On Interplay Between Network Topology and Alternating CSIT for Multi-Receiver Wiretap Channel

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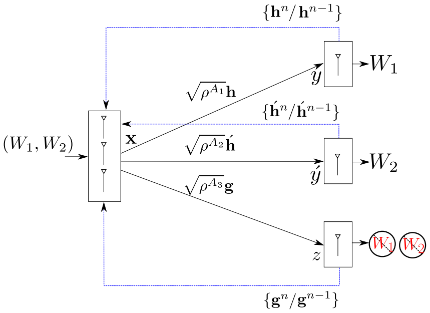
***Abstract*—We study the problem of secure transmission over a Gaussian multi-input single-output (MISO) two receiver channel with an external eavesdropper under the assumption that links connecting the transmitter to three receivers may have unequal strength *statistically*. In addition to this, the state of the channel to each receiver is conveyed either perfectly (P) or with delay (D) to the transmitter. Let S_1, S_2, and S_3 be the channel state information at the transmitter (CSIT) of user 1, user 2, and eavesdropper, respectively. The overall CSIT can then alternate between eight possible states, i.e., $(S_1, S_2, S_3) \in \{P, D\}^3$. We denote by $\lambda_{S_1 S_2 S_3}$ the fraction of time during which the state $S_1 S_2 S_3$ occurs; and, focus on a two state topological setting of strong v.s. weak links with symmetric alternating CSIT, $\lambda_{PDD} = \lambda_{DPD}$. For this setting, we establish an inner bound on the Generalized Secure Degrees of Freedom (GSDoF) region with different topology states. The encoding scheme sheds light on the usage of both resources, i.e., alternating — topology and CSIT; and, show that as opposed to coding independently over different states, joint encoding across the CSIT and topological states, enables strictly better secure rates.**

I. Introduction

Transmission of information over a wireless channel is particularly sensitive to eavesdropping, due to the inherent openness of the medium. Wyner in [1], introduced a basic wiretap channel and showed that by utilizing the randomness of the wireless channel, the confidential information can be securely sent to the legitimate receiver. The wiretap channel introduced by Wyner has attracted significant attention in the research community and is extended to a variety of multi-antenna [2] and multi-users setting [3]–[5]. Due to the difficulty in characterizing the complete secrecy capacity region, a number of recent contributions have focused on characterizing the *approximate* capacity of these networks. The approximate capacity is measured by the notion of secure degrees of freedom (SDoF), that captures the asymptotic behavior of secure data rates in high signal-to-noise ratio (SNR) regime [6], [7]. In these models, generally it is assumed that the CSI which is available at the receivers is conveyed in a timely fashion to the transmitter. In practice, due to the random fading experienced by the feed back link, the quality of CSI that is available at the transmitter can vary over time. This lack of precise and varying quality of CSI may lead to information leakage. Towards this direction, building on [8], Yang *et al.* in [9] study the two-user MIMO broadcast channel where strictly causal CSI (delayed) is provided to the transmitter from both receivers. For this model the authors characterize the SDoF region. Zaidi *et al.* in [10] study the two-user MIMO X-channel with asymmetric feedback and delayed CSIT, and characterize the complete sum SDoF region. In [11] and [12], the authors studied the MISO broadcast and multi-receiver wiretap channel, respectively, and assume that CSI conveyed by the receivers can vary over time.


$\{\mathbf{h}^n/\mathbf{h}^{n-1}\}$
$\sqrt{\rho^{A_1}}\mathbf{h}$
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W_1
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$\sqrt{\rho^{A_3}}\mathbf{g}$
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W_1
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Fig. 1. Multi receiver wiretap channel, where the link power exponent to — receiver 1 is $A_1 \in \{1, \alpha\}$, receiver 2 is $A_2 \in \{1, \alpha\}$ and eavesdropper is $A_3 \in \{1, \alpha\}$, where $0 \leq \alpha \leq 1$.

In mobile networks due to mobility, communication links are subjected to different topological effects, e.g., jamming, path loss, interference, which can influence the channel in an *asymmetric* manner. A fundamental issue with DoF (SDoF) analysis is that it ignores the diversity of links strength and implicitly assumes that all non-zero channels are equally strong, irrespective of the magnitude of channel coefficients. The GDoF metric, introduced by [13], [14], solves this limitation by taking diversity of links strength into account. In [15], Chen *et al.* study a two-user MISO broadcast channel by considering the two state topological setting of strong v.s. weak links and assume that CSI conveyed by both receivers can vary over time. For this model the authors establish bounds on the GDoF region.

In this work, we consider a Gaussian multi-receiver wiretap channel which consists of three nodes — a transmitter and three receivers as shown in Figure 1. The transmitter is equipped with three antennas and each receiver is equipped with a single antenna. The transmitter wants to reliably transmit message W_1 to the receiver 1 and message W_2 to the receiver 2 and wishes to conceal them from the eavesdropper. In investigating this model we assume that, 1) each receiver knows the perfect instantaneous CSI; and, is allowed to convey either the instantaneous (P) or past delayed (D) CSI to the transmitter, with both of them being perfect; and, 2) links connecting three receivers may have different strength, statistically. We restrict our attention to the two state topological setting of stronger ($A_i = 1$) v.s. weaker ($A_i = \alpha$) links $\forall\, i$; thus, the topology and CSIT of this network is allowed to alternate between eight possible $(A_1, A_2, A_3) \in \{1, \alpha\}^3$ and $(S_1, S_2, S_3) \in \{P, D\}^3$ states, respectively. For this general model, we focus our attention on the symmetric alternating

— CSIT and topology setting — where the time spend in state (P, D, D) with topology state $(1, \alpha, \alpha)$ and (D, P, D) with topology state $(\alpha, 1, \alpha)$ are equal. For this setting, we establish an inner bound on the GSDoF region. The encoding scheme sheds lights on how to carefully utilizes the topology of the network and quality of CSIT. We show that as opposed to coding independently over states and topology, jointly encoding across them, provide strictly better secure rates. The setting that we study in this work can be used to better understand the connection between topology and CSIT for the general class of K-user models.

