Energy Efficient Full Duplex Massive MIMO Multi-carrier Bidirectional Communication with Hardware Impairments

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***Abstract*—In this paper, we address the power allocation problem for a bi-directional communication system, where a full duplex (FD) massive multiple-input-multiple-output (mMIMO) multi-carrier (MC) node communicates with multiple FD MC single antenna nodes. We consider orthogonal frequency division multiplexing (OFDM) as our MC strategy. The impact of hardware distortions resulting in residual self-interference and inter-carrier leakage (ICL), and also imperfect channel state information (CSI) are jointly taken into account. We formulate a joint sub-carrier and power allocation problem to maximize the spectral efficiency (sum rate maximization) and an iterative optimization method is proposed, which follows successive inner approximation (SIA) framework to reach the convergence point that satisfies the Karush–Kuhn–Tucker (KKT) conditions. Then, we extend it to an energy efficiency (EE) maximization problem which is solved using a two stage iterative algorithm which follows the SIA and Dinkelbach algorithm. Numerical results show the significance of distortion aware design for such systems and also the significant gain in terms of sum rate and energy efficiency compared to its half duplex (HD) counterpart.**

***Index Terms*—Massive MIMO, Full duplex, Multi-carrier, Power allocation, Imperfect CSI**

I. Introduction

Full duplex and massive MIMO are two of the most promising technologies that are in consideration for future communication systems, to tackle capacity crunch and spectrum scarcity. In an FD system the transceiver nodes are allowed to transmit and receive at the same frequency-time channel, which is not the case in the current traditional wireless systems, where the transmission and reception at a transceiver node are isolated either in time or frequency. This simultaneous transmission and reception helps to improve the spectral efficiency of the system compared to an HD system [1]. The main drawback of such a system is that it suffers from self-interference because the transceiver node receives its own transmitted signal which is the same frequency-time channel to that of the desired receive signal. In recent years, various techniques [2]–[4] were developed in order to mitigate this self-interference and some studies are conducted in this regards [5], [6]. The main idea is to remove the known transmitted signal from the received ones. This is difficult because of the imperfect transmitter/receiver chain components, ageing of the components, imperfect knowledge of the self interference channel, etc.

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In an mMIMO communication system, the nodes are equipped with a large number of antennas, which provides large spatial diversity resulting in improved spectral efficiency. In these systems, less power is required to transmit data as the antennas can operate in conjunction with each other to improve the gain of transmitted signals at the receiver, which makes it a more energy efficient system. Due to the large number of antenna arrays, the hardware cost for such a system becomes expensive. In order to reduce the cost, the inexpensive or less efficient transmitter/receiver chain components such as low-resolution ADC, DAC [7], low-cost power amplifiers are preferred. Usage of these less efficient components and their ageing over time will introduce more hardware distortion to the system. In particular, for an FD MC system these non-linear hardware distortions leads to inter-carrier leakage, i.e., a higher residual self-interference is introduced in all of the subcarrier channels even one of the subcarriers is employed with a high-power transmission. So, it is essential to have distortion-aware design which considers the distortions caused by the hardware impairments especially in an FD mMIMO system.

In [8]–[10], the resource allocation is addressed for an FD MC system while considering single antenna transceivers. A Linear precoder and decoder (receive filter) is designed to enhance system performance following minimum-mean-squared-error (MMSE) and sumrate maximization strategies for a bidirectional FD MIMO OFDM system [11], for an FD MIMO MC decode-and-forward relay in [12]. In recent years, some studies were done in the context of FD mMIMO cellular networks [13]–[15], but there is no consideration of hardware distortions. In case of FD massive MIMO relay with consideration of hardware impairments, the resource allocation problem is addressed in [16], [17], however assuming single carrier systems.

In this paper, we investigate an MC bidirectional communication between an FD mMIMO transceiver base-station and multiple FD user nodes with single antenna, where residual interference due to the impact of hardware distortions, inter carrier leakage and imperfect CSI are taken into account. In Section II, we model the operation of a bidirectional OFDM communication system and formulate the impact of imperfect CSI as well as the impact of hardware distortions. In Section III, we frame an optimization problem to maximize the system sum-rate, which falls into the class of smooth difference-of-convex (DC) optimization problems. We propose an SIA iterative optimization solution, which converges to a point that satisfies KKT conditions. We also extend it to an EE maximization problem and solve it using SIA and Dinkelbach algorithm [18]. In Section IV, we evaluate the performance of our proposed algorithms for both the EE and sum-rate maximization problem using numerical simulations. It is observed that, by considering distortion-aware design a significant gain can be achieved, especially in the high transceiver inaccuracy scenario. In section V, we summarize our main results.

A. Mathematical Notation

Throughout this paper, we denote the vectors and matrices by lower-case and upper-case bold letters, respectively. We use $\mathbb{E}\{.\}$, tr$(.)$, $(.)^{-1}$, $(.)^*$, $(.)^T$, and $(.)^H$ for mathematical expectation, trace, inverse, conjugate, transpose, and Hermitian transpose, respectively. We use diag$(.)$ for the diag operator, which returns a diagonal matrix by setting off-diagonal elements to zero. We denote an all zero matrix of size $m \times n$ by $\mathbf{0}_{m\times n}$. We represent the Euclidean norm as $\|.\|_2$. We denote the set of real, positive real, and complex numbers as $\mathbb{R}$, $\mathbb{R}^+$, and $\mathbb{C}$ respectively. We use $|.|$ for the cardinality of a set.

II. System Model

We consider an asymmetrical MC bidirectional communication setup, where an mMIMO FD BS communicates with L FD single antenna user nodes. The number of transmit and receive antennas at the BS node is represented as N_{BS} and M_{BS}, respectively. We denote the index set of all user nodes and subcarriers by $\mathbb{N}$ and $\mathbb{K}$, respectively. Let us define a set of nodes including the BS as $\tilde{\mathbb{N}} = \{\mathbb{N} \cup 0\}$, where 0 is the index of the BS. Furthermore, $\mathbf{h}_{0i}^k \in \mathbb{C}^{M_{\text{BS}}}$ and $\mathbf{h}_{i0}^k \in \mathbb{C}^{1\times N_{\text{BS}}}$ represent the k-th subcarrier uplink and downlink channel, respectively. $h_{ii} \in \mathbb{C}^1$ is the self interference of the user node $i \in \mathbb{N}$. The self interference channel of the BS is denoted by $\mathbf{H}_{00}^k \in \mathbb{C}^{M_{\text{BS}}\times N_{\text{BS}}}$. $h_{ij}^k \in \mathbb{C}^1$ represents the co-channel interference channel from the j-th node to the i-th node, when $i \neq j$.

