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Sparse Framework for Hybrid TDoA/DoA Multiple Emitter Localization

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***Abstract*—In this paper, we consider the problem of localizing multiple non-collaborative transmitters by a network of distributed sensor nodes. The nodes are equipped with versatile sensing capabilities allowing them to estimate the time differences of arrival (TDoAs) and/or the directions of arrival (DoAs) of the incoming waves. We formulate the localization task as a joint block-sparse recovery problem and develop a framework that allows to accommodate different types of measures, such as the beamformer outputs in case of DoAs or cross-correlation functions in case of TDoAs. We then propose a reduced-size location recovery approach in which we perform multiple location estimations from partial combinations of measures that are later fused together. Our results indicate that in doing so we can achieve estimation performance superior to that of the fully joint recovery, while keeping a lower computational complexity.**

I. Introduction

Localization of spatially distributed transmitters is an important task in a number of practical applications. In radio-localization, one aims to determine the unknown locations of transmitters by exploiting propagation parameters of the emitted signals, e.g., based on time, phase, or power measures [1]. One can also apply hybrid approaches that combine multiple signal measures, such as time- and phase differences for instance, as these are known to provide improved estimation performance [2], [3]. Along with traditional localization methods it has been recently proposed to use the ideas from compressed sensing to formulate the radio-localization task as a sparse recovery problem [4]–[11]. Compressed sensing is a recently emerged sampling paradigm that allows to reduce signal sampling rate without loss of information, provided that the signal possesses a sparse representation [12]. This said, it is especially appealing for localization schemes where, due to centralized baseband signal processing, a high volume of data has to be transported through the backhaul.

In order to be able to formulate the localization problem based on the notion of sparsity, it is necessary to find a suitable sparsity-promoting representation. In search for such sparse formulations, the majority of the work in this area considers one particular type of measure. Thus, the seminal work [4], representative of a number of related papers on super-resolution direction finding such as [13], [14] for instance, focuses on a sparse version of a DoA estimation problem. Sparsity-promoting formulations of the time difference of arrival (TDoA) based localization can be found in [7], [8], [11], while [5], [6] and, more recently, [9] provide an example of similar approaches based on received signal strength (RSS) measurements. Finally, [10] discusses a sparsity based approach applied to radar processing that combines TDoA and DoA measurements. It assumes a collaborative scheme where an accurate knowledge of the transmitted signal is available. In all of these, one represents the localization space (the angular space in case of DoA estimation) by a discrete grid, which is used to devise a sparsifying dictionary.

Inspired by these developments, in the current work we adopt the sparse recovery framework for non-collaborative hybrid TDoA/DoA localization of multiple sources using a combination of different measures. To do so, we split the spatial location area into a number of location bins and create sparsifying dictionaries for each considered measure. We then take advantage of the shared structure of the active atoms in the individual dictionaries to formulate the location estimation task as a joint block-sparse recovery problem. As such, it can then be solved by applying available recovery algorithms such as the block orthogonal matching pursuit (BOMP) from [15] for instance. However, the problem size and hence the estimation complexity increases rapidly with the number of sensors and the number of different measures used [10], making a direct recovery from all available measures computationally inefficient. To tackle this problem, we propose a reduced-size method in which we perform multiple location estimations from (different) partial combinations of measures. The resulting candidate estimates are then fused together by applying 2D Gaussian smoothing. This allows us to significantly reduce the size of each individual recovery problem while still enjoying the benefits of the joint block-sparse signal structure.

We evaluate the performance of the proposed estimation method with respect to the SNR and the compression rate. Our numerical results indicate that a reduced-size recovery from partial combinations provides an overall location estimation performance similar to the direct fully joint recovery, yet outperforming it in the lower SNRs. An additional advantage of the proposed method is that it allows for a distributed approach to location recovery which means that one can potentially avoid fully centralized processing.

II. System Model

A. Sensing scenario

Consider a system of N sensing nodes (Rx_n, $n \in [1, N]$) and M signal sources (Tx_m, $m \in [1, M]$) distributed over some spatial area $\mathcal{F}$, as shown in Figure 1. The sensing nodes are equipped with one or more sensing capabilities, such as precise time synchronization for time difference of arrival (TDoA) estimation and/or antenna arrays for direction of arrival (DoA) estimation. The sensors Rx_n are collaborative in that they share the received data with a central processing unit, referred to as the fusion center (FC). To enable the data exchange, it is assumed that the nodes are inter-connected via high speed data links. The sources Tx_m, on the other hand, are non-collaborative, meaning that they do not provide their synchronization information, while the signals from different transmitters are independent and uncorrelated. However, Tx_m are assumed to operate within the same standard in the sense of using the same modulation, bandwidth, and possibly central frequency which are known to Rx_n.