II. System Model and Definitions

We consider a multi-receiver wiretap channel as shown in Figure 1. In this model, the transmitter is equipped with three antennas and each of the receiver is equipped with a single antenna. The transmitter wants to send message $W_1 \in \mathcal{W}_1 = \{1, \ldots, 2^{nR_1(A_1,\rho)}\}$ to receiver 1, and message $W_2 \in \mathcal{W}_2 = \{1, \ldots, 2^{nR_2(A_2,\rho)}\}$ to receiver 2, and wishes to conceal both messages from the eavesdropper.

As mentioned before, due to random fluctuations of the wireless medium, the variance of the channel coefficients may vary over time. This induces two classes of links, where few links are stronger than others statistically. We denote $A_i \in \{1, \alpha\}$ be the link power exponent from the transmitter-to-receiver 1, 2, and eavesdropper, $\forall i \in \{1, 2, 3\}$, respectively, $0 \leq \alpha \leq 1$. Then, based on the topology of the network, the model can be classified into eight possible states, $(A_1, A_2, A_3) \in \{1, \alpha\}^3$. The channel input-output relationship at time instant t is then given by

$$y_t = \sqrt{\rho^{A_{1,t}}}\mathbf{h}_t\mathbf{x}_t + n_{1t} \tag{1a}$$

$$\acute{y}_t = \sqrt{\rho^{A_{2,t}}}\acute{\mathbf{h}}_t\mathbf{x}_t + n_{2t} \tag{1b}$$

$$z_t = \sqrt{\rho^{A_{3,t}}}\mathbf{g}_t\mathbf{x}_t + n_{3t}, \quad t = 1, \ldots, n \tag{1c}$$

where $\mathbf{x} \in \mathbb{C}^{3\times1}$ is the channel input vector, $\mathbf{h} \in \mathcal{H} \subseteq \mathbb{C}^{1\times3}$ is the channel vector connecting receiver 1 to the transmitter, $\acute{\mathbf{h}} \in \mathcal{H} \subseteq \mathbb{C}^{1\times3}$ is the channel vector connecting receiver 2 to the transmitter and $\mathbf{g} \in \mathcal{G} \subseteq \mathbb{C}^{1\times3}$ is the channel vector connecting eavesdropper to the transmitter. The parameter ρ is subject to input power constraint and the channel output noise n_i is assumed to be independent and identically distributed (i.i.d.) white Gaussian noise, with $n_i \sim \mathcal{CN}(0, 1)\ \forall i$. For the sake of conciseness, we normalize the channel input vector, $||\mathbf{x}_t||^2 \leq 1$, then the average received signal-to-noise ratio (SNR) for each link at time instant t is given by

$$\mathbb{E}_{\mathbf{h}_t,\mathbf{x}_t}\left[||\sqrt{\rho^{A_{1,t}}}\mathbf{h}_t\mathbf{x}_t||^2\right] = \rho^{A_{1,t}}$$

$$\mathbb{E}_{\acute{\mathbf{h}}_t,\mathbf{x}_t}\left[||\sqrt{\rho^{A_{2,t}}}\acute{\mathbf{h}}_t\mathbf{x}_t||^2\right] = \rho^{A_{2,t}}$$

$$\mathbb{E}_{\mathbf{g}_t,\mathbf{x}_t}\left[||\sqrt{\rho^{A_{3,t}}}\mathbf{g}_t\mathbf{x}_t||^2\right] = \rho^{A_{3,t}}.$$

Let $\mathbf{S}_t = [\mathbf{h}_t\ \acute{\mathbf{h}}_t\ \mathbf{g}_t]^T$ as the channel state matrix and $\mathbf{S}^{t-1} = \{\mathbf{S}_1, \ldots, \mathbf{S}_{t-1}\}$ captures the collection of channel state matrices over the past $(t-1)$ symbols, where $\mathbf{S}^0 = \emptyset$. Furthermore, at each time instant t, the past states of the channel matrix $\mathbf{S}^{t-1}$ are known to all nodes. However, the instantaneous states $\mathbf{h}_t$, $\acute{\mathbf{h}}_t$, and $\mathbf{g}_t$ are known only to receiver 1, receiver 2, and eavesdropper, respectively.

Information transmission over the wireless channel is particularly sensitive to the nature of CSIT. In this work, we assume that the receiver can feed back the CSIT with infinite precision. Although, there are numerous forms of CSIT, we focus our attention on two of them as follows.

- **Perfect CSIT**: denoted by 'P', refers to those instances in which the transmitter has perfect knowledge of the instantaneous CSI.
- **Delayed CSIT**: denoted by 'D', refers to those instances in which at time t, the transmitter has perfect knowledge of *only* the past $(t-1)$ channel states. Furthermore, at time instant t, the current channel state is independent of the past $(t-1)$ channel states.

Let S_1 denotes the CSIT state of user 1, S_2 denotes the CSIT state of user 2 and S_3 denotes the CSIT state of the eavesdropper. Then, based on the availability of the CSIT, the model that we study (1) can be classified into any of the following eight states

$$\begin{aligned}(S_1, S_2, S_3) \in \{&PPP, PPD, PDP, PDD, DPP, DPD,\\ &DDP, DDD\}.\end{aligned} \tag{2}$$

We denote $\lambda_{S_1S_2S_3}^{A_1A_2A_3}$ be the fraction of time state $S_1S_2S_3$ occurs with $A_1A_2A_3$ topology state, such that

$$\sum_{\substack{(S_1,S_2,S_3)\in\{P,D\}^3\\(A_1,A_2,A_3)\in\{1,\alpha\}^3}} \lambda_{S_1S_2S_3}^{A_1A_2A_3} = 1. \tag{3}$$

For simplicity of analysis, we assume that $\lambda_{PDD} = \lambda_{DPD}$, i.e., the fractions of time spent in states PDD and DPD are equal. We refer $(S_1, S_2, S_3, A_1, A_2, A_3)$ be the state in which the model chooses $(S_1, S_2, S_3) \in \{P, D\}^3$ with topology $(A_1, A_2, A_3) \in \{1, \alpha\}^3$. In this work, we focus our attention on the case $(\lambda_{PDD}^{1\alpha\alpha}, \lambda_{DPD}^{\alpha1\alpha}) = (\frac{1}{2}, \frac{1}{2})$, where the model is allowed to alternate between $(P, D, D, 1, \alpha, \alpha)$ and $(D, P, D, \alpha, 1, \alpha)$ states equal fractions of communication time.