In this work, we assume all the channels are constant for each frame, frequency-flat in each subcarrier and only the imperfect CSI is known. We adapt the channel error model used in [19], [20], where the true channel can be decomposed into the estimated channel plus estimation error, can be stated as

$$\begin{aligned} \mathbf{h}_{i0}^k &= \hat{\mathbf{h}}_{i0}^k + \tilde{\mathbf{h}}_{i0}^k, \;\; \hat{\mathbf{h}}_{i0}^k \perp \tilde{\mathbf{h}}_{i0}^k, \\ \mathbf{h}_{0i}^k &= \hat{\mathbf{h}}_{0i}^k + \tilde{\mathbf{h}}_{0i}^k, \;\; \hat{\mathbf{h}}_{i0}^k \perp \tilde{\mathbf{h}}_{i0}^k, \\ h_{ij}^k &= \hat{h}_{ij}^k + \tilde{h}_{ij}^k, \;\; \hat{h}_{ij}^k \perp \tilde{h}_{ij}^k, \\ \mathbf{H}_{00}^k &= \hat{\mathbf{H}}_{00}^k + \tilde{\mathbf{H}}_{00}^k, \;\; \hat{\mathbf{H}}_{00}^k \perp \tilde{\mathbf{H}}_{00}^k, \;\; \forall i,j \in \mathbb{N}, \forall k \in \mathbb{K}, \end{aligned} \tag{1}$$

where $\hat{\mathbf{h}}_{i0}^k, \hat{\mathbf{h}}_{0i}^k, \hat{h}_{ij}^k$ and $\hat{\mathbf{H}}_{00}^k$ represents the estimated channels. The entries of channel estimation error $\tilde{\mathbf{h}}_{i0}^k, \tilde{\mathbf{h}}_{0i}^k, \tilde{h}_{ij}^k$ and $\tilde{\mathbf{H}}_{00}^k$ are i.i.d. complex Gaussian with zero mean and variance $(\sigma_{e,i0}^k)^2, (\sigma_{e,0i}^k)^2, (\sigma_{e,ij}^k)^2$ and $(\sigma_{e,00}^k)^2$, respectively. We consider the receiver performs channel estimation which allows us to assume the estimated channel and estimation error become statistically uncorrelated, for example, via the minimum mean square error (MMSE) channel estimation strategy.

The source symbol to BS from the i-th user node using the k-th subcarrier can be represented as $s_{i,k}^{\text{UL}} \in \mathbb{C}^1$. Whereas $s_{i,k}^{\text{DL}} \in \mathbb{C}^1$ denotes the source symbol from the BS to the i-th node. We assume the symbols are independent and identically distributed (i.i.d.) with unit power, i.e. $\mathbb{E}\{s_{i,k}^{\text{UL}}(s_{i,k}^{\text{UL}})^*\} = 1$ and $\mathbb{E}\{s_{i,k}^{\text{DL}}(s_{i,k}^{\text{DL}})^*\} = 1$. Let $\mathbf{v}_i^k = \tilde{\mathbf{v}}_i^k\sqrt{p_{i,k}^{\text{DL}}}$ and $\mathbf{u}_i^k$ represent the transmit precoders and receive decoders (linear filters) at the BS for the i-th node, respectively, $\tilde{\mathbf{v}}_i^k$ denotes the normalised transmit precoders and $i \in \mathbb{N}$. The transmit power of i-th node is denoted by $p_{i,k}^{\text{UL}}$, where $i \in \mathbb{N}$. The transmit power dedicated to the i-th node for downlink at the BS is $p_{i,k}^{\text{DL}}$ and the total transmit power at the BS is p_0. The transmit signal of the i-th user node and the BS can be written as

$$\begin{aligned} x_i^k &= \underbrace{\sqrt{p_{i,k}^{\text{UL}}}(s_{i,k}^{\text{UL}})}_{:=\tilde{x}_i^k} + e_{\text{t},i}^k, \;\; i \in \mathbb{N}, \forall k \in \mathbb{K}, \\ \mathbf{x}_0^k &= \underbrace{\sum_i \tilde{\mathbf{v}}_i^k \sqrt{p_{i,k}^{\text{DL}}}(s_{i,k}^{\text{DL}})}_{:\tilde{\mathbf{x}}_0^k} + \mathbf{e}_{\text{t},0}^k, \;\; i \in \mathbb{N}, \forall k \in \mathbb{K}, \end{aligned} \tag{2}$$

where $e_{\text{t},i}^k$ and $\mathbf{e}_{\text{t},0}^k$ are the transmitter distortion at the i-th node and BS respectively. The $\tilde{x}_i^k$ and $\tilde{\mathbf{x}}_0^k$ represents the intended transmit signal at the i-th node and BS, respectively.

Correspondingly, the received signal at the i-th node and BS can be obtained as

$$\begin{aligned} y_i^k &= \underbrace{\mathbf{h}_{i0}^k \mathbf{x}_0^k + \sum_{j\in\mathbb{N}} h_{ij}^k x_j^k}_{:=\tilde{y}_i^k} + n_i^k + e_{\text{r},i}^k, \;\; i \in \mathbb{N}, \forall k \in \mathbb{K}, \\ \mathbf{y}_0^k &= \underbrace{\sum_{i\in\mathbb{N}} \mathbf{h}_{0i}^k x_i^k + \mathbf{H}_{00}^k \mathbf{x}_0^k}_{:=\tilde{\mathbf{y}}_0^k} + \mathbf{n}_0^k + \mathbf{e}_{\text{r},0}^k, \;\; i \in \mathbb{N}, \forall k \in \mathbb{K}, \end{aligned} \tag{3}$$

where $e_{\text{r},i}^k$ and $\mathbf{e}_{\text{r},0}^k$ are the receiver distortion at the i-th node and BS respectively, and $n_i^k \sim \mathcal{CN}\left(0, (\sigma_{n_i}^k)^2\right)$ and $\mathbf{n}_0^k \sim \mathcal{CN}\left(0_{M_{\text{BS}}}, (\sigma_{n_0}^k)^2 \mathbf{I}_{M_{\text{BS}}}\right)$ are the noise at the i-th node and BS respectively. The intended received signal at the i-th node and BS is denoted by $\tilde{y}_i^k$ and $\tilde{\mathbf{y}}_0^k$, respectively. The SIC is applied to the received signal, i.e., only the known part of the transmit signal can be mitigated. The received signal after SIC can be expressed as

$$\begin{aligned} \bar{y}_i^k &= y_i^k - \hat{h}_{ii}^k \sqrt{p_{i,k}^{\text{UL}}}(s_{i,k}^{\text{UL}}), \;\; i \in \mathbb{N}, \forall k \in \mathbb{K}, \\ \bar{\mathbf{y}}_0^k &= \mathbf{y}_0^k - \sum_{i\in\mathbb{N}} \hat{\mathbf{H}}_{00}^k \tilde{\mathbf{v}}_i^k \sqrt{p_{i,k}^{\text{DL}}}(s_{i,k}^{\text{DL}}), \;\; i \in \mathbb{N}, \forall k \in \mathbb{K},. \end{aligned} \tag{4}$$

The inaccuracy of hardware components such as ADC and DAC error, noises caused by power amplifiers, AGC and oscillator on transmit and receive chain are jointly modelled for FD MIMO transceiver in [21], [22], based on [23]–[26]. The distortion terms are proportional to the intensity of the intended signals. In this work, we consider OFDM as our MC strategy. Hence, the impact of these hardware distortions in the frequency domain can be characterised as in [11]:

Lemma II.1. *Let's define $\tilde{x}_l^m$ and $\tilde{y}_l^m$ as the intended transmit and receive signal via m-th subcarrier at the l-th transmit/receive chain. The impact of hardware distortions in the frequency domain is characterized as*

$$e_{t,l}^k \sim \mathcal{CN}\left(0, \frac{\tilde{\kappa}_l}{K}\sum_{m\in\mathbb{F}_K}\mathbb{E}\left\{|\tilde{y}_l^m|^2\right\}\right),\;\; e_{t,l}^k \perp \tilde{y}_l^k,\;\; e_{t,l}^k \perp e_{t,l'}^k, \tag{5}$$

$$e_{r,l}^k \sim \mathcal{CN}\left(0, \frac{\tilde{\beta}_l}{K}\sum_{m\in\mathbb{F}_K}\mathbb{E}\left\{|\tilde{x}_l^m|^2\right\}\right),\;\; e_{r,l}^k \perp \tilde{x}_l^k,\;\; e_{r,l}^k \perp e_{r,l'}^k, \tag{6}$$

transforming the statistical independence, as well as the proportional variance properties from the time domain. Here, K represents the total number of subcarriers. The $\tilde{\kappa}_l$ and $\tilde{\beta}_l$ corresponds to the transmit and receive distortion coefficient at the l-th transmit/receive chain.

Proof. Please refer to the appendix of [11] □

We use a similar model for the transmit and receiver distortions as that of [11], [12]. The statistics of the distortion terms can be written as

$$\mathbf{e}_{t,i}^k \sim \mathcal{CN}\left(0, \frac{\tilde{\kappa}_i}{K}\sum_{k\in\mathbb{K}}\left(\mathbb{E}\{\tilde{x}_i^k(\tilde{x}_i^k)^H\}\right)\right), \tag{7}$$

$$\mathbf{e}_{t,0}^k \sim \mathcal{CN}\left(\mathbf{0}_{N_{\mathrm{BS}}}, \frac{1}{K}\tilde{\mathbf{\Theta}}_{\mathrm{t},0}\sum_{k\in\mathbb{K}}\mathrm{diag}\left(\mathbb{E}\{\tilde{\mathbf{x}}_0^k(\tilde{\mathbf{x}}_0^k)^H\}\right)\right), \tag{8}$$

$$\mathbf{e}_{r,i}^k \sim \mathcal{CN}\left(0, \frac{\tilde{\beta}_i}{K}\sum_{k\in\mathbb{K}}\left(\mathbb{E}\{\tilde{y}_i^k(\tilde{y}_i^k)^H\}\right)\right), \tag{9}$$

$$\mathbf{e}_{r,0}^k \sim \mathcal{CN}\left(\mathbf{0}_{M_{\mathrm{BS}}}, \frac{1}{K}\tilde{\mathbf{\Theta}}_{\mathrm{r},0}\mathrm{diag}\left(\mathbb{E}\{\tilde{\mathbf{y}}_0^k(\tilde{\mathbf{y}}_0^k)^H\}\right)\right), \tag{10}$$

where the transmit and receiver distortion coefficient of the i-th user node are given by $\tilde{\kappa}_i$ and $\tilde{\beta}_i$, respectively. The diagonal matrices $\tilde{\mathbf{\Theta}}_{\mathrm{t},0}$ and $\tilde{\mathbf{\Theta}}_{\mathrm{r},0}$ consist of transmit (receive) distortion coefficients for the corresponding chains at the BS. For the detailed definition of the used distortion model, please refer to the [11, Section II.A]. The equations (7), (8), (9), and (10) explicitly indicate the impact of the ICL, i.e., that the distortion signal variance at each subcarrier is related to the total distortion power at the corresponding chain.

Let's define $\kappa_i = \frac{\tilde{\kappa}_i}{K}$, $\beta_i = \frac{\tilde{\beta}_i}{K}$, $\mathbf{\Theta}_{\mathrm{t},0} = \frac{1}{K}\tilde{\mathbf{\Theta}}_{\mathrm{t},0}$, and $\mathbf{\Theta}_{\mathrm{r},0} = \frac{1}{K}\tilde{\mathbf{\Theta}}_{\mathrm{r},0}$ for further calculations. By employing Lemma II.1, and Equations (7), (8), (9), and (10) on (4), the covariance of received collective interference-plus-noise signal at the i-th user node can be calculated as

$$\begin{aligned}
\Sigma_i^k \approx &\sum_{\substack{j\in\mathbb{N}\\ j\neq i}}\hat{h}_{ij}^k p_{j,k}^{\mathrm{UL}}(\hat{h}_{ij}^k)^* + \sum_{\substack{j\in\mathbb{N}\\ j\neq i}}\hat{\mathbf{h}}_{i0}^k\tilde{\mathbf{v}}_j^k p_{j,k}^{\mathrm{DL}}(\tilde{\mathbf{v}}_j^k)^H(\hat{\mathbf{h}}_{i0}^m)^H \\
&+ \sum_{j\in\mathbb{N}}\kappa_j\sum_{m\in\mathbb{K}}\hat{h}_{ij}^k p_{j,m}^{\mathrm{UL}}(\hat{h}_{ij}^k)^* + \beta_i\sum_{j\in\mathbb{N}}\sum_{m\in\mathbb{K}}(\hat{h}_{ij}^m p_{j,m}^{\mathrm{UL}}(\hat{h}_{ij}^m)^H) \\
&+ \beta_i\sum_{j\in\mathbb{N}}\sum_{m\in\mathbb{K}}(\hat{\mathbf{h}}_{i0}^m\tilde{\mathbf{v}}_j^m p_{j,m}^{\mathrm{DL}}(\tilde{\mathbf{v}}_j^m)^H(\hat{\mathbf{h}}_{i0}^m)^H) + \beta_i\sum_{m\in\mathbb{K}}(\sigma_{n_i}^m)^2 \\
&+ (\sigma_{n_i}^k)^2 + \sum_{j\in\mathbb{N}}\hat{\mathbf{h}}_{i0}^k\mathbf{\Theta}_{\mathrm{t},0}\mathrm{diag}\left(\sum_{m\in\mathbb{K}}\tilde{\mathbf{v}}_j^m p_{j,m}^{\mathrm{DL}}(\tilde{\mathbf{v}}_j^m)^H\right)(\hat{\mathbf{h}}_{i0}^k)^H \\
&+ \beta_i\sum_{j\in\mathbb{N}}\sum_{m\in\mathbb{K}}(\sigma_{e,ij}^m)^2 p_{j,m}^{\mathrm{UL}} + \sum_{j\in\mathbb{N}}(\sigma_{e,ij}^k)^2\kappa_j\sum_{m\in\mathbb{K}}p_{j,m}^{\mathrm{UL}} \\
&+ \sum_{j\in\mathbb{N}}(\sigma_{e,i0}^k)^2 p_{j,k}^{\mathrm{DL}} + \sum_{j\in\mathbb{N}}(\sigma_{e,ij}^k)^2 p_{j,k}^{\mathrm{UL}} \\
&+ \sum_{j\in\mathbb{N}}(\sigma_{e,i0}^k)^2\mathrm{tr}\left(\mathbf{\Theta}_{\mathrm{t},0}\sum_{m\in\mathbb{K}}\tilde{\mathbf{v}}_j^m p_{j,m}^{\mathrm{DL}}(\tilde{\mathbf{v}}_j^m)^H\right) \\
&+ \beta_i\sum_{j\in\mathbb{N}}\sum_{m\in\mathbb{K}}(\sigma_{e,i0}^m)^2 p_{j,m}^{\mathrm{DL}}.
\end{aligned} \tag{11}$$