We suppose that $\mathcal{F}$ is divided into a set of location bins $k = \{1, 2, 3, \ldots, K\}$. Each bin corresponds to certain spatial coordinates (x_{k_1}, y_{k_2}) with $k_1 = 1, \ldots, K_1$ and $k_2 = 1, \ldots, K_2$ where $K = K_1 K_2$. Furthermore, we assume that the sensor nodes are placed at some known coordinates $(x_n^{\mathrm{r}}, y_n^{\mathrm{r}}), n = 1, \ldots, N$, while the signal sources are located at $(x_m^{\mathrm{s}}, y_m^{\mathrm{s}}), m = 1, \ldots, M$, which are unknown. Next, we associate a binary value $c_{\mathrm{b},k} \in \{0, 1\}$ to each location bin, depending on the presence ($c_{\mathrm{b},k} = 1$) or absence ($c_{\mathrm{b},k} = 0$) of the signal source at this location. By collecting $c_{\mathrm{b},k}$ together, we can represent the location plane $\mathcal{F}$ via a length-K binary vector $\boldsymbol{c}_{\mathrm{b}} = [c_{\mathrm{b},1}, \ldots, c_{\mathrm{b},K}]^{\mathrm{T}}$. Assuming that the number of sources is low with respect to the number of locations bins, $\boldsymbol{c}_{\mathrm{b}}$ has $M \ll K$ non-zero entries only and hence it is M-sparse.

B. Signal model

In the presence of M signal sources, each sensor Rx_i receives a superposition of M transmitted signals $s_m(t)$. Let $h_{m,i}(t)$ be an impulse response of the propagation channel between the m-th signal source and the i-th sensor. Then, the input signal of the m-th sensor is given by

$$r_i(t) = \sum_{m=1}^{M} h_{m,i}(t) * s_m(t) = \sum_{m=1}^{M} s_{m,i}(t), \qquad (1)$$

where $s_{m,i}(t) = h_{m,i}(t) * s_m(t)$ and $*$ denotes the convolution operation. Assuming time-invariant channels, at different nodes one observes different (scaled and delayed) copies of $s_m(t)$. Given (1), the localization task is to infer the source locations $(x_m^{\mathrm{s}}, y_m^{\mathrm{s}})$ from the set of measurements $\{r_i(t)\}_{i=1}^{N}$. In terms of the discrete vector of locations $\boldsymbol{c}_{\mathrm{b}}$, this is equivalent to determining the support $\mathcal{S}(\boldsymbol{c}_{\mathrm{b}})$, where $\mathcal{S}(\boldsymbol{x}) \triangleq \{i : x_i \neq 0\}$ for any vector $\boldsymbol{x}$.

Without loss of generality, in the following we represent $r_i(t)$ by its Nyquist sampled baseband version $r_i[n] = \sum_{m=1}^{M} s_{m,i}[n]$. Note that for the sake of brevity we describe the proposed joint localization framework in detail on an example of a single-path scenario where $h_{m,i}(t)$ consists of a single (line of sight (LoS)) component such that

$$r_i[n] = \sum_{m=1}^{M} \alpha_{m,i} s_m[n - \tau_{m,i}]. \qquad (2)$$

Here, $\alpha_{m,i}$ and $\tau_{m,i}$ are the attenuation and the (sample) delay of the LoS path from the mth signal source to the ith sensor, respectively. We then briefly discuss a more generic case of multipath propagation afterwards in Section V.

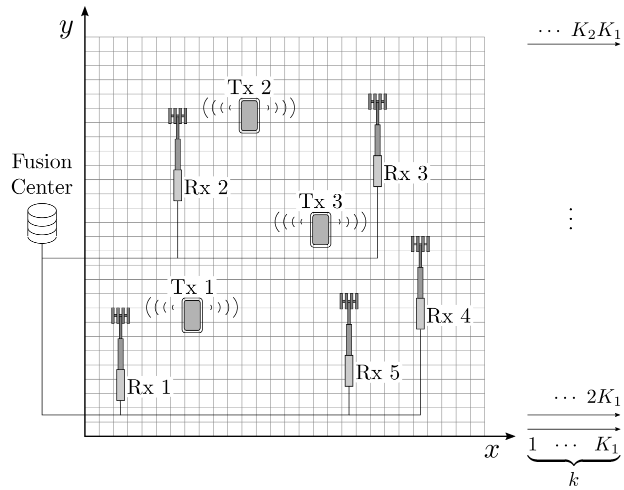

Figure 1. A sensing scenario with $M = 3$ signal sources Tx_m and $N = 5$ sensing nodes Rx_n connected via a backhaul to the fusion center.

III. Joint Sparse Localization Framework

A. Generic framework description

Given N inputs $r_i[n]$ to N sensors Rx_i, we can obtain a total of L signal measures $\boldsymbol{y}_\ell \in \mathbb{C}^{Q_\ell \times 1}$, each a functional of the location vector $\boldsymbol{c}_{\mathrm{b}}$, i.e.,

$$\boldsymbol{y}_\ell = \Phi_\ell(\boldsymbol{c}_{\mathrm{b}}). \qquad (3)$$

Note that the actual value of L depends on which particular measure is used. In (3), Φ_ℓ represents a mapping from the binary location space to the ℓ-th (complex) measure vector while $\ell = 1, 2, \ldots, L$. Note that the mapping is defined by the type of measure as well (e.g., time, phase, power, etc.). It can also be dimensionality reducing such that $Q_\ell < Q_{\mathrm{Nyq},\ell}$ where $Q_{\mathrm{Nyq},\ell}$ denotes the size of the ℓ-th measure computed at the Nyquist rate of the corresponding input signal.