Definition 1: A rate pair $(R_1(A_1, \rho), R_2(A_2, \rho))$ is said to be achievable if there exists a sequence of codes such that

$$\limsup_{n\to\infty} \Pr\{\hat{W}_i \neq W_i\} = 0, \quad \forall\ i \in \{1, 2\}. \tag{4}$$

Definition 2: A GSDoF pair $(d_1(A_1), d_2(A_2))$ is said to be achievable if there exists a sequence of codes satisfying following

1) Reliability condition:

$$\limsup_{n\to\infty} \Pr\{\hat{W}_i \neq W_i\} = 0, \qquad \forall\ i \in \{1, 2\}, \tag{5}$$

2) Perfect secrecy condition:

$$\limsup_{n\to\infty} \frac{I(W_1, W_2; z^n, \mathbf{S}^n)}{n} = 0, \tag{6}$$

3) and communication rate condition:

$$\lim_{\rho\to\infty} \liminf_{n\to\infty} \frac{\log|\mathcal{W}_i(n, \rho, A_i)|}{n\log\rho} \geq d_i(A_i), \quad \forall\ i \in \{1, 2\}. \tag{7}$$

Due to the space limitations, some proofs in this work are only outlined or omitted. Detailed proofs are provided in [16].

III. Main Results

In this section, we establish an inner bound on the multi-receiver wiretap channel, where $(\lambda^{1\alpha\alpha}_{PDD}, \lambda^{\alpha 1\alpha}_{DPD}) = (\frac{1}{2}, \frac{1}{2})$.

Theorem 1: An inner bound on the GSDoF region of the multi-receiver wiretap channel with alternating CSIT and topology state, $(\lambda^{1\alpha\alpha}_{PDD}, \lambda^{\alpha 1\alpha}_{DPD}) = (\frac{1}{2}, \frac{1}{2})$, is given by the set of all non-negative pairs (d_1, d_2) satisfying

$$(18 + 18\alpha)d_1 + (6 + 16\alpha)d_2 \leq (3 + 3\alpha)(3 + 2\alpha) \tag{8a}$$

$$(6 + 16\alpha)d_1 + (18 + 18\alpha)d_2 \leq (3 + 3\alpha)(3 + 2\alpha). \tag{8b}$$

Proof: The region in (8) is characterized by the corner points $(\frac{3+2\alpha}{6}, 0)$, $(0, \frac{3+2\alpha}{6})$ and the point $\left(\frac{(3+3\alpha)(3+2\alpha)}{2(12+17\alpha)}, \frac{(3+3\alpha)(3+2\alpha)}{2(12+17\alpha)}\right)$ obtained by the intersection of line equations in (8). The achievability of the two corner points $(\frac{3+2\alpha}{6}, 0)$, $(0, \frac{3+2\alpha}{6})$ follow by the coding schemes developed, in [9], where $2\alpha/3$ GSDoF is achievable; and, in [12] where 1 GSDoF is achievable, equal fractions of communication time. The achievability of the point $\left(\frac{(3+3\alpha)(3+2\alpha)}{2(12+17\alpha)}, \frac{(3+3\alpha)(3+2\alpha)}{2(12+17\alpha)}\right)$ is provided in subsection III-A. □

Remark 1 (Synergistic benefits of alternating CSIT and topology): We observe that the sum GSDoF in Theorem 1 can be larger than the one obtained by fixed topology and CSIT state setting. For fixed topology and CSIT setting, i.e., $\lambda^{1\alpha\alpha}_{PDD} = 1$ and $\lambda^{\alpha 1\alpha}_{DPD} = 1$, the sum GSDoF is given by 1 [12]. By using these fixed states equal fractions of communication time, the set-up that we study in Theorem 1 is equivalent to it, in the sense that the time duration for stronger and weaker links and CSIT states are equal. The sum GSDoF with fixed topology and CSIT is given by

$$\begin{aligned}\text{GSDoF} &= \frac{1}{2} \times \underbrace{1}_{\lambda^{1\alpha\alpha}_{PDD}=\frac{1}{2}} + \frac{1}{2} \times \underbrace{1}_{\lambda^{\alpha 1\alpha}_{DPD}=\frac{1}{2}} \\ &= \frac{1}{2} \leq \underbrace{\frac{(3+3\alpha)(3+2\alpha)}{12+17\alpha}}_{(8)}\end{aligned} \tag{9}$$

which is smaller than Theorem 1.

A. Coding scheme using alternating CSIT and topology

Before proceeding to the formal proof of the coding scheme, we first provide an auxiliary scheme which will be useful to establish the results in this work.

Theorem 2: An inner bound on the GDoF region of the two-user MISO broadcast channel with alternating CSIT and topology, $(\lambda^{1\alpha}_{PD}, \lambda^{\alpha 1}_{DP}) = (\frac{1}{2}, \frac{1}{2})$ is given by the set of all non-negative pairs (d_1, d_2) satisfying

$$2\alpha d_1 + 2(3 + 2\alpha)d_2 \leq (1 + \alpha)(3 + 2\alpha) \tag{10a}$$

$$2(3 + 2\alpha)d_1 + 2\alpha d_2 \leq (1 + \alpha)(3 + 2\alpha). \tag{10b}$$

Proof: The region in (10) is characterized by the corner points $(\frac{1+\alpha}{2}, 0)$, $(0, \frac{1+\alpha}{2})$ and the point $(\frac{3+2\alpha}{6}, \frac{3+2\alpha}{6})$ obtained by the intersection of line equations in (10). The GDoF pairs $(\frac{1+\alpha}{2}, 0)$, and $(0, \frac{1+\alpha}{2})$ are readily achievable by sending one symbol to each receiver. The achievability of the point $(\frac{3+2\alpha}{6}, \frac{3+2\alpha}{6})$ is provided in Appendix I. □

