf{r}}$, such that $\mathcal{E}(\hat{\mathbf{r}} - \mathbf{r}) = \mathbf{0}$ and its error variance is minimum. Unfortunately, for such an estimator we cannot perform power allocation, since the underlying mathematical problem is intractable, cf. [7].

We, therefore, propose an alternative estimator which is not only unbiased but also its variance of error is further reduced by doing power allocation and optimal fusion. Moreover, the estimator is tractable. If the power allocation and fusion strategy are chosen such that the value h_l in the observation (9) is always one, then the estimator

$$\hat{r}_l = \tilde{r}_l = r_l + w_l \,, \tag{12}$$

is obviously unbiased, since w_l is zero-mean. Such an unbiased estimator delivers the estimation error $\mathcal{E}(|w_l|^2)$ in estimating target r_l. It is easy to prove that $f(\mathbf{u}_l, \mathbf{v}_l) \coloneqq \mathcal{E}(|w_l|^2)$ is

$$f(\mathbf{u}_l, \mathbf{v}_l) = \sum_{k \in \mathbb{F}_K} |v_{lk}|^2 (N_{lk} + |h_{lk}|^2 |u_{lk}|^2 M_{lk}). \tag{13}$$

It is also useful to represent $\mathcal{E}(|w_l|^2)$ in vector form as

$$f(\mathbf{u}_l, \mathbf{v}_l) = \mathbf{v}_l^* \mathbf{\Lambda}_{\mathbf{d}_l} \mathbf{v}_l \,, \tag{14}$$

where

$$\mathbf{\Lambda}_{\mathbf{d}_l} \coloneqq \operatorname{diag}(\mathbf{d}_l) \,, \tag{15a}$$

$$[\mathbf{d}_l]_k \coloneqq N_{lk} + |h_{lk}|^2 |u_{lk}|^2 M_{lk} \,. \tag{15b}$$

At the same time, we can minimize the total estimation error by solving the proposed optimization problem

$$\min_{\substack{\mathbf{u}_l \in \mathbb{C}^{K\times 1} \\ \mathbf{v}_l \in \mathbb{C}^{K\times 1} \\ l \in \mathbb{F}_L}} \sum_{l \in \mathbb{F}_L} \sum_{k \in \mathbb{F}_K} |v_{lk}|^2 (N_{lk} + |h_{lk}|^2 |u_{lk}|^2 M_{lk}) \tag{16a}$$

$$\text{s.t.} \quad \sum_{k \in \mathbb{F}_K} v_{lk} h_{lk} u_{lk} g_{lk} = 1, \forall l \in \mathbb{F}_L, \tag{16b}$$

$$\sum_{l \in \mathbb{F}_L} |u_{lk}|^2 (M_{lk} + R_l |g_{lk}|^2) \le P_k, \; k \in \mathbb{F}_K, \tag{16c}$$

$$\sum_{k \in \mathbb{F}_K} \sum_{l \in \mathbb{F}_L} |u_{lk}|^2 (M_{lk} + R_l |g_{lk}|^2) \le P_{\text{tot}}. \tag{16d}$$

The unbiasedness is provided by the constraint in (16b) which is derived from (9) and (10). Also, individual and sum-power constraints are guaranteed by (16c) and (16d), resulting from equations (4) and (11).

IV. Optimizing Fusion Rule

Note, that the power constraints (16c) and (16d) are independent from fusion rules v_{lk}. Hence, the optimal fusion strategy is achieved by minimizing the objective function (16a) subject to (16b). Interestingly, there is no interdependence between fusion rules for different targets, i.e., $\mathbf{v}_l$. More precisely, the objective function is the summation of different independent terms, i.e., $f(\mathbf{u}_l, \mathbf{v}_l)$ on the one hand and on the other hand the constraint for each target, i.e., $h_l = 1$ depends only $\mathbf{v}_l$. Having given the mentioned property, we can break the original problem down into L independent optimization problem. Therefore, the optimal $\mathbf{v}_l^\star,\ l \in \mathbb{F}_L$ is the solution of