Representing the operation of Φ_ℓ as an inner product between some $Q_\ell \times K$ dictionary $\boldsymbol{\Phi}_\ell$ and a K-length coefficient vector $\boldsymbol{c}_\ell$, we obtain

$$\boldsymbol{y}_\ell = \boldsymbol{\Phi}_\ell \boldsymbol{c}_\ell, \qquad (4)$$

where each $\boldsymbol{c}_\ell$ is M_ℓ-sparse[1]. Expression (4) presents a typical sparse recovery problem that can be solved by applying available recovery techniques. As a result, from (4) one could obtain estimates $\mathcal{S}(\hat{\boldsymbol{c}}_\ell)$ for each $\boldsymbol{c}_\ell$ individually. Once these are found, the (vector) indices of the source

[1] In general, $\mathcal{S}(\boldsymbol{c}_\ell)$ is not necessarily equal to $\mathcal{S}(\boldsymbol{c}_{\mathrm{b}})$ but rather $\mathcal{S}(\boldsymbol{c}_\ell) \supseteq \mathcal{S}(\boldsymbol{c}_{\mathrm{b}})$ due to the possible presence of multipath, hence $M_\ell \geq M$.

locations could be inferred from the set $\{\mathcal{S}(\hat{\boldsymbol{c}}_\ell)\}_{\ell=1}^{L}$, e.g., by taking an intersection over the individual support estimates.

Alternatively, we can collect all individual measures $\boldsymbol{y}_\ell$ in a single Q-long vector $\boldsymbol{y} = [\boldsymbol{y}_1^{\mathrm{H}}, \ldots, \boldsymbol{y}_L^{\mathrm{H}}]^{\mathrm{H}}$, where $Q = \sum_{\ell=1}^{L} Q_\ell$ and $(\cdot)^{\mathrm{H}}$ is the Hermitian transpose, to obtain

$$\boldsymbol{y} = \begin{pmatrix} \boldsymbol{\Phi}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Phi}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & & \boldsymbol{\Phi}_L \end{pmatrix} \begin{bmatrix} \boldsymbol{c}_1 \\ \boldsymbol{c}_2 \\ \vdots \\ \boldsymbol{c}_L \end{bmatrix} = \boldsymbol{\Phi}\boldsymbol{c}. \tag{5}$$

The $Q \times LK$ matrix $\boldsymbol{\Phi}$ here is a block diagonal matrix that contains individual dictionaries $\boldsymbol{\Phi}_\ell$, whereas $\boldsymbol{c}$ is an LK-length coefficient vector that collects vectors $\boldsymbol{c}_\ell$.

Consider now the structure of $\boldsymbol{c}$. In a single-path scenario (2), we can always construct $\boldsymbol{\Phi}_\ell$ such that all $\boldsymbol{c}_\ell$ have the same support set irrespective of the measure used, i.e., $\mathcal{S}(\boldsymbol{c}_\ell) = \mathcal{S}(\boldsymbol{c}_{\mathrm{b}})$ for all $\ell \in [1, L]$. In this case, the non-zeros in $\boldsymbol{c}$ appear in a periodic structure that admits a block-sparse representation. Note that we can rearrange $\boldsymbol{c}$ into another length-LK vector $\bar{\boldsymbol{c}}$ by aggregating every K-th element as

$$\bar{\boldsymbol{c}} = [\underbrace{c_1 \cdots c_{(L-1)K+1}}_{\bar{\boldsymbol{c}}_1} \cdots \underbrace{c_K \cdots c_{(L-1)K+K}}_{\bar{\boldsymbol{c}}_L}]^{\mathrm{T}},$$

where $(\cdot)^{\mathrm{T}}$ denotes the vector/matrix transpose. For a rearranged coefficient vector $\bar{\boldsymbol{c}}$, (5) transforms into

$$\boldsymbol{y} = [\bar{\boldsymbol{\Phi}}_1 \; \bar{\boldsymbol{\Phi}}_2 \cdots \bar{\boldsymbol{\Phi}}_L] \begin{bmatrix} \bar{\boldsymbol{c}}_1 \\ \bar{\boldsymbol{c}}_2 \\ \vdots \\ \bar{\boldsymbol{c}}_L \end{bmatrix} = \bar{\boldsymbol{\Phi}}\bar{\boldsymbol{c}}, \tag{6}$$

where $\bar{\boldsymbol{\Phi}} = [\bar{\boldsymbol{\Phi}}_1 \; \bar{\boldsymbol{\Phi}}_2 \cdots \bar{\boldsymbol{\Phi}}_L]$ is a column-wise rearranged version of $\boldsymbol{\Phi}$ such that

$$\bar{\boldsymbol{\Phi}}_\ell = \begin{pmatrix} \boldsymbol{\varphi}_{1,\ell} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \boldsymbol{\varphi}_{L,\ell} \end{pmatrix}$$

with $\boldsymbol{\varphi}_{i,j}$ denoting the j-th column of $\boldsymbol{\Phi}_i$. In (6), the vector $\bar{\boldsymbol{c}}$ is block M-sparse meaning that only M out of L blocks $\bar{\boldsymbol{c}}_\ell$ are non-identically zero. Furthermore, the block support of $\bar{\boldsymbol{c}}$, denoted by Ω, is equal to the support of $\boldsymbol{c}_{\mathrm{b}}$. This being said, it is known that exploiting the joint block-sparse structure of (6), by using block-sparse adaptation of OMP for instance [15]–[17], allows improving overall recovery performance compared to the individual recovery according to (4). In both cases however, we need to find appropriate sparsity-promoting dictionaries first. In the following, we show how one can construct $\boldsymbol{\Phi}_\ell$ and $\boldsymbol{\Phi}$ on the example of the phase and time delay based measures $\boldsymbol{y}_\ell$.