Here, we ignore the higher order terms of the transmit and receive distortion since the tranmsit and receive distortion coefficients $\tilde{\kappa}_j$ and $\tilde{\beta}_i$ lie within the range of 0 and 1 and mostly have very small values. The first two terms in the above equation (11), correspond to the co-channel interference. The third term represents the transmit distortion at the usernode j. The next three terms relate to the receive distortion. The seventh term is the thermal noise at the i-th user node. The eighth term corresponds to the transmit distortion at the BS. The remaining terms correspond to interference/noise due to the channel estimation error. Similarly, the covariance of received collective interference-plus-noise signal at the BS for user node i can be expressed as

$$\begin{aligned}
\mathbf{\Sigma}_{0,i}^k \approx &\sum_{\substack{j\in\mathbb{N}\\ j\neq i}}(\hat{\mathbf{h}}_{0j}^k p_{j,m}^{\mathrm{UL}}(\hat{\mathbf{h}}_{0j}^k)^H) + \sum_{j\in\mathbb{N}}\kappa_j\sum_{m\in\mathbb{K}}\hat{\mathbf{h}}_{0j}^k p_{j,m}^{\mathrm{UL}}(\hat{\mathbf{h}}_{0j}^k)^H \\
&+ \sum_{j\in\mathbb{N}}\hat{\mathbf{H}}_{00}^k\mathbf{\Theta}_{\mathrm{t},0}\mathrm{diag}\left(\sum_{m\in\mathbb{K}}\tilde{\mathbf{v}}_j^m p_{j,m}^{\mathrm{DL}}(\tilde{\mathbf{v}}_j^m)^H\right)(\hat{\mathbf{H}}_{00}^k)^H \\
&+ \mathbf{\Theta}_{\mathrm{r},0}\sum_{j\in\mathbb{N}}\mathrm{diag}\left(\sum_{m\in\mathbb{K}}\hat{\mathbf{H}}_{00}^m\tilde{\mathbf{v}}_j^m p_{j,m}^{\mathrm{DL}}(\tilde{\mathbf{v}}_j^m)^H(\hat{\mathbf{H}}_{00}^m)^H\right) \\
&+ \mathbf{\Theta}_{\mathrm{r},0}\sum_{j\in\mathbb{N}}\mathrm{diag}\left(\sum_{m\in\mathbb{K}}\hat{\mathbf{h}}_{0j}^m p_{j,m}^{\mathrm{UL}}(\hat{\mathbf{h}}_{0j}^m)^H\right) + \mathbf{\Theta}_{\mathrm{r},0}\sum_{m\in\mathbb{K}}(\sigma_{n_0}^m)^2 \\
&+ (\sigma_{n_0}^k)^2\mathbf{I}_{M_{\mathrm{BS}}} + \sum_{j\in\mathbb{N}}(\sigma_{e,00}^k)^2 p_{j,k}^{\mathrm{DL}}\mathbf{I}_{M_{\mathrm{BS}}} \\
&+ \sum_{j\in\mathbb{N}}(\sigma_{e,0j}^k)^2 p_{j,k}^{\mathrm{UL}}\mathbf{I}_{M_{\mathrm{BS}}} + \sum_{j\in\mathbb{N}}\kappa_j\sum_{m\in\mathbb{K}}(\sigma_{e,0j}^k)^2 p_{j,m}^{\mathrm{UL}}\mathbf{I}_{M_{\mathrm{BS}}} \\
&+ \sum_{j\in\mathbb{N}}(\sigma_{e,00}^k)^2\mathrm{tr}\left(\mathbf{\Theta}_{\mathrm{t},0}\sum_{m\in\mathbb{K}}\tilde{\mathbf{v}}_j^m p_{j,m}^{\mathrm{DL}}(\tilde{\mathbf{v}}_j^m)^H\right)\mathbf{I}_{M_{\mathrm{BS}}} \\
&+ \mathbf{\Theta}_{\mathrm{r},0}\sum_{j\in\mathbb{N}}\sum_{m\in\mathbb{K}}\left((\sigma_{e,00}^m)^2 p_{j,m}^{\mathrm{DL}} + (\sigma_{e,0j}^m)^2 p_{j,m}^{\mathrm{UL}}\right).
\end{aligned} \tag{12}$$

Here the first term in the above equation (12), represents the co-channel interference. The second term represents the transmit distortion at the user node j. The third term represents the transmit distortion at the BS. The Fourth, fifth and sixth terms correspond to the receive distortion at the BS. The seventh term is the thermal noise at the i-th user node. The remaining terms correspond to interference/noise due to the channel estimation error.

A. Achievable information rate

The achievable information rate for the uplink of the i-th node using subcarrier k can be expressed as

$$R_{i,k,\mathrm{UL}}^{\mathrm{UB}} = \gamma_0 \log_2 \left(1 + \frac{|(\mathbf{u}_i^k)^H \hat{\mathbf{h}}_{0i}^k|^2 p_{i,k}^{\mathrm{UL}}}{\alpha_{n_0}^k + \sum_{m\in\mathbb{K}} \sum_{j\in\mathbb{N}} (\tilde{\gamma}_{ij}^{km} p_{j,m}^{\mathrm{UL}} + \hat{\gamma}_{ij}^{km} p_{j,m}^{\mathrm{DL}})} \right) \tag{13}$$

where

$$\begin{aligned} \tilde{\gamma}_{ij}^{km} =& \delta_{km}(1-\delta_{ij})(\mathbf{u}_i^k)^H (\hat{\mathbf{h}}_{0j}^m (\hat{\mathbf{h}}_{0j}^m)^H)(\mathbf{u}_i^k) \\ &+ \delta_{km}(\mathbf{u}_i^k)^H (\sigma_{e,0j}^m)^2 (\mathbf{u}_i^k) \\ &+ (\mathbf{u}_i^k)^H \kappa_j \hat{\mathbf{h}}_{0j}^k (\hat{\mathbf{h}}_{0j}^k)^H (\mathbf{u}_i^k) + (\mathbf{u}_i^k)^H \kappa_j (\sigma_{e,0j}^k)^2 (\mathbf{u}_i^k) \\ &+ (\mathbf{u}_i^k)^H \boldsymbol{\Theta}_{\mathrm{r},0} \mathrm{diag}(\hat{\mathbf{h}}_{0j}^m (\hat{\mathbf{h}}_{0j}^m)^H)(\mathbf{u}_i^k) \\ &+ (\mathbf{u}_i^k)^H \boldsymbol{\Theta}_{\mathrm{r},0} (\sigma_{e,0j}^m)^2 (\mathbf{u}_i^k), \end{aligned} \tag{14}$$