Remark 2 (GDoF Gains with Topological Diversity): Theorem 2, provides an inner bound on the GDoF region of the two user MISO broadcast channel with symmetric alternating — topology and states, where each receiver observes a strong link half of the duration of communication time and model alternates between (P, D) and (D, P) states. We note that sum GDoF in Theorem 2, can be larger than the one obtained by a similar model with alternating CSIT and fixed topology state. For fixed setting, i.e., $\lambda^{1\alpha}_{PD/DP}$, in [15, Theroem 8], the authors established an inner bound on the GDoF region of alternating CSIT model with fixed topology given by

$$\text{GDoF} = \underbrace{\frac{(2+3\alpha)(1+\alpha)}{2(1+2\alpha)}}_{\text{Sum GDoF}(\lambda^{1\alpha}_{PD/DP})} \leq \underbrace{1 + \frac{2\alpha}{3}}_{\text{Sum GDoF (10)}} \tag{11}$$

which is clearly smaller than the sum GDoF of Theorem 2. This result shows the benefits of topological diversity.

We now provide some coding schemes that provide the main ingredients to establish the inner bound in Theorem 1. The following schemes achieve the sum GSDoF of $\frac{(3+3\alpha)(3+2\alpha)}{(12+17\alpha)}$.

1) S_1 – using $(P, D, D, 1, \alpha, \alpha)$ and $(D, P, D, \alpha, 1, \alpha)$ states for $\left(\frac{15+19\alpha}{24+34\alpha}, \frac{9+15\alpha}{24+34\alpha}\right)$ fractions of time, $(d_1, d_2) = \left(\frac{(3+3\alpha)(3+2\alpha)}{2(12+17\alpha)}, \frac{(3+3\alpha)(3+2\alpha)}{2(12+17\alpha)}\right)$ GSDoF is achievable.
2) S_2 – using $(P, D, D, 1, \alpha, \alpha)$ and $(D, P, D, \alpha, 1, \alpha)$ states for $\left(\frac{9+15\alpha}{24+34\alpha}, \frac{15+19\alpha}{24+34\alpha}\right)$ fractions of time, $(d_1, d_2) = \left(\frac{(3+3\alpha)(3+2\alpha)}{2(12+17\alpha)}, \frac{(3+3\alpha)(3+2\alpha)}{2(12+17\alpha)}\right)$ GSDoF is achievable.

The achievability of the corner point $\left(\frac{(3+3\alpha)(3+2\alpha)}{2(12+17\alpha)}, \frac{(3+3\alpha)(3+2\alpha)}{2(12+17\alpha)}\right)$ in Theorem 1 follows by using S_1 and S_2 schemes equal fractions of communication time.

1) S_1 — Coding scheme using $(P, D, D, 1, \alpha, \alpha)$ and $(D, P, D, \alpha, 1, \alpha)$ states: We now show that by using$(P, D, D, 1, \alpha, \alpha)$ and $(D, P, D, \alpha, 1, \alpha)$ states for $\left(\frac{15+19\alpha}{24+34\alpha}, \frac{9+15\alpha}{24+34\alpha}\right)$ fractions of time, $(d_1, d_2) = \left(\frac{(3+3\alpha)(3+2\alpha)}{2(12+17\alpha)}, \frac{(3+3\alpha)(3+2\alpha)}{2(12+17\alpha)}\right)$ GSDoF is achievable. In this scheme, the transmitter wants to send four confidential symbols (v_1, v_2, v_3, v_4) to receiver 1 and (w_1, w_2, w_3, w_4) to receiver 2, respectively. The communication takes place in two steps, i.e., data dissemination phase and broadcasting of common information.

A) Data dissemination phase: In this step, the transmitter sends desired information to both receivers. In the first time slot, the transmitter chooses $(P, D, D, 1, \alpha, \alpha)$ state and injects artificial noise $\mathbf{u} := [u_1, u_2, u_3]^T$. At the end of this phase, the channel input-output relationship is given by

$$y_1 = \underbrace{\sqrt{\rho}\mathbf{h}_1\mathbf{u}}_{\mathcal{O}(\rho)}, \tag{12a}$$

$$\acute{y}_1 = \underbrace{\sqrt{\rho^\alpha}\acute{\mathbf{h}}_1\mathbf{u}}_{\mathcal{O}(\rho^\alpha)}, \tag{12b}$$

$$z_1 = \underbrace{\sqrt{\rho^\alpha}\mathbf{g}_1\mathbf{u}}_{\mathcal{O}(\rho^\alpha)}. \tag{12c}$$

At the end of first time slot, the receiver 2 and eavesdropper feed back the delayed CSI to the transmitter.

In the second time slot, the transmitter remains in $(P, D, D, 1, \alpha, \alpha)$ state and sends $\mathbf{v} := [v_1, v_2, v_3]^T$ to the receiver 1 along with channel output y_1 at the receiver 1. The transmitter can learn y_1, since it already knows the perfect CSI $\mathbf{h}_1$ and $\mathbf{u}$ and sends $\mathbf{x}_1 = \mathbf{v} + [y_1 \, \phi \, \phi]^T$. The channel input-output relationship, at the end of second time slot is given by

$$y_2 = \underbrace{\sqrt{\rho}(\mathbf{h}_2\mathbf{v} + h_{21}y_1)}_{\mathcal{O}(\rho)}, \tag{13a}$$

$$\acute{y}_2 = \underbrace{\sqrt{\rho^{\alpha}}(\acute{\mathbf{h}}_2\mathbf{v} + \acute{h}_{21}y_1)}_{\text{side information } (\mathcal{O}(\rho^{\alpha}))}, \tag{13b}$$

$$z_2 = \underbrace{\sqrt{\rho^{\alpha}}(\mathbf{g}_2\mathbf{v} + g_{21}y_1)}_{\text{side information } (\mathcal{O}(\rho^{\alpha}))}. \tag{13c}$$

At the end of second time slot, since the receiver 1 knows the CSI $(\mathbf{h}_2)$ and the channel output at receiver 1 in time slot 1 (y_1), it subtracts out the contribution of y_1 from y_2 to get one equation with 3 symbols $(\mathbf{v})$; and, requires 2 extra equations — being available as side information at receiver 2 and eavesdropper — to decode the intended symbols. Notice that, the side information at receiver 2 and eavesdropper is available at a reduced power level $(\mathcal{O}(\rho^{\alpha}))$ compared to the receiver 1.