B. Dictionary building

1) Phase delay dictionary: Suppose the sensor nodes are equipped with antenna arrays for DoA estimation, each with n_i elements. Then, for an input of form (2) the (noise-free) baseband signal at the sensor output can be expressed as

$$\boldsymbol{r}_i[n] = \sum_{m=1}^{M} \boldsymbol{a}_i(\theta_{m,i}) s_{m,i}[n], \tag{7}$$

where $\boldsymbol{r}_i[n] \in \mathbb{C}^{n_i \times 1}$ is a vector of antenna outputs, $s_{m,i}[n] = \alpha_{m,i} s_m[n - \tau_{m,i}]$ and $\boldsymbol{a}(\theta)$ denotes the antenna response as a function[2] of the azimuth angle, where $\theta_{m,i}$ is the (azimuth) angle of arrival of the m-th source signal at the i-th sensor. We write (7) in a matrix form as

$$\boldsymbol{r}_i[n] = \boldsymbol{A}_i \boldsymbol{s}_i[n], \tag{8}$$

where $\boldsymbol{A}_i = [\boldsymbol{a}_i(\theta_{1,i}), \boldsymbol{a}_i(\theta_{2,i}), \cdots, \boldsymbol{a}_i(\theta_{K,i})] \in \mathbb{C}^{n_i \times K}$ is the array steering matrix of the i-th sensor and $\theta_{k,i}$ is the (azimuth) LoS direction from the k-th location bin to the i-th sensor. A length-K vector $\boldsymbol{s}_i[n]$ here contains complex amplitudes $s_{m,i}[n]$ at the indices corresponding to $\theta_{m,i}$.

Given (8), we can introduce a $p_i \times n_i$ compression matrix $\boldsymbol{W}_i$ that combines n_i antenna outputs into $p_i < n_i$ receiver channels [18] such that

$$\tilde{\boldsymbol{r}}_i[n] = \boldsymbol{W}_i \boldsymbol{A}_i \boldsymbol{s}_i[n] = \tilde{\boldsymbol{A}}_i \boldsymbol{s}_i[n], \tag{9}$$

where $\tilde{\boldsymbol{A}}_i = \boldsymbol{W}_i \boldsymbol{A}_i \in \mathbb{C}^{p_i \times n_i}$ is the effective array manifold after the combining. We notice that (9) has the desired form of (4) where $\tilde{\boldsymbol{A}}_i$ constitutes the dictionary $\boldsymbol{\Phi}_\ell$, $\boldsymbol{y}_\ell = \tilde{\boldsymbol{r}}_i[n]$, and $\boldsymbol{s}_i[n]$ is the M-sparse coefficient vector. Note that for a single dictionary $\tilde{\boldsymbol{A}}_i$ one can form multiple measures $\boldsymbol{y}_\ell$, one for each sample index n. Therefore, from N DoA sensors we can have $L_{\mathrm{doa}} = NT$ measures $\boldsymbol{y}_\ell$, where T denotes the number of samples taken in (9).

2) Time delay dictionary: Consider now time-synchronized receiver nodes that are capable of time delay measurements for TDoA estimation. In this case, we compute the (discrete) cross-correlation functions of the signals received at two receivers with indices $i_1 \neq i_2$ as

$$R_{i_1,i_2}[p] = \frac{1}{T} \sum_{n=n_1}^{n_1+T} r_{i_1}^*[n] r_{i_2}[n+p] = \frac{1}{T} \boldsymbol{r}_{i_1}^{\mathrm{H}} \boldsymbol{r}_{i_2}^{(p)}, \tag{10}$$

where $\boldsymbol{r}_{i_1} = [r_{i_1}[n_1], \ldots, r_{i_1}[n_1+T]]^{\mathrm{H}}$, $\boldsymbol{r}_{i_2}^{(p)} = [r_{i_2}[n_1 + p], \ldots, r_{i_2}[n_1+p+T]]^{\mathrm{H}}$ and $p \in [0, T-1]$ represents the time delay in samples.