$$\begin{aligned} \hat{\gamma}_{ij}^{km} =& \delta_{km}(\mathbf{u}_i^k)^H (\sigma_{e,00}^k)^2 (\mathbf{u}_i^k) \\ &+ (\mathbf{u}_i^k)^H \hat{\mathbf{H}}_{00}^k \boldsymbol{\Theta}_{t,0} \mathrm{diag}\big(\tilde{\mathbf{v}}_j^m (\tilde{\mathbf{v}}_j^m)^H\big)(\hat{\mathbf{H}}_{00}^k)^H \mathbf{u}_i^k \\ &+ (\mathbf{u}_i^k)^H (\sigma_{e,00}^k)^2 \mathrm{tr}\big(\boldsymbol{\Theta}_{t,0} \tilde{\mathbf{v}}_j^m (\tilde{\mathbf{v}}_j^m)^H\big)(\mathbf{u}_i^k) \\ &+ (\mathbf{u}_i^k)^H \boldsymbol{\Theta}_{\mathrm{r},0} \mathrm{diag}\big(\hat{\mathbf{H}}_{00}^m \tilde{\mathbf{v}}_j^m (\tilde{\mathbf{v}}_j^m)^H (\hat{\mathbf{H}}_{00}^m)^H\big)(\mathbf{u}_i^k) \\ &+ (\mathbf{u}_i^k)^H \boldsymbol{\Theta}_{\mathrm{r},0} (\sigma_{e,00}^m)^2 (\mathbf{u}_i^k), \end{aligned} \tag{15}$$

$$\alpha_{n_0}^k = \big(\mathbf{u}_i^k\big)^H \big((\sigma_{n_0}^k)^2 \mathbf{I}_{M_{\mathrm{BS}}} + \boldsymbol{\Theta}_{\mathrm{r},0} \sum_{m\in\mathbb{K}} (\sigma_{n_0}^m)^2\big)(\mathbf{u}_i^k) \tag{16}$$

and $\delta_{km} = 1$ when $k = m$ and otherwise $\delta_{km} = 0$.

Similarly, the achievable information rate for the downlink of the i-th node using subcarrier k can be expressed as

$$R_{i,k,\mathrm{DL}}^{\mathrm{UB}} = \gamma_0 \log_2 \left(1 + \frac{|\hat{\mathbf{h}}_{i0}^k \tilde{\mathbf{v}}_i^k|^2 p_{i,k}^{\mathrm{DL}}}{\alpha_{n_i}^k + \sum_{m\in\mathbb{K}} \sum_{j\in\mathbb{N}} \big(\bar{\gamma}_{ij}^{km} p_{j,m}^{\mathrm{UL}} + \acute{\gamma}_{ij}^{km} p_{j,m}^{\mathrm{DL}}\big)} \right) \tag{17}$$

where

$$\begin{aligned} \bar{\gamma}_{ij}^{km} =& \delta_{km}\big((1-\delta_{ij})(\hat{h}_{ij}^m (\hat{h}_{ij}^m)^*) + (\sigma_{e,ij}^m)^2\big) + \kappa_j \hat{h}_{ij}^k (\hat{h}_{ij}^k)^* \\ &+ \kappa_j (\sigma_{e,ij}^k)^2 + \beta_i (\hat{h}_{ij}^m (\hat{h}_{ij}^m)^H) + \beta_i (\sigma_{e,ij}^m)^2, \end{aligned} \tag{18}$$

$$\begin{aligned} \acute{\gamma}_{ij}^{km} =& \delta_{km}(1-\delta_{ij}) \hat{\mathbf{h}}_{i0}^m \tilde{\mathbf{v}}_j^m (\tilde{\mathbf{v}}_j^m)^H (\hat{\mathbf{h}}_{i0}^m)^H + \delta_{km} (\sigma_{e,i0}^m)^2 \\ &+ \hat{\mathbf{h}}_{i0}^k \boldsymbol{\Theta}_{\mathrm{t},0} \mathrm{diag}\big(\tilde{\mathbf{v}}_j^m (\tilde{\mathbf{v}}_j^m)^H\big)(\hat{\mathbf{h}}_{i0}^k)^H \\ &+ (\sigma_{e,i0}^k)^2 \mathrm{tr}\big(\boldsymbol{\Theta}_{\mathrm{t},0} \sum_{j\in\mathbb{N}} \tilde{\mathbf{v}}_j^m (\tilde{\mathbf{v}}_j^m)^H\big) \\ &+ \beta_i (\hat{\mathbf{h}}_{i0}^m \tilde{\mathbf{v}}_j^m (\tilde{\mathbf{v}}_j^m)^H (\hat{\mathbf{h}}_{i0}^m)^H) + \beta_i (\sigma_{e,i0}^m)^2 \end{aligned} \tag{19}$$

and

$$\alpha_{n_i}^k = (\sigma_{n_i}^k)^2 \mathbf{I}_{M_{\mathrm{BS}}} + \beta_i \sum_{m\in\mathbb{K}} (\sigma_{n_i}^m)^2. \tag{20}$$

Since we are using a large antenna array at the BS, there are different well-studied linear precoder-decoder filtering strategies available for the selection of transmit precoders and receive decoders at the BS, such as, maximum ratio transmission/maximum ratio combining (MRT/MRC), zero forcing (ZF), MMSE etc. If some common assumptions from mMIMO studies on the channel covariance matrices such as Hermitian, Teoplitz, etc., as discussed in [17] are assumed, the computational complexity to obtain the achievable rates can be reduced. Now, the upper bound on the total achievable information rate for the i-th user node can be written as

$$R_{i,k}^{\mathrm{UB}} = R_{i,k,\mathrm{UL}}^{\mathrm{UB}} + R_{i,k,\mathrm{DL}}^{\mathrm{UB}}. \tag{21}$$

III. Optimization Problem

In this section, we formulate the joint subcarrier and power allocation optimization problem in terms of sum rate and energy efficiency maximization for a bidirectional FD mMIMO communication system. In our formulation, the subcarrier allocation is incorporated into the power allocation problem. We can consider the user is not transmitting/receiving signal in a given subcarrier, if the power allocated to that subcarrier is zero.