$$f(\mathbf{u}_l, \mathbf{v}_l^\star) = \min_{\mathbf{v}_l \in \mathbb{C}^{K\times 1}} \ \mathbf{v}_l^* \mathbf{\Lambda}_{\mathbf{d}_l} \mathbf{v}_l \tag{17a}$$

$$\text{s.t.} \ \ \mathbf{v}_l' \mathbf{c}_l = 1 \,. \tag{17b}$$

By composing the Lagrange dual function of the problem (17) and using the corresponding KKT conditions, it is straightforward to achieve the optimal fusion strategy $\mathbf{v}_l^\star$ for target $l \in \mathbb{F}_L$. The resulting solution is the global optimum due to the convexity of the problem (17). As an outcome, the duality gap between primal and dual problem is zero [9]. Hence, the solution of the dual problem, i.e.,

$$\mathbf{v}_l^\star = \frac{\mathbf{\Lambda}_{\mathbf{d}_l}^{-1} (\mathbf{c}_l^*)'}{\mathbf{c}_l^* \mathbf{\Lambda}_{\mathbf{d}_l}^{-1} \mathbf{c}_l} \,, \tag{18a}$$

$$f(\mathbf{u}_l, \mathbf{v}_l^\star) = \frac{1}{\mathbf{c}_l^* \mathbf{\Lambda}_{\mathbf{d}_l}^{-1} \mathbf{c}_l} \,, \tag{18b}$$

is identical to the optimum of the primal problem (17). Using the definition of $\mathbf{c}_l$ and $\mathbf{\Lambda}_{\mathbf{d}_l}$ in (10) and (15), we can rewrite $f(\mathbf{u}_l, \mathbf{v}_l^\star)$ as

$$f(\mathbf{u}_l, \mathbf{v}_l^\star) = \frac{1}{\sum\limits_{k\in\mathbb{F}_K} \frac{|h_{lk}|^2 |u_{lk}|^2 |g_{lk}|^2}{N_{lk} + |h_{lk}|^2 |u_{lk}|^2 M_{lk}}} \,. \tag{19}$$

V. Power Allocation

To further minimize the total estimation error, we need to perform optimal power allocation since the error function $f(\mathbf{u}_l, \mathbf{v}_l^\star)$ still depends on power allocation. Let

$$\alpha_{lk} \coloneqq \sqrt{\frac{|g_{lk}|^2}{M_{lk}}} \,, \tag{20a}$$

$$\beta_{lk} \coloneqq \sqrt{\frac{N_{lk}\big(M_{lk} + R_l |g_{lk}|^2\big)}{|h_{lk}|^2 M_{lk}}} \,, \tag{20b}$$

then, by replacing (3) in (19) the estimation error of target l can be stated as a function of X_{lk}:

$$f(\mathbf{u}_l, \mathbf{v}_l^\star) = \frac{1}{\sum\limits_{k\in\mathbb{F}_K} \frac{\alpha_{lk}^2 X_{lk}}{X_{lk} + \beta_{lk}^2}} \,. \tag{21}$$

The total estimation error (16a), thus, reads

$$\sum_{l\in\mathbb{F}_L} f(\mathbf{u}_l, \mathbf{v}_l^\star) = \sum_{l\in\mathbb{F}_L} \frac{1}{\sum\limits_{k\in\mathbb{F}_K} \frac{\alpha_{lk}^2 X_{lk}}{X_{lk} + \beta_{lk}^2}} \,. \tag{22}$$

Therefore, the resulting power allocation problem reads

$$\min_{\substack{X_{lk}\in\mathbb{R} \\ l\in\mathbb{F}_L, k\in\mathbb{F}_K}} \sum_{l\in\mathbb{F}_L} \frac{1}{\sum\limits_{k\in\mathbb{F}_K} \frac{\alpha_{lk}^2 X_{lk}}{X_{lk} + \beta_{lk}^2}} \tag{23a}$$

$$\text{s.t.} \ \ X_{lk} \geq 0 \,,\ l \in \mathbb{F}_L \,,\ k \in \mathbb{F}_K \,, \tag{23b}$$

$$\sum_{l\in\mathbb{F}_L} X_{lk} \leq P_k \,,\ k \in \mathbb{F}_K \,, \tag{23c}$$

$$\sum_{k\in\mathbb{F}_K} \sum_{l\in\mathbb{F}_L} X_{lk} \leq P_{\text{tot}} \,. \tag{23d}$$