In the third time slot, the transmitter chooses $(D, P, D, \alpha, 1, \alpha)$ state and sends fresh information $\mathbf{w} := [w_1, w_2, w_3]^T$ to the receiver 2 along with channel output $\acute{y}_1$ at the receiver 2. The transmitter can learn $\acute{y}_1$, since it knows the past CSI $\acute{\mathbf{h}}_1$ and $\mathbf{u}$ and sends $\mathbf{x}_2 = \mathbf{w} + [\acute{y}_1 \, \phi \, \phi]^T$. The channel input-output relationship is given by

$$y_3 = \underbrace{\sqrt{\rho^{\alpha}}(\mathbf{h}_3\mathbf{w} + h_{31}\acute{y}_1)}_{\text{side information } \mathcal{O}(\rho^{\alpha})}, \tag{14a}$$

$$\acute{y}_3 = \underbrace{\sqrt{\rho}(\acute{\mathbf{h}}_3\mathbf{w} + \acute{h}_{31}\acute{y}_1)}_{\mathcal{O}(\rho)}, \tag{14b}$$

$$z_3 = \underbrace{\sqrt{\rho^{\alpha}}(\mathbf{g}_3\mathbf{w} + g_{31}\acute{y}_1)}_{\text{side information } (\mathcal{O}(\rho^{\alpha}))}. \tag{14c}$$

At the end of third time slot, since the receiver 2 knows the CSI $(\acute{\mathbf{h}}_3)$ and the channel output at receiver 2 in time slot 1 $(\acute{y}_1)$, it subtracts out the contribution of $\acute{y}_1$ from $\acute{y}_3$ to obtain one equation with 3 variables $(\mathbf{w})$ and requires 2 extra equations being available as side information at receiver 1 and eavesdropper to decode the intended variables. The information leaked to eavesdropper after 3 time slots can be readily shown to be bounded by

$$I(\mathbf{v}, \mathbf{w}; z_1, z_2, z_3 | \mathbf{S}^n) \leq o(\log(P)). \tag{15}$$

At the end of three time slots, the receiver i requires side information available at the receiver j, $\forall i \neq j$ and eavesdropper — at reduced power levels $(\mathcal{O}(\rho^{\alpha}))$ — to decode the desired symbols. Due to the availability of strictly causal CSI, the transmitter can learn these side informations. Next, by using Theorem 2, z_2 $(\mathcal{O}(\rho^{\alpha}))$ is send to the receiver 1 and z_3 $(\mathcal{O}(\rho^{\alpha}))$ is send to the receiver 2 over a total of $\frac{2\alpha}{\text{GDoF}^{1\alpha/\alpha 1}_{PD/DP}} = \frac{6\alpha}{3+2\alpha}$ time slots.

After, conveying z_2 and z_3 to the desired receivers, the next step is to convey $\acute{y}_2$ to receiver 1 and y_3 to the receiver 2. One can not simply multicast these side informations in the spirit of Theorem 2, since it will leak extra information to the eavesdropper. Recall that, at the end of time slot 3, the side information at both receivers are available at $\mathcal{O}(\rho^{\alpha})$. The transmitter then performs following operations: 1) it quantizes the channel outputs $\acute{y}_2$ and y_3 to $\alpha\log(\rho) + o(\log\rho)$ bits within bounded noise distortion, respectively, and 2) performs a bit wise XOR operation to generate $\alpha\log(\rho) + o(\log\rho)$ bits which are then mapped to a common message c where $c \in \mathcal{C} = \{1, \ldots, \rho^{\alpha}\}$. After receiving the common message, both receivers can construct the desired side information within bounded noise distortion, that suffices to decode their respective symbols.

The resulting GSDoF at each receiver can be concisely written as

$$d'_i = \frac{1 + 2\alpha}{3 + \frac{2\alpha}{\text{GDoF}^{1\alpha/\alpha 1}_{PD/DP}} + \frac{\alpha}{\text{GSDoF}_{\text{common}}}}, \quad i = 1, 2 \tag{16}$$

where $\text{GSDoF}_{\text{common}}$ denotes the GSDoF of the common message c. Note that, three symbols are send to each receiver, where only $(1 + 2\alpha)\log(\rho)$ bits can be decoded.

In what follows, we provide the description of the coding scheme that conveys a common message to both receivers securely.

B.2) Multicasting common information: In this scheme, the transmitter sends 2 common symbols (c_1, c_2) to both receivers along with two confidential symbols (v_4, w_4) securely. In the first time slots, the transmitter chooses $(P, D, D, 1, \alpha, \alpha)$ state and sends c_1 and v_4 along with artificial noise q_1 as

$$\mathbf{x}_1 = \Big[c_1 + \rho^{-\alpha/2}v_4 \;\; \phi \;\; \phi\Big]^{\mathrm{T}} + \boldsymbol{\Theta}_1 q_1,$$

where $\boldsymbol{\Theta}_1 \in \mathbb{C}^{3\times 1}$ is the precoding vector chosen such that $\mathbf{h}_1\boldsymbol{\Theta}_1 = 0$. The channel input-output relationship is given by

$$y_1 = \underbrace{\sqrt{\rho}h_{11}c_1}_{\mathcal{O}(\rho)} + \underbrace{\sqrt{\rho^{1-\alpha}}h_{11}v_4}_{\mathcal{O}(\rho^{1-\alpha})}, \tag{17a}$$