A. Weighted Sum Rate Maximization

The sum rate maximization problem for a FD Bidirectional communication can be formulated as

$$\begin{aligned} \max_{\substack{p_{i,k}^{\mathrm{DL}} > 0, \\ p_{i,k}^{\mathrm{UL}} > 0}} \; & \sum_{i\in\mathbb{N}} w_i \sum_{k\in\mathbb{K}} R_{i,k}^{\mathrm{UB}} \\ \text{subject to} & \sum_{k\in\mathbb{K}} p_{i,k}^{\mathrm{UL}} \le p_i, i \in \mathbb{N} \\ & \sum_{i\in\mathbb{N}} \sum_{k\in\mathbb{K}} p_{i,k}^{\mathrm{DL}} \le p_0, \end{aligned} \tag{22}$$

where $w_i \in \mathbb{R}^+$ is the weight associated to communication with i-th node, p_i and p_0 are the available transmit power at the user node i and the BS. It can be clearly seen that the above optimization problem belongs to the class of smooth difference-of-convex optimization problems. We propose an iterative algorithm that utilizes the SIA framework [27], which reaches a converging point that satisfies the KKT optimality conditions. Let us first select $p_{i,k,\mathrm{o}}^{\mathrm{UL}}$ and $p_{i,k,\mathrm{o}}^{\mathrm{DL}}$ as a feasible transmit power value at the i-th user node and BS, respectively. Next, we use Taylor's approximation on the concave terms of $R_{i,k,\mathrm{UL}}^{\mathrm{UB}}$, and a lower-bound of $R_{i,k,\mathrm{UL}}^{\mathrm{UB}}$, which can be expressed as

$$
\begin{aligned}
R^{\mathrm{UB}}_{i,k,\mathrm{UL}} \geq\ & \gamma_0 \log_2\Bigg(\alpha^k_{n_0} + \sum_{m\in\mathbb{K}}\sum_{j\in\mathbb{N}}\big(\tilde{\gamma}^{km}_{ij} p^{\mathrm{UL}}_{j,m} + \hat{\gamma}^{km}_{ij} p^{\mathrm{DL}}_{j,m}\big) + \\
& |(\mathbf{u}^k_i)^H \hat{\mathbf{h}}^k_{0i}|^2 p^{\mathrm{UL}}_{i,k}\Bigg) \\
& - \gamma_0 \log_2\Bigg(\alpha^k_{n_0} + \sum_{m\in\mathbb{K}}\sum_{j\in\mathbb{N}}\big(\tilde{\gamma}^{km}_{ij} p^{\mathrm{UL}}_{j,m,\mathrm{o}} + \hat{\gamma}^{km}_{ij} p^{\mathrm{DL}}_{j,m,\mathrm{o}}\big)\Bigg) \\
& - \frac{\gamma_0 \sum\limits_{m\in\mathbb{K}}\sum\limits_{j\in\mathbb{N}}\big(\tilde{\gamma}^{km}_{ij}(p^{\mathrm{UL}}_{j,m} - p^{\mathrm{UL}}_{j,m,\mathrm{o}}) + \hat{\gamma}^{km}_{ij}(p^{\mathrm{DL}}_{j,m} - p^{\mathrm{DL}}_{j,m,\mathrm{o}})\big)}{\log(2)\Bigg(\alpha^k_{n_0} + \sum\limits_{m\in\mathbb{K}}\sum\limits_{j\in\mathbb{N}}\big(\tilde{\gamma}^{km}_{ij} p^{\mathrm{UL}}_{j,m} + \hat{\gamma}^{km}_{ij} p^{\mathrm{DL}}_{j,m}\big)\Bigg)} \\
& =: \bar{R}^{\mathrm{UB}}_{i,k,\mathrm{UL}}
\end{aligned} \tag{23}
$$

Similarly, the lower bound $\bar{R}^{\mathrm{UB}}_{i,k,\mathrm{DL}}$ of $R^{\mathrm{UB}}_{i,k,\mathrm{DL}}$, after applying Taylor's approximation, can be written as

$$
\begin{aligned}
R^{\mathrm{UB}}_{i,k,\mathrm{DL}} \geq\ & \gamma_0 \log_2\Bigg(\alpha^k_{n_i} + \sum_{m\in\mathbb{K}}\sum_{j\in\mathbb{N}}\big(\bar{\gamma}^{km}_{ij} p^{\mathrm{UL}}_{j,m} + \acute{\gamma}^{km}_{ij} p^{\mathrm{DL}}_{j,m}\big) + \\
& |\hat{\mathbf{h}}^k_{i0} \tilde{\mathbf{v}}^k_i|^2 p^{\mathrm{DL}}_{i,k}\Bigg) \\
& - \gamma_0 \log_2\Bigg(\alpha^k_{n_i} + \sum_{m\in\mathbb{K}}\sum_{j\in\mathbb{N}}\big(\bar{\gamma}^{km}_{ij} p^{\mathrm{UL}}_{j,m,\mathrm{o}} + \acute{\gamma}^{km}_{ij} p^{\mathrm{DL}}_{j,m,\mathrm{o}}\big)\Bigg) \\
& - \frac{\gamma_0 \sum\limits_{m\in\mathbb{K}}\sum\limits_{j\in\mathbb{N}}\big(\bar{\gamma}^{km}_{ij}(p^{\mathrm{UL}}_{j,m} - p^{\mathrm{UL}}_{j,m,\mathrm{o}}) + \acute{\gamma}^{km}_{ij}(p^{\mathrm{DL}}_{j,m} - p^{\mathrm{DL}}_{j,m,\mathrm{o}})\big)}{\log(2)\Bigg(\alpha^k_{n_i} + \sum\limits_{m\in\mathbb{K}}\sum\limits_{j\in\mathbb{N}}\big(\bar{\gamma}^{km}_{ij} p^{\mathrm{UL}}_{j,m} + \acute{\gamma}^{km}_{ij} p^{\mathrm{DL}}_{j,m}\big)\Bigg)} \\
& =: \bar{R}^{\mathrm{UB}}_{i,k,\mathrm{DL}}
\end{aligned} \tag{24}
$$

Using this approximation, we can write $\bar{R}^{\mathrm{UB}}_{i,k} = \bar{R}^{\mathrm{UB}}_{i,k,\mathrm{UL}} + \bar{R}^{\mathrm{UB}}_{i,k,\mathrm{DL}}$, which is a jointly concave function over $p^{\mathrm{UL}}_{i,k}$ and $p^{\mathrm{DL}}_{i,k}$. We propose an iterative algorithm, where for each iterative update, we now solve the convex problem:

$$
\begin{aligned}
\max_{\substack{p^{\mathrm{DL}}_{i,k}>0,\\ p^{\mathrm{UL}}_{i,k}>0}}\ & \sum_{i\in\mathbb{N}} w_i \sum_{k\in\mathbb{K}} \bar{R}^{\mathrm{UB}}_{i,k} \\
\text{subject to} & \sum_{k\in\mathbb{K}} p^{\mathrm{UL}}_{i,k} \leq p_i, i \in \mathbb{N} \\
& \sum_{i\in\mathbb{N}}\sum_{k\in\mathbb{K}} p^{\mathrm{DL}}_{i,k} \leq p_0.
\end{aligned} \tag{25}
$$

to optimality. The iterative update is continued until a stable point is reached. Since we use a first order Taylor approximation on a smooth convex function, we can conclude that $\bar{R}^{\mathrm{UB}}_{i,k}$ represents a global and tight lower bound to $R^{\mathrm{UB}}_{i,k}$, with a shared slope at the point of approximation [28]. The proposed iterative update also fulfils the requirements set in [27, Theorem 1], so that the solution can achieve a convergence point that satisfies KKT conditions. Algorithm 1 defines the detailed algorithm procedure.