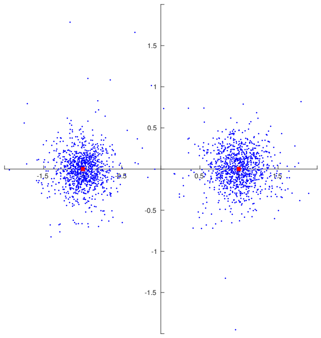
The problem (23) is convex for its objective function and feasible set are convex. Let $g(x) = \frac{ax}{x+b}$, then we know that its second derivative $g''(x) = \frac{-2ab}{(x+b)^3}$. It is easy to see that $g''(x) \leq 0$, if x, a, and b are positive which makes the function $g(x)$ concave. Now, let $x = X_{lk} \geq 0$, $a = \alpha_{lk}^2 \geq 0$ and $b = \beta_{lk}^2 > 0$, then $g(X_{lk})$ is concave and non-negative. Consequently, the denominator of each function $f(\mathbf{u}_l, \mathbf{v}_l^\star)$ in (21) is the sum of non-negative concave functions and thus concave and positive itself. As $f(\mathbf{u}_l, \mathbf{v}_l^\star)$ is the inverse of a positive concave function, it is a convex function. This makes the whole function (23a) convex, since it is the sum of convex functions, cf. [9].

The main challenge in minimizing the total estimation error of the proposed estimator was solving the complex problem in (16). Now, after simplifying the problem into the closed form solution for fusion rules in (18) and ending up in the convex power allocation problem in (23), we can easily utilize any convex solver to have the optimal solution of (16). It is obvious the solution is the global optimum due to convexity of the problem.

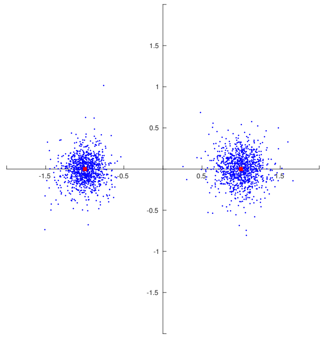
VI. Simulations

Having come up with the solution of the proposed optimization problem in (16), we now provide some numerical results to justify the performance of the sensor network. In the current simulations, the estimation of the targets are performed for several realizations of channel and noise, each of which hands in a different observation of the targets. We shall refer to these realizations as *estimation instances*. The channel coefficients and noise terms are complex-valued, iid with Gaussian distribution with zero mean and of given variances. The variance of sensing and communication channels as well as the power of all targets are set to one. The sensing and communication noise terms are generated with variances of σ_s^2 and σ_c^2, respectively. The definition of signal-to-noise ration (SNR) is a challenging task in order to capture the realistic situation of the network. Nonetheless, for sake of simplicity we define SNR by $-10\log(\sigma_s\sigma_c)$, with $\sigma_s = \sigma_c$.

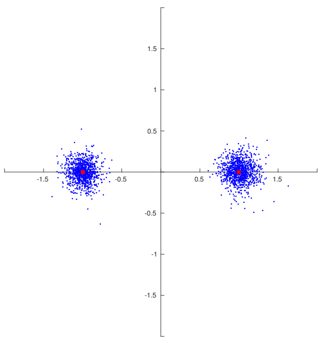
In Fig. 2 and Fig. 3 we have plotted the estimations of constellation points of $L = 2$ and 4 target signals, respectively.