Noting that $r_i[n] = \sum_{m=1}^{M} s_{m,i}[n]$ and taking into account the propagation model (2), we can obtain

$$\begin{aligned} R_{i_1,i_2}[p] = &\sum_{m=1}^{M} \alpha_m^{(i_1,i_2)} \underbrace{\frac{1}{T} \sum_{n=n_1}^{n_1+T} s_m^*[n] s_m[n+p-\tau_m^{(i_1,i_2)}]}_{R_m[p-\tau_m^{(i_1,i_2)}]} \\ &+ \sum_{m_1 \neq m_2} \alpha_{m_1,m_2}^{(i_1,i_2)} \underbrace{\frac{1}{T} \sum_{n=n_1}^{n_1+T} s_{m_1}^*[n] s_{m_2}[n+p-\tau_{m_1,m_2}^{(i_1,i_2)}]}_{R_{m_1,m_2}[p-\tau_{m_1,m_2}^{(i_1,i_2)}]}. \end{aligned} \tag{11}$$

In (11), $R_m[p] = \frac{1}{T} \sum_{n=n_1}^{n_1+T} s_m^*[n] s_m[n+p]$ denotes the (discrete) autocorrelation function of the m-th source signal, while $R_{m_1,m_2}[p] = \frac{1}{T} \sum_{n=n_1}^{n_1+T} s_{m_1}^*[n] s_{m_2}[n+p]$

[2]Note that $\boldsymbol{a}$ is generally a function of both azimuth and elevation angles of arrival as well as the polarization state of the incident plain wave. Since in this work we are interested in 2D localization, we assume that the sources are located in the azimuthal plane of the receiver antenna arrays and the impinging waves are co-polarized with them.

is the cross-correlation term between two signals from two sources Rx_{m_1} and Rx_{m_2}, where $m_1, m_2 \in [1, M] : m_1 \neq m_2$. This said, $\tau_m^{(i_1,i_2)} = \tau_{m,i_1} - \tau_{m,i_2}$ and $\tau_{m_1,m_2}^{(i_1,i_2)} = \tau_{m_1,i_1} - \tau_{m_2,i_2}$ are the relative delays in samples between different signal copies at two receivers, whereas $\alpha_m^{(i_1,i_2)} = \alpha_{m,i_1}^* \alpha_{m,i_2}$ and $\alpha_{m_1,m_2}^{(i_1,i_2)} = \alpha_{m_1,i_1}^* \alpha_{m_1,i_2}$. Since the individual signals $s_m(t)$ are uncorrelated, $R_{m_1,m_2}[p] \approx 0$ and, therefore[3],

$$R_{i_1,i_2}[p] \cong \sum_{m=1}^{M} \alpha_m^{(i_1,i_2)} R_m[p - \tau_m^{(i_1,i_2)}]. \tag{12}$$

Furthermore, Tx_m are assumed to be operating within the same standard which means that $R_m[p] = R[p]$ for all $m = 1, 2, \ldots, M$ and

$$R_{i_1,i_2}[p] \cong \sum_{m=1}^{M} \alpha_m^{(i_1,i_2)} R[p - \tau_m^{(i_1,i_2)}], \tag{13}$$

where $R[p]$ denotes the basic discrete baseband auto-correlation function of the system's waveform.

Given (13), we introduce $P \times T$ matrices $\boldsymbol{B}_i$ that represent the sub-sampling operation such that $\tilde{\boldsymbol{r}}_i = \boldsymbol{B}_i \boldsymbol{r}_i$ and

$$\begin{aligned} \tilde{R}_{i_1,i_2}[p] &= \frac{1}{P} \tilde{\boldsymbol{r}}_{i_1}^{\mathrm{H}} \tilde{\boldsymbol{r}}_{i_2}^{(p)} = \frac{1}{P} \boldsymbol{r}_{i_1}^{\mathrm{H}} \boldsymbol{B}_{i_1}^{\mathrm{T}} \boldsymbol{B}_{i_2} \boldsymbol{r}_{i_2}^{(p)} \\ &\cong \sum_{m=1}^{M} \alpha_m^{(i_1,i_2)} \tilde{R}_{i_1,i_2}[p - \tau_m^{(i_1,i_2)}], \end{aligned} \tag{14}$$

where $\tilde{\boldsymbol{r}}_i^{(p)} = [\tilde{r}_i[n_1 + p], \ldots, \tilde{r}_i[n_1 + p + T]]^{\mathrm{T}}$, $\tilde{R}_{i_1,i_2}[p]$ is the basic discrete baseband auto-correlation function that corresponds to the sub-sampling operation of $\boldsymbol{B}_{i_1}^{\mathrm{T}} \boldsymbol{B}_{i_2}$ and $p \in [0, P-1]$. The matrices $\boldsymbol{B}_i$ in (14) can be obtained from a $T \times T$ identity matrix $\mathbf{I}_T$ by randomly selecting P rows from it, for instance. Note that we arrive at the Nyquist-rate relation (12) by simply choosing $\boldsymbol{B}_i = \mathbf{I}_T$.