Algorithm 1 For sum rate maximization

```
a ← 0 (set iteration number to zero)
p^UL_{i,k,o}, p^DL_{i,k,o} ← uniform (equal) power initialization
repeat
    a ← a + 1
    p^UL_{i,k}, p^DL_{i,k} ← solve (25)
    p^UL_{i,k,o}, p^DL_{i,k,o} ← p^UL_{i,k,o}=p^UL_{i,k} and p^DL_{i,k,o}=p^DL_{i,k}, respectively
until a stable point, or maximum number of a reached
return {p^UL_{i,k}, p^DL_{i,k}}
```

B. Energy Efficiency Maximization

Due to the rapid expansion of wireless networks, ecological concerns and economical benefits makes energy efficiency (EE) an important metric for designing a future wireless system. In this section, the EE is defined as the ratio of the spectral efficiency (sumrate) to the total power consumption of all the user nodes and BS. The total power consumption P_{tot} can be expressed as

$$
P_{\mathrm{tot}} = \sum_{i\in\tilde{\mathbb{N}}} P_i. \tag{26}
$$

where $P_i := \frac{1}{\mu_i}\sum_{k\in\mathbb{K}} \mathbb{E}\{\|\boldsymbol{x}^k_i\|^2\} + P_{i_{\mathrm{zero}}} + P_{i_{\mathrm{FD}}}$. μ_i are the efficiency of the power amplifier at the i-th user node and BS when $i = 0$. $P_{i_{\mathrm{zero}}}$ is the power dissipated by other circuit blocks at the transmitter chain of each node. $P_{i_{\mathrm{FD}}}$ is the power required for self-interference cancellation. By using the above definition, the EE maximization problem can be expressed as

$$
\begin{aligned}
\max_{\substack{p^{\mathrm{DL}}_{i,k}>0,\\ p^{\mathrm{UL}}_{i,k}>0}}\ & \sum_{i\in\mathbb{N}} w_i \sum_{k\in\mathbb{K}} \frac{R^{\mathrm{UB}}_{i,k}}{P_{\mathrm{tot}}} \\
\text{subject to} & \sum_{k\in\mathbb{K}} p^{\mathrm{UL}}_{i,k} \leq P_{i,\max}, i \in \mathbb{N} \\
& \sum_{i\in\mathbb{N}}\sum_{k\in\mathbb{K}} p^{\mathrm{DL}}_{i,k} \leq P_{0,\max},
\end{aligned} \tag{27}
$$

where $P_{i,\max}$ and $P_{0,\max}$ are the total available power for consumption at the user node i and the BS. To solve the above problem, we propose a two stage (loop) iterative algorithm (Algorithm 2). Let us first select $p^{\mathrm{UL}}_{i,k,\mathrm{o}}$ and $p^{\mathrm{DL}}_{i,k,\mathrm{o}}$ as a feasible transmit power value at the i-th user node and BS, respectively. In the first or the outer loop, we calculate the rate approximations using (24) and (23) for the point of approximation $p^{\mathrm{UL}}_{i,k,\mathrm{o}}$ and $p^{\mathrm{DL}}_{i,k,\mathrm{o}}$. We fix these values for the inner loop. In the inner loop, we use Dinckelbach algorithm [18], using which we can rewrite the optimization problem as

$$
\begin{aligned}
\max_{\substack{p^{\mathrm{DL}}_{i,k}>0,\\ p^{\mathrm{UL}}_{i,k}>0}}\ & \sum_{i\in\mathbb{N}} w_i \sum_{k\in\mathbb{K}} \bar{R}^{\mathrm{UB}}_{i,k} - \lambda P_{\mathrm{tot}} \\
\text{subject to} & \sum_{k\in\mathbb{K}} p^{\mathrm{UL}}_{i,k} \leq P_{i,\max}, i \in \mathbb{N} \\
& \sum_{i\in\mathbb{N}}\sum_{k\in\mathbb{K}} p^{\mathrm{DL}}_{i,k} \leq P_{0,\max},
\end{aligned} \tag{28}
$$

For fixed $p_{i,k,\mathrm{o}}^{\mathrm{UL}}$ and $p_{i,k,\mathrm{o}}^{\mathrm{DL}}$, we iteratively solve for λ , $p_{i,k}^{\mathrm{UL}}$ and $p_{i,k}^{\mathrm{DL}}$. The λ can be determine from

$$\sum_{i\in\mathbb{N}} w_i \sum_{k\in\mathbb{K}} \bar{R}_{i,k}^{\mathrm{UB}} - \lambda P_{tot} = 0. \tag{29}$$

Since Dinckelbach algorithm is applied to the concave-affine fractional problem, we can conclude that a global optimal can be achieved [29, Section 3.2]. Then in the outerloop, we update $p_{i,k,\mathrm{o}}^{\mathrm{UL}}$ and $p_{i,k,\mathrm{o}}^{\mathrm{DL}}$ in order to calculate the new rate approximations and solve the optimization problem until a stable point is reached. Algorithm 2 defines the detailed algorithm procedure.

Algorithm 2 For energy efficiency maximization

```
p^UL_{i,k,o}, p^DL_{i,k,o} ← uniform (equal) power initialization
a ← 0 (set iteration number to zero for outer loop)
repeat
    a ← a + 1
    λ = 0 ← Lambda initialization
    repeat
        p^UL_{i,k}, p^DL_{i,k} ← solve (28)
        λ ← solve (29)
    until a stable point is reached
    p^UL_{i,k,o} ← p^UL_{i,k} and p^DL_{i,k,o} ← p^DL_{i,k}
until a stable point, or maximum number of a reached
return {p^UL_{i,k}, p^DL_{i,k}}
```

IV. Simulation Results

By using numerical simulations, we evaluate the performance of the proposed algorithms introduced in Section III for a bidirectional communication setup between an FD MC mMIMO BS and multiple MC single antenna FD nodes. We compare our proposed algorithms (Opt) with other benchmarks, such as the no-distortion (ND) algorithm, where the hardware distortions are not considered, and half-duplex (HD) algorithm. For the simulations, we consider the MRT/MRC strategy for our transmit precoder and receive filters at the BS. All communication channels follow an uncorrelated Rayleigh flat fading model. The self-interference channel follows the characterization reported in [26], i.e., $\mathbf{H}_{00} \sim \mathcal{CN}\left(\sqrt{\frac{\rho_{si}K_{\mathrm{R}}}{1+K_{\mathrm{R}}}}\mathbf{H}_0, \frac{\rho_{si}}{1+K_{\mathrm{R}}}\mathbf{I}_{M_{\mathrm{BS}}} \otimes \mathbf{I}_{N_{\mathrm{BS}}}\right)$ and $h_{ii} \sim \mathcal{CN}\left(\sqrt{\frac{\rho_{si}K_{\mathrm{R}}}{1+K_{\mathrm{R}}}}, \frac{\rho_{si}}{1+K_{\mathrm{R}}}\right)$, where ρ_{si} is the self-interference channel strength, $\mathbf{H}_0$ is a deterministic matrix of all-1 elements, and K_R is the Rician coefficient. The overall system performance is then averaged over 100 channel realizations. During our simulations, the following values are used to define the default setup: $|\mathbb{K}| = 4$, $K_{\mathrm{R_{BS}}} = K_{\mathrm{R_i}} = 10$ $N_{\mathrm{BS}} = 2M_{\mathrm{BS}} = 32$, $|\mathbb{N}| = 3$, $\rho = \rho_{ij} = \rho_{io} = \rho_{oi} = -20dB$, $\rho_{si} = 1$, $\sigma_n^2 = (\sigma_{n0}^k)^2 = (\sigma_{n_i}^k)^2 = -30dB$, $\sigma_e^2 = (\sigma_{e,ij}^k)^2 = (\sigma_{e,i0}^k)^2 = (\sigma_{e,0i}^k)^2 = (\sigma_{e,00}^k)^2 = -30dB$, $P_i = 1$, $\mu_i = 0.9$, $P_{i_{\mathrm{zero}}} + P_{i_{\mathrm{FD}}} = \frac{P_i}{10}$ for $i \in \tilde{\mathbb{N}}$, $\kappa = \beta = -50dB$ where $\mathbf{\Theta}_{\mathrm{t},0} = \kappa\mathbf{I}_N$ and $\mathbf{\Theta}_{\mathrm{r},0} = \kappa\mathbf{I}_N$. For asymmetricity, we assume the third user has no uplink.