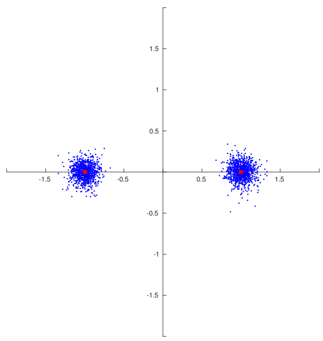
(a) $K = 2$



(b) $K = 3$

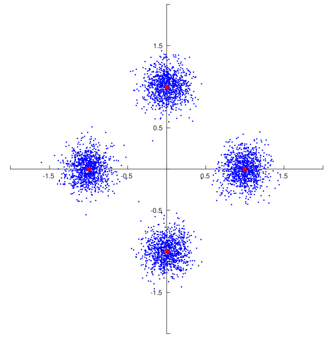


(c) $K = 5$

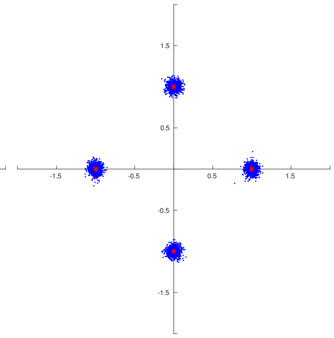


(d) $K = 8$

Fig. 2: Estimation of target signals with proposed estimator ($L = 2$, $P_k = 2$, $P_{\text{tot}} = 10$, $\sigma_s = \sigma_c = 0.2$, SNR ≈ 14 dB). Estimation is done for 1000 estimation instances (different realizations) whose estimation values are shown with blue dots, while the red dots correspond to the constellation points of target signals.



(a) $K = 8$



(b) $K = 16$

Fig. 3: Estimation of target signals with proposed estimator ($L = 4$, $P_k = 2$, $P_{\text{tot}} = 10$, $\sigma_s = \sigma_c = 0.1$, SNR ≈ 20 dB). Estimation is done for 1000 estimation instances (different realizations) whose estimation values are shown with blue dots, while the red dots correspond to the constellation points of target signals.

The figure are generated for estimation instances. It is significant to remark the following properties of these figures. The estimation (blue points) are scattered around red constellation points in a symmetric fashion, since they are located inside circles which are co-centered with constellation points. The centers of these balls, compared to constellation points, are not rotated, not shifted and not scaled, which all together emphasize the fact that the proposed estimator is unbiased and suitable for classification purposes. Obviously, the increase of radii of these balls corresponds to higher noise power , or equivalently, lower SNR in the system.

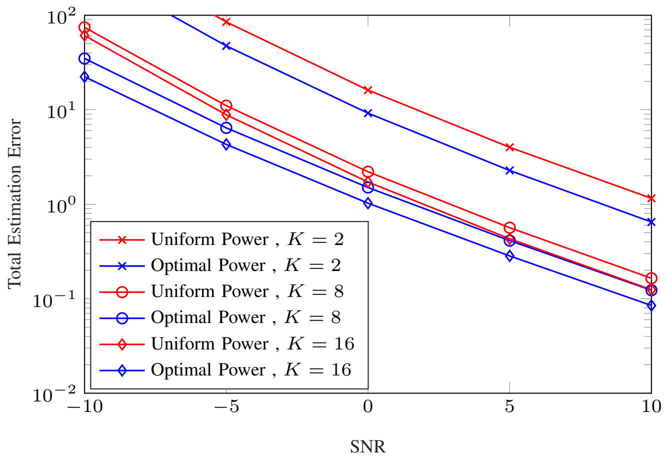
An interesting fact about Fig. 2 and Fig. 3 is that by increase of K, the estimations become more accurate. This is also evidenced in Fig. 4 where the total estimation error (TEE) and symbol error rate (SER) are plotted against SNR for different values of K.

The reason of such improvement in case of $K \leq 5$ (unlike $K > 5$) is that the total available power increases when the number of sensor nodes increases. But, this is not the only reason. The main reason which also applies to the case of $K > 5$ is that, by adding more sensor nodes for a given total power, i.e., $\min(KP_k, P_{\text{tot}})$, the number of sensors nodes with good quality of observation (lower estimation noise) increases and thus the overall quality of estimation enhances. The second reason is the only general cause for networks with $K \geq \frac{P_{\text{tot}}}{P_k}$, i.e., $K \geq 5$ in Fig. 4a, given the same power allocation strategy. More importantly, we can easily see, optimal power allocation increases the estimation quality. For instance, a system with $K = 8$ sensor nodes under optimal power allocation outperforms $K = 16$ sensors with uniform power. Even though the difference between the TEE in both system seems to be negligible, but one can see in Fig. 4b that SER for $K = 16$ at SNR$=10$ dB is reduces by 99.5%, i.e., from 2×10^{-3} to 10^{-5} by optimal power allocation. The underlying reason is that while power allocation gives more power to sensors with higher quality of observation, non-optimal strategies, e.g., uniform power allocation, amplify the effective observation noise, in general. Note that SER is calculated by doing detection of the symbols, since we have assumed, without loss of generality, that targets transmit QPSK symbols. The results are averaged over 10^5 estimation instances. Each symbol is considered as correct, only if both bits are detected error-freely.