In order to obtain a formulation compatible with (4), we represent (14) in a matrix form by staking individual $\tilde{R}_{i_1,i_2}[p]$ into a length-P vector $\boldsymbol{r}_q = [\tilde{R}_{i_1,i_2}[0], \ldots, \tilde{R}_{i_1,i_2}[P-1]]^{\mathrm{T}}$ and constructing a $P \times K$ dictionary $\boldsymbol{R}_q$ from $\tilde{R}_{i_1,i_2}[p]$ computed for K relative delays corresponding to K location bins. Then,

$$\boldsymbol{r}_q = \boldsymbol{R}_q \boldsymbol{\alpha}_q, \tag{15}$$

where $q = (i_2 - i_1) + C_N^2 - C_{N-i_1+1}^2$ for $i_1 \in [1, N]$ and $i_2 \in [i_1 + 1, N]$ while C_N^K is the binomial coefficient. According to (14), the k-th column of $\boldsymbol{R}_q$ contains $\tilde{R}_{i_1,i_2}[p - \tau_k^{(i_1,i_2)}]$, where $\tau_k^{(i_1,i_2)}$ is the relative delay in samples of the LoS paths from the k-th location bin to i_1-th and i_2-th sensors while $\boldsymbol{\alpha}_q$ is a K-length vector that contains elements $\alpha_m^{(i_1,i_2)}$ at indices $k_m : \tau_{k_m}^{(i_1,i_2)} = \tau_m^{(i_1,i_2)}$. Hence, in this case we have that $\boldsymbol{\Phi}_\ell = \boldsymbol{R}_q$, $\boldsymbol{c}_\ell = \boldsymbol{\alpha}_q$ and $\boldsymbol{y}_\ell = \boldsymbol{r}_q$. In conclusion, we note that when all Rx_i are TDoA-capable sensors, one can obtain a total of $L_{\mathrm{tdoa}} = C_N^2$ measures $\boldsymbol{y}_\ell$.

[3]Note that the approximation here is due to the limited number of samples taken for computation of the cross-correlation function in (10).

IV. Location Recovery

A. Simultaneous joint recovery

In previous section, we have constructed two different sparsifying dictionaries, $\tilde{\boldsymbol{A}}_i$ for the DoA-type measures and $\boldsymbol{R}_q$ for the TDoA-type measures where $i = 1, \ldots, L_{\mathrm{doa}}$ and $q = 1, \ldots, L_{\mathrm{tdoa}}$. Note that $L_{\mathrm{doa}} \leq NT$ is the number of sensors that are equipped for DoA estimation times the number of (time) snapshots taken, whereas $L_{\mathrm{tdoa}} \leq C_N^2$ is the number of sensor pairs that are equipped for TDoA estimation. Given the relations (9), (15), we can now form $\boldsymbol{y}$ by collecting all available $\boldsymbol{r}_i$, $\boldsymbol{r}_q$ together and build $\boldsymbol{\Phi}$ from the corresponding dictionaries $\tilde{\boldsymbol{A}}_i$, $\boldsymbol{R}_q$ according to (5). This yields a total vector $\boldsymbol{y}$ of length $Q = \sum_{i=1}^{N_{\mathrm{doa}}} p_i + PN_{\mathrm{tdoa}}$ and $\boldsymbol{\Phi}$ of size $Q \times LK$ where $L = L_{\mathrm{doa}} + L_{\mathrm{tdoa}}$. After solving (6), we can obtain the vector of source locations $\boldsymbol{c}_{\mathrm{b}}$ by evaluating the block support Ω and then computing the location bin states $c_{\mathrm{b},k}$ as

$$c_{\mathrm{b},k} = \begin{cases} 1 & , k \in \Omega \\ 0 & , \text{otherwise} \end{cases}.$$

Subsequently, we can represent the source locations on a plane by rearranging $\boldsymbol{c}_{\mathrm{b}}$ back into an $K_2 \times K_1$ matrix $\boldsymbol{F}$.

However such an approach can potentially result in a significant computational complexity due to the large dimensions Q and LK. To circumvent this problem while still being able to enjoy the benefits of the joint block-sparse structure of (6), in the following we propose a reduced-sized approach based on partial joint recovery and subsequent estimate fusion.

B. Reduced-size joint recovery

Instead of collecting all available measures into a single vector $\boldsymbol{y}$, we can form partial combinations by stacking (e.g., random) subsets of $L_j \leq L$ individual measures[4] into length-$(Q_j \leq Q)$ vectors $\boldsymbol{y}^{(j)}$ such that

$$\boldsymbol{y}^{(j)} = \boldsymbol{\Psi}_j \boldsymbol{b}_j, \tag{16}$$

where $\boldsymbol{\Psi}_j$ is an $Q_j \times L_j K$ block-diagonal sub-matrix of $\boldsymbol{\Phi}$, while $\boldsymbol{b}_j$ is a corresponding length-$L_j K$ sub-vector of $\boldsymbol{c}$. Note that $L_j = L$ corresponds to the original full-size formulation (5). Denoting by Ω_j the block support of $\boldsymbol{b}_j$, after solving (16) we obtain a set of support estimates $\{\Omega_j\}_{j=1}^{L_{\mathrm{c}}}$ and a corresponding set of vector location estimates $\{\boldsymbol{c}_{\mathrm{b},j}\}_{j=1}^{L_{\mathrm{c}}}$ where L_{c} is the number of different subset combinations. Now we need to superpose $\boldsymbol{c}_{\mathrm{b},j}$ into the final estimate of the source locations. To do so, we first rearrange each $\boldsymbol{c}_{\mathrm{b},j}$ into a $K_2 \times K_1$ location matrix $\boldsymbol{F}_j$. We then apply a 2D Gaussian filter with an impulse response

$$g(k_1, k_2) = \frac{1}{2\pi\sigma^2} e^{-\frac{k_1^2 + k_2^2}{2\sigma^2}}, \tag{17}$$