$$\acute{y}_1 = \underbrace{\sqrt{\rho^{\alpha}}\acute{h}_{11}c_1 + \sqrt{\rho^{\alpha}}\acute{\mathbf{h}}_1\boldsymbol{\Theta}_1 q_1}_{\mathcal{O}(\rho^{\alpha})} + \underbrace{\sqrt{\rho^{0}}\acute{h}_{11}v_4}_{\mathcal{O}(\rho^{0})}, \tag{17b}$$

$$z_1 = \underbrace{\sqrt{\rho^{\alpha}}g_{11}c_1 + \sqrt{\rho^{\alpha}}\mathbf{g}_1\boldsymbol{\Theta}_1 q_1}_{\text{side information } \mathcal{O}(\rho^{\alpha})} + \underbrace{\sqrt{\rho^{0}}g_{11}v_4}_{\mathcal{O}(\rho^{0})}. \tag{17c}$$

At the end of time slot 1, the receiver 1 can recover c_1 and v_4. Receiver 2 gets c_1 embedded in with artificial noise q_1 and requires one extra equation available as side information at the eavesdropper (z_1).

In the second time slot, the transmitter chooses $(D, P, D, \alpha, 1, \alpha)$ state and sends c_2 and w_4 along with artificial noise q_2 as

$$\mathbf{x}_2 = \Big[c_2 + \rho^{-\alpha/2}w_4 \;\; \phi \;\; \phi\Big]^{\mathrm{T}} + \boldsymbol{\Theta}_2 q_2, \tag{18}$$

where $\boldsymbol{\Theta}_2 \in \mathbb{C}^{3\times 1}$ is the precoding vector chosen such that $\acute{\mathbf{h}}_2\boldsymbol{\Theta}_2 = 0$. The channel input-output relationship is given by

$$y_2 = \underbrace{\sqrt{\rho^{\alpha}}h_{21}c_2 + \sqrt{\rho^{\alpha}}\mathbf{h}_2\boldsymbol{\Theta}_2 q_2}_{\mathcal{O}(\rho^{\alpha})} + \underbrace{\sqrt{\rho^{0}}h_{21}w_4}_{\mathcal{O}(\rho^{0})}, \tag{19a}$$

$$\acute{y}_2 = \underbrace{\sqrt{\rho}\acute{h}_{21}c_2}_{\mathcal{O}(\rho)} + \underbrace{\sqrt{\rho^{1-\alpha}}\acute{h}_{21}w_4}_{\mathcal{O}(\rho^{0})}, \tag{19b}$$

$$z_2 = \underbrace{\sqrt{\rho^{\alpha}}g_{21}c_2 + \sqrt{\rho^{\alpha}}\mathbf{g}_2\boldsymbol{\Theta}_2 q_2}_{\text{side information } \mathcal{O}(\rho^{\alpha})} + \underbrace{\sqrt{\rho^{0}}g_{21}w_4}_{\mathcal{O}(\rho^{0})}. \tag{19c}$$

Similar to time slot 1, at the end of time slot 2, the receiver 2 can readily recover c_2 and w_4. Receiver 1 gets c_2 embedded in with artificial noise q_2 and requires one extra equation available as side information at the eavesdropper (z_2).

At the end of two time slots, both receivers require one extra equation available as side information at the eavesdropper with $\mathcal{O}(\rho^\alpha)$. These side informations are send in the spirit of Theorem 2, over a total of $\frac{2\alpha}{\text{GDoF}^{1\alpha/\alpha 1}_{PD/DP}} = \frac{6\alpha}{3+2\alpha}$ time slots. Thus, 2 common symbols (2α) are securely send to each receiver over a total of $2+\frac{6\alpha}{3+2\alpha}$ time slots yielding a common GSDoF of

$$\begin{aligned}\text{GSDoF}_{\text{common}} &= \frac{2\alpha}{2+\frac{6\alpha}{3+2\alpha}} \\ &= \frac{2\alpha(3+2\alpha)}{2(3+2\alpha)+6\alpha}. \end{aligned} \tag{20}$$

Note, that in conveying two common messages to both receivers, the transmitter can also send two confidential symbols (v_4, w_4) *securely*, since the eavesdropper receives both of these symbols below noise floor and can not decode them. The over all GSDoF is then given by

$$d_i = d_i' + \frac{(1-\alpha)/2}{3+\frac{2\alpha}{\text{GDoF}^{1\alpha/\alpha 1}_{PD/DP}}+\frac{\alpha}{\text{GSDoF}_{\text{common}}}}, \quad i=1,2. \tag{21}$$

Finally replacing (20) in (21) yields

$$d_i = \frac{(3+3\alpha)(3+2\alpha)}{2(12+17\alpha)}, \quad i=1,2. \tag{22}$$

2) S_2 — Coding scheme using $(P,D,D,1,\alpha,\alpha)$ and $(D,P,D,\alpha,1,\alpha)$ states: The coding scheme in this case follows along similar lines as mentioned above by reversing the roles of receiver 1 and receiver 2, and is omitted for brevity.

Appendix I
Coding Scheme Achieving $(\frac{3+2\alpha}{6}, \frac{3+2\alpha}{6})$ GDoF Pair in Theorem 2