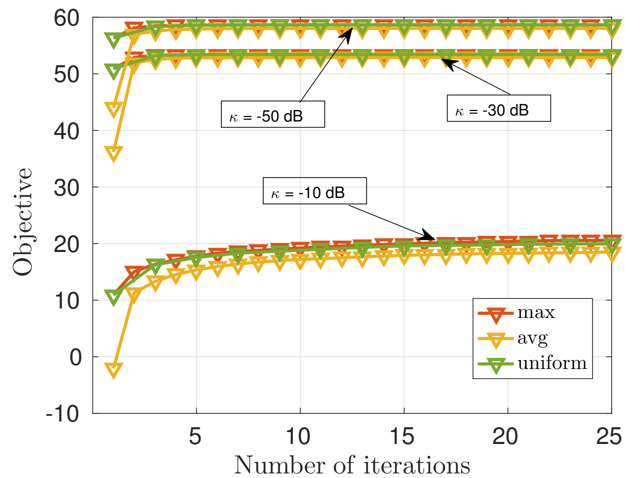

Figure 1. Average convergence behaviour of algorithm 1 for different values of hardware distortion coefficient.

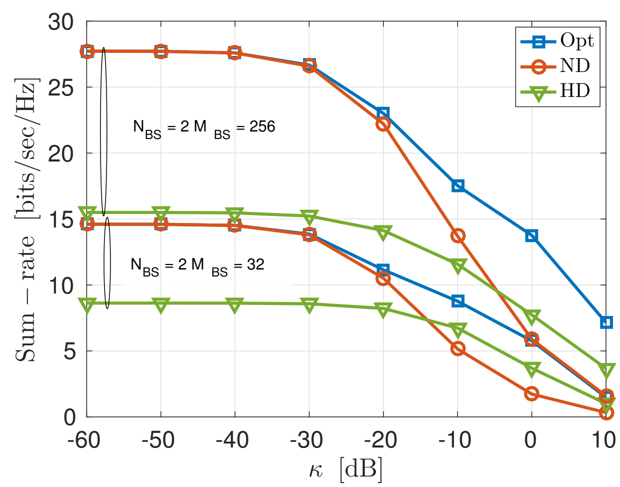

Figure 2. Sum rate vs. hardware inaccuracy.

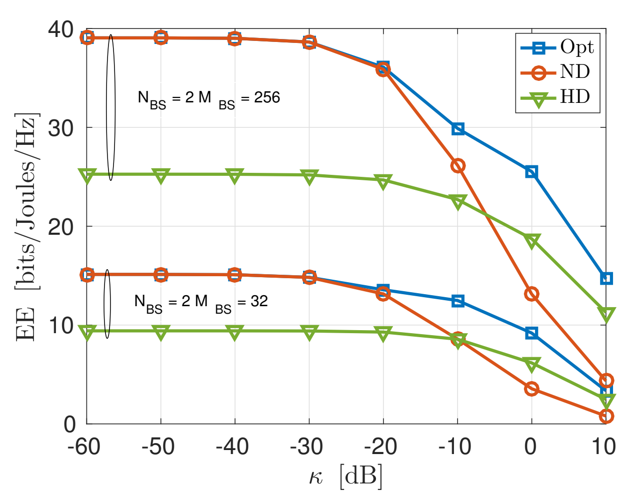

Figure 3. Energy Efficiency vs. hardware inaccuracy.

In Fig. 1, the average convergence behaviour of algorithm 1 is plotted for different values of hardware inaccuracy κ dB. The curves 'avg' and 'max' represent the maximum, and average values of the algorithm objective at the corresponding optimization step over the choice of 100 random initializations. The 'uniform' represents uniform power distribution. It can be observed that the curves 'uniform' and 'max' coincides with each other, hence we consider uniform power distribution as our initialization strategy. It is observed that the algorithm converges within 10-25 iterations. It can be seen that the objective has higher values for smaller hardware inaccuracy.

In Fig. 2, the performance of the algorithm 1 in terms of system sum rate, is evaluated for different values of transceiver accuracy, and also for different number of antennas at the BS. As we can observe, the sum rate decreases as the transceiver inaccuracy increases, i.e., higher the κ smaller the sum rate. It is clear that the proposed algorithm outperforms all the other benchmarks.

In Fig. 3, the performance of the algorithm 2 is plotted in terms of the energy efficiency, for different values of transceiver accuracy, and also for different number of antennas at the BS. A similar trend to that of the sum rate is observed, i.e., the energy efficiency decreases as the transceiver inaccuracy increases. It is clear that the proposed algorithm outperforms all the other benchmarks.

From both the Figures 2 and 3, it is interesting to observe that the HD algorithm performs better than the ND algorithm for higher values of κ. It shows how essential it is to consider the impact of hardware inaccuracies in an FD communication system.

V. CONCLUSIONS

In this paper, we addressed a joint subcarrier and power allocation problem for an MC bidirectional communication setup between an mMIMO FD transceiver basestation and multiple FD user nodes with a single antenna. We modelled the operation of the system by jointly considering the impact of hardware distortion, ICL, and imperfect CSI. We formulated the optimization problems to maximize the sum rate and EE of the system. An iterative optimization approach, which follows SIA framework is proposed for the sum rate maximization problem, which converges to a point that satisfies the KKT conditions. Another optimization, which utilizes SIA framework and Dinkelbach algorithm, is also proposed for the EE maximization problem. We evaluated the proposed algorithms using numerical simulations. It is observed that the proposed distortion-aware design attains a significant gain, especially when the transceiver accuracy decreases and ICL becomes dominant. Also for higher hardware inaccuracy HD performs better than the ND design, indicating the importance of considering hardware inaccuracies.

REFERENCES

[1] J. I. Choi, M. Jain, K. Srinivasan, P. Levis, and S. Katti, "Achieving Single Channel, Full Duplex Wireless Communication," in *Proceedings of the Sixteenth Annual International Conference on Mobile Computing and Networking*, ser. MobiCom '10, 2010, pp. 1–12.

[2] T. Riihonen and R. Wichman, "Analog and digital self-interference cancellation in full-duplex MIMO-OFDM transceivers with limited resolution in A/D conversion," in *2012 Conference Record of the Forty Sixth Asilomar Conference on