[4]Note that the subset size L_j can be different for different vectors $\boldsymbol{y}^{(j)}$, while the individual measures $\boldsymbol{y}_\ell$ can be used more then once in different subset combinations.

where σ^2 denotes the variance in grid points, to obtain smoothed location plane estimates $\tilde{\boldsymbol{F}}_j$, which we afterwards combine together as

$$\hat{\boldsymbol{F}} = \frac{1}{L_{\mathrm{c}}} \sum_{j=1}^{L_{\mathrm{c}}} \tilde{\boldsymbol{F}}_j. \tag{18}$$

In the last step, we estimate the source locations as the ones that correspond to M largest peaks[5] in $\hat{\boldsymbol{F}}$.

V. A NOTE ON MODEL EXTENSIONS

For the sake of simplicity, so far we have considered a single-path LoS propagation scenario and TDoA dictionary built on the assumption of known transmitter waveforms. In the following, we briefly discuss how the proposed framework can be applied with multipath and in a completely blind scenario when the transmitted waveforms are not known at the receivers.

A. Multipath propagation scenario

In the case of multipath, the individual coefficient vectors $\boldsymbol{c}_\ell$ do not necessarily share exactly the same support, but rather $\mathcal{S}(\boldsymbol{c}_\ell) \subseteq \mathcal{S}(\boldsymbol{c}_{\mathrm{b}})$ for any ℓ, where $\bigcap_\ell \mathcal{S}(\boldsymbol{c}_\ell) = \mathcal{S}(\boldsymbol{c}_{\mathrm{b}})$. Therefore, the rearranged vector $\bar{\boldsymbol{c}}$ will now have $\bar{M} = |\bigcup_\ell \mathcal{S}(\boldsymbol{c}_\ell)|$ non-identically zero blocks where $M \leq \bar{M} \leq \sum_\ell M_\ell - ML$. However, only M of them will contain significant number of elements, i.e., the ones corresponding to the support of $\boldsymbol{c}_{\mathrm{b}}$, while the rest are likely to contain only a few ($\ll L$) non-zeros. This is because they originate from the multipath components whose spatial "locations" differ depending on the positions of both sensors and sources. Furthermore, the non-zero coefficients corresponding to the multipath components are also likely to be of lower magnitude. Altogether, this means that in the case of multipath the coefficient vector $\bar{\boldsymbol{c}}$ will preserve a quasi-block sparse structure, albeit approximately.

B. TDoA dictionary with unknown signal waveforms

While building the dictionary for time delay based measures in Section III-B2, we have assumed that the transmitters operate within the same standard that is known to the receivers. This implies that the basic autocorrelation function $R[p]$ is the same for all Tx_m and it is known. In a scenario where the transmitted waveforms are unknown or can vary from transmitter to transmitter, instead of using the exact basic autocorrelation function to compose $\boldsymbol{R}_q$ in (15) we can compute its bandlimited approximation, provided that the bandwidth is (at least approximately) known.

VI. NUMERICAL RESULTS

To evaluate the proposed localization framework, we simulate a scenario with $M = 4$ sources and $N = 5$ sensors distributed over an 1000×1000 m area. The positions of the sensing nodes are fixed and constant throughout the evaluation at the locations schematically illustrated in Figure 2,

[5]Note that when M is not known a prior one could apply a thresholding approach and pick those locations that correspond to the peaks exceeding some pre-defined threshold value.

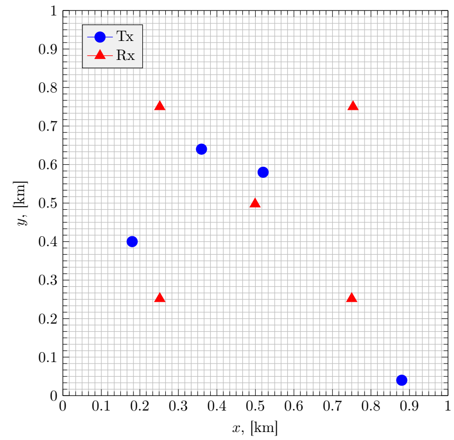

Figure 2. A single realization of a sensing scenario for numerical evaluation with $M = 4$ signal sources Tx_m positioned randomly and $N = 5$ sensing nodes Rx_n located at coordinates $x_n^{\mathrm{r}}, y_n^{\mathrm{r}} \in \{0.25, 0.5, 0.75\}$ km.

while the Tx coordinates are generated uniformly at random. The location area is divided into $K = 2500$ grid points placed at $d_{\mathrm{grd}} = 20$ m. Each sensor is capable of providing both TDoA and DoA measures $\boldsymbol{y}_\ell$ such that the total number of measures available is $L = \frac{N!}{2(N-2)!} + N = 15$. Out of these, we either select $L_{\mathrm{c}} = 50$ (different) random subsets of size $L_j \in [2, 4]$ for partial joint recovery according to (16) or use all measures at once as in (6). In both cases, to solve the associated sparse recovery problem we use the BOMP algorithm from [15]. For the DoA part, the sensors employ uniform circular arrays (UCAs), each with $n_i = 10$ elements. For the TDoA part, it is assumed that the sources use QPSK-modulation with a 0.5 roll-off factor raised cosine filter, an oversampling factor of 10 and a bandwidth of 10 MHz. Additionally, when computing the TDoA measures $\boldsymbol{r}_q$ we apply compression matrices $\boldsymbol{B}_i = \boldsymbol{B}$ composed of a random selection of P rows from $\mathbf{I}_T$. The resulting compression rate is defined as $\frac{T-P}{T}$ where the total number of samples taken is $T = 10^4$.