We now provide the proof of the coding scheme which gives the GDoF pair $(\frac{3+2\alpha}{6}, \frac{3+2\alpha}{6})$. In this scheme, the transmitter uses 6 time slots to send 8 symbols $\{a_i\}_{i=1}^8$ to receiver 1 and $\{b_i\}_{i=1}^8$ to receiver 2, respectively. In time slots 1, 3, 5, the transmitter chooses $(P,D,1,\alpha)$ state and in time slots 2, 4, 6, it chooses $(D,P,\alpha,1)$ state. In the first time slot, the transmitter sends (a_1,a_2,a_3) along with (b_1) as

$$\mathbf{x}_1 = [a_1 \;\; a_2]^{\mathrm{T}} + \left[a_3\rho^{-\alpha/2} \;\; \phi\right]^{\mathrm{T}} + \Theta_1 b_1,$$

where $\Theta_1 \in \mathbb{C}^{2\times 1}$ is the precoding vector chosen such that $\mathbf{h}_1\Theta_1 = 0$. The channel input-output relationship is given by

$$y_1 = \underbrace{\sqrt{\rho}L_1(a_1,a_2)}_{\mathcal{O}(\rho)} + \underbrace{\sqrt{\rho^{1-\alpha}}h_{11}a_3}_{\mathcal{O}(\rho^{1-\alpha})}, \tag{23a}$$

$$\acute{y}_1 = \underbrace{\sqrt{\rho^\alpha}L_2(a_1,a_2)}_{\text{side information } \mathcal{O}(\rho^\alpha)} + \underbrace{\sqrt{\rho^0}\acute{h}_{11}a_3}_{\mathcal{O}(\rho^0)} + \underbrace{\sqrt{\rho^\alpha}\acute{\mathbf{h}}_1\Theta_1 b_1}_{\mathcal{O}(\rho^\alpha)}. \tag{23b}$$

At the end of time slot 1, the receiver 1 gets a linear combination of (a_1,a_2) denoted by $L_1(a_1,a_2)$ along with symbol a_3. The receiver 1 can readily recover a_3 by treating $L_1(a_1,a_2)$ as noise with $(1-\alpha)\log(\rho)$ bits and requires one extra equation, $L_2(a_1,a_2)$, being available as side information at receiver 2 to decode the symbols (a_1,a_2). Receiver 2 gets the desired symbol embedded in with interference $L_2(a_1,a_2)$. Notice that, conveying this interference to both receivers is useful in two ways, 1) it provides the side information to receiver 1 to decode the symbols (a_1,a_2), and 2) also helps the receiver 2 to remove the contribution of $L_2(a_1,a_2)$ from $\acute{y}_1$ to recover b_1.

In the second time slot the transmission scheme is similar to time slot 1 with the roles of receiver 1 and 2 being reversed. The transmitter sends,

$$\mathbf{x}_2 = [b_2 \;\; b_3]^{\mathrm{T}} + \left[b_4\rho^{-\alpha/2} \;\; \phi\right]^{\mathrm{T}} + \Theta_2 a_4,$$

where $\Theta_2 \in \mathbb{C}^{2\times 1}$ is the precoding vector chosen such that $\acute{\mathbf{h}}_2\Theta_2 = 0$. The channel input-output relationship is given by

$$y_2 = \underbrace{\sqrt{\rho^\alpha}L_3(b_2,b_3)}_{\text{side information } \mathcal{O}(\rho^\alpha)} + \underbrace{\sqrt{\rho^0}h_{21}b_4}_{\mathcal{O}(\rho^0)} + \underbrace{\sqrt{\rho^\alpha}\mathbf{h}_2\Theta_2 a_4}_{\mathcal{O}(\rho^\alpha)}, \tag{24a}$$

$$\acute{y}_2 = \underbrace{\sqrt{\rho}L_4(b_2,b_3)}_{\mathcal{O}(\rho)} + \underbrace{\sqrt{\rho^{1-\alpha}}\acute{h}_{21}b_4}_{\mathcal{O}(\rho^{1-\alpha})}. \tag{24b}$$

At the end of time slot 2, the receiver 2 gets a linear combination of (b_2,b_3) along with symbol b_4. The receiver 2 can decode b_4 by treating $L_4(b_2,b_3)$ as noise with $(1-\alpha)\log(\rho)$ bits; and, requires one extra equation, $L_3(b_2,b_3)$, available as side information at receiver 1 to decode the symbols (b_2,b_3). Receiver 1 gets the symbol a_4 embedded in with interference $L_3(b_2,b_3)$. Thus, by conveying $L_3(b_2,b_3)$ to both receivers will suffice to decode the desired symbols.

At the end of time slot 2, the transmitter can *learn* the side informations $L_2(a_1,a_2)$ and $L_3(b_2,b_3)$ by means of past CSI; and, the next step is to communicate them for interference alignment. Recall that, both side informations are available at power level $\mathcal{O}(\rho^\alpha)$. After constructing the side informations, the transmitter quantizes $L_2(a_1,a_2)$ and $L_3(b_2,b_3)$ into $2\alpha\log(\rho)+o(\log\rho)$ bits within bounded noise distortion, which are then mapped to a common symbol $\{c_i\}$ where $c_i \in \mathcal{C} = \{1,\dots,\rho^\alpha\}$ $\forall i \in \{1,2\}$ and sends

$$\mathbf{x}_3 = \left[c_1 + \rho^{-\alpha/2}a_5 \;\; \phi\right]^{\mathrm{T}} + \Theta_3 b_5,$$

where $\Theta_2 \in \mathbb{C}^{3\times 1}$ is the precoding vector chosen such that $\mathbf{h}_3\Theta_3 = 0$. The channel input-output relationship is given by

$$y_3 = \underbrace{\sqrt{\rho}h_{31}c_1}_{\mathcal{O}(\rho)} + \underbrace{\sqrt{\rho^{1-\alpha}}h_{31}a_5}_{\mathcal{O}(\rho^{1-\alpha})}, \tag{25a}$$

$$\acute{y}_3 = \underbrace{\sqrt{\rho^\alpha}\acute{h}_{31}c_1}_{\mathcal{O}(\rho^\alpha)} + \underbrace{\sqrt{\rho^0}\acute{h}_{31}a_5}_{\mathcal{O}(\rho^0)} + \underbrace{\sqrt{\rho^\alpha}\acute{\mathbf{h}}_3\Theta_3 b_5}_{\mathcal{O}(\rho^\alpha)}. \tag{25b}$$

At the end of time slot 3, the receiver 1 first decodes a_5, by treating c_1 as noise, which contains $(1-\alpha)\log(\rho)$ bits. Afterwards, it recovers c_1 by subtracting the contribution of a_5 from y_3. Next, after decoding c_1 the receiver 1 can reconstruct $L_2(a_1,a_2)$ within bounded noise. Finally, with $(y_1, L_2(a_1,a_2))$ it decodes (a_1,a_2) with 2α bits via channel inversion. Receiver 2, gets b_5 along with common symbol c_1.