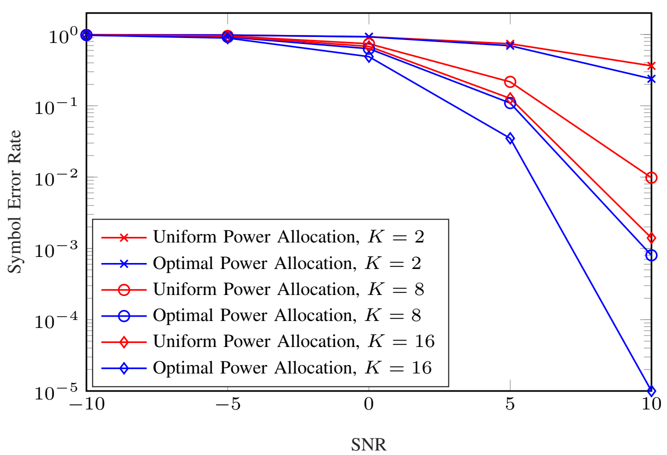
Finally, in order to provide a comparison between the present and the previous work [7], Fig. 5 plots the SER of both systems against SNR for different numbers of sensors. As figure shows the current sensor network, as expected, considerably outperforms the previous system, since the targets are separable at sensor nodes. In addition, the estimation of $L = 4$ targets is possible for low numbers of sensors, i.e., $K = 2$, while the same problem is infeasible in case sensing channels are not orthogonal. In other words, symbol error rate regardless of the value of SNR is 1, as the $K = 2$ sensors do not observe the $L = 4$ targets orthogonally. The reason, cf. [7, Table I], is that the underlying system of equations is overdetermined and thus insolvable for $K < L$.

VII. Conclusion

This paper studies a multi-target wireless sensor network for estimating true values of target signals. We have proposed an unbiased estimator of minimized estimation error. This is done by doing data fusion at the fusion center and power allocation



(a) TEE



(b) SER

Fig. 4: TEE and SER against different values of SNR for several numbers of sensors K. It is assumed that $L = 4$, $P_k = 2$, $P_{\text{tot}} = 10$. The results are averaged over 1000 realization of the channel and noise.

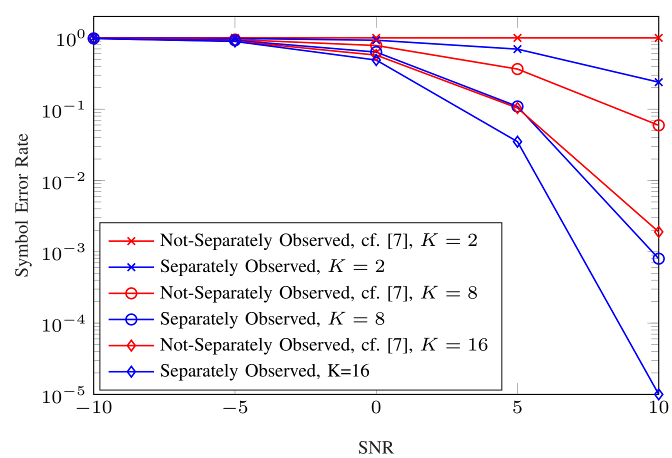


Fig. 5: SER against different values of SNR for several numbers of sensors K. It is assumed that $L = 4$, $P_k = 2$, $P_{\text{tot}} = 10$. The blue lines correspond to the system of the current paper, while red lines represent SER of a system whose sensor nodes cannot separately observe the targets, [7]. Power allocation is performed in both cases.

among sensor nodes. It is assumed that the targets can be separately, i.e., interference-freely, observed due to either orthogonality in frequency or segregation in their physical nature. The results prove that such a system with orthogonal observation, as expected, outperforms networks in which all targets transmit over the same frequency band.

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