Figure 3 presents an average (among 600 Monte Carlo realizations) localization error in terms of the root mean squared error (RMSE) between the true and the estimated source locations in units of the grid step. It shows the RMSE as a function of the SNR and the compression rate in a form of a color-plot for the cases when we use i) only a single type of measures (denoted by TDoA-only or DoA-only), or ii) a mixture of both measures (denoted by hybrid TDoA/DoA). Furthermore, we display the results for the full simultaneous ($L_j = L$, upper row) and the reduced-size partial ($L_j \in [2, 4]$, lower row) joint recovery. We observe that, while the overall trend for both recovery approaches is similar, namely the hybrid TDoA/DoA approach expectedly outperforms the localization based on a single DoA or TDoA measure, the reduced-size recovery in each case provides better estimation accuracy in the lower SNRs. Note that for the TDoA-based localization the RMSE naturally deteriorates at higher compression rates, whereas it exhibits no such behavior for the DoA-only based localization, as no compression in the spatial domain has been applied (i.e., $\boldsymbol{W}_i = \mathbf{I}_{n_i}$ for all $i = 1, \ldots, N$).

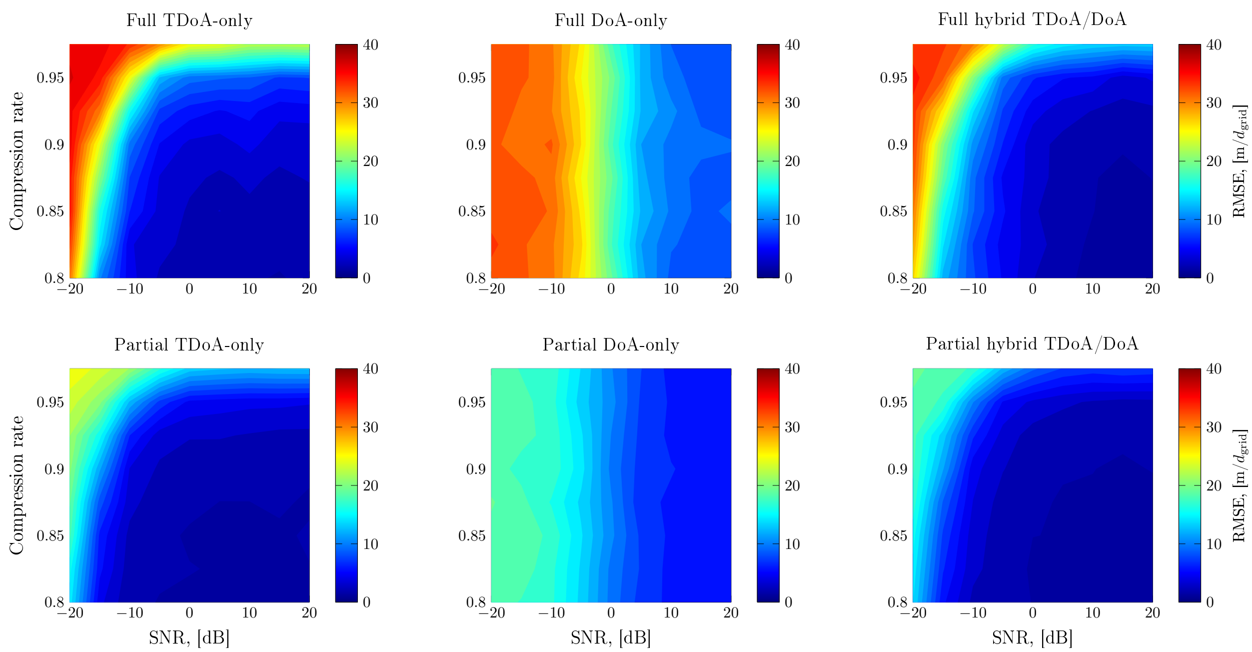

Figure 3. RMSE vs. SNR vs. compression rate for 3 considered scenarios and two recovery approaches. The first row shows the results for the simultaneous recovery where $L_j = L$, whereas the second row presents corresponding figures for reduced-size approach with partial recovery with $L_j \in [3, 5]$ and fusion.

VII. Conclusions

In this work, we considered the task of localizing multiple non-collaborative transmitters by a network of distributed sensor nodes that are capable of TDoA and DoA estimation. We developed a sparse localization framework that formulates the localization task as a joint block-sparse recovery problem allowing us to perform location estimation from both types of measures simultaneously. Our results indicate that the proposed reduced-size estimation method outperforms fully joint recovery, while keeping a lower computational complexity.

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