In the fourth time slot, the transmission scheme follows along similar lines as in time slot 3, by interchanging the roles of

Bits	Receiver 1	Receiver 2
$\alpha\log(\rho)$	a_1, a_2, a_4, a_6, a_8	b_1, b_2, b_3, b_5, b_7
$(1-\alpha)\log(\rho)$	a_3, a_5, a_7	b_4, b_6, b_8

TABLE I
INFORMATION CONTENT IN EACH SYMBOL.

receiver 1 and receiver 2. The transmitter sends common symbol c_2 along with new symbols a_6 and b_6 to receiver 1 and 2 as,

$$\mathbf{x}_4 = \left[c_2 + \rho^{-\alpha/2} b_6 \ \ \phi\right]^{\mathrm{T}} + \Theta_4 a_6,$$

where $\Theta_4 \in \mathbb{C}^{2\times 1}$ is the precoding vector chosen such that $\acute{\mathbf{h}}_4\Theta_4 = 0$. The channel input-output relationship is given by

$$y_4 = \underbrace{\sqrt{\rho^{\alpha}} h_{41} c_2}_{\mathcal{O}(\rho^{\alpha})} + \underbrace{\sqrt{\rho^{0}} h_{41} b_6}_{\mathcal{O}(\rho^{0})} + \underbrace{\sqrt{\rho^{\alpha}} \mathbf{h}_4 \Theta_4 a_6}_{\mathcal{O}(\rho^{\alpha})}, \tag{26a}$$

$$\acute{y}_4 = \underbrace{\sqrt{\rho} \acute{h}_{41} c_2}_{\mathcal{O}(\rho)} + \underbrace{\sqrt{\rho^{1-\alpha}} \acute{h}_{41} b_6}_{\mathcal{O}(\rho^{1-\alpha})}. \tag{26b}$$

At the end of time slot 4, the receiver 2 can decode both b_6 and c_2. After recovering c_2, with $(\acute{y}_2, L_3(b_2, b_3))$ the receiver 2 can then decode (b_2, b_3) with $2\alpha\log(\rho)$ bits.

At the end of fourth time slot, receiver 1 requires c_2 to decode a_6 and receiver 2 requires c_1 to decode b_5. In the fifth time slot, the transmitter sends common symbol c_2 along with new symbols a_7 and b_7 to receiver 1 and 2 as,

$$\mathbf{x}_5 = \left[c_2 + \rho^{-\alpha/2} a_7 \ \ \phi\right]^{\mathrm{T}} + \Theta_5 b_7,$$

where $\Theta_5 \in \mathbb{C}^{2\times 1}$ is the precoding vector chosen such that $\mathbf{h}_5\Theta_5 = 0$. The channel input-output relationship is given by

$$y_5 = \underbrace{\sqrt{\rho} h_{51} c_2}_{\mathcal{O}(\rho)} + \underbrace{\sqrt{\rho^{1-\alpha}} h_{51} a_7}_{\mathcal{O}(\rho^{1-\alpha})}, \tag{27a}$$

$$\acute{y}_5 = \underbrace{\sqrt{\rho^{\alpha}} \acute{h}_{51} c_2}_{\mathcal{O}(\rho^{\alpha})} + \underbrace{\sqrt{\rho^{0}} \acute{h}_{51} a_7}_{\mathcal{O}(\rho^{0})} + \underbrace{\sqrt{\rho^{\alpha}} \acute{\mathbf{h}}_5 \Theta_5 b_7}_{\mathcal{O}(\rho^{\alpha})}. \tag{27b}$$

At the end of fifth time slot, the receiver 1 can readily recover c_2 and a_7 from y_5. After decoding c_2, the receiver 1 can construct $L_3(b_2, b_3)$ within bounded noise and can remove its contribution from (24a), (26a) to decode a_4 and a_6 respectively. Similarly, since receiver 2 knows c_2 from (26b), it can subtracts out the contribution of c_2 from (27b) to decode b_7.

In the sixth time slot, the transmitter sends common symbol c_1 along with new symbols a_8 and b_8 to receiver 1 and 2 as,

$$\mathbf{x}_6 = \left[c_1 + \rho^{-\alpha/2} b_8 \ \ \phi\right]^{\mathrm{T}} + \Theta_6 a_8,$$

where $\Theta_6 \in \mathbb{C}^{2\times 1}$ is the precoding vector chosen such that $\acute{\mathbf{h}}_6\Theta_6 = 0$. The channel input-output relationship is given by

$$y_6 = \underbrace{\sqrt{\rho^{\alpha}} h_{61} c_1}_{\mathcal{O}(\rho^{\alpha})} + \underbrace{\sqrt{\rho^{0}} h_{61} b_8}_{\mathcal{O}(\rho^{0})} + \underbrace{\sqrt{\rho^{\alpha}} \mathbf{h}_6 \Theta_6 a_8}_{\mathcal{O}(\rho^{\alpha})}, \tag{28a}$$

$$\acute{y}_6 = \underbrace{\sqrt{\rho} \acute{h}_{61} c_1}_{\mathcal{O}(\rho)} + \underbrace{\sqrt{\rho^{1-\alpha}} \acute{h}_{61} b_8}_{\mathcal{O}(\rho^{1-\alpha})}. \tag{28b}$$

At the end of sixth time slot, since the receiver 1 knows c_1 from (25a), it subtracts out the contribution of c_1 from (28a) to decode a_8. Similarly from (28b), the receiver 2 can recover c_1 and b_8. Next, it subtracts out the contribution of c_1 from (23b), (25b) to decode b_1 and b_5.

Thus, at the end of 6 time slots all the symbols are decoded at both receivers. Table I summarizes the information content in each symbol send by transmitter to both receivers which yields a GDoF pair $(\frac{3+2\alpha}{6}, \frac{3+2\alpha}{6})$.

This concludes the proof.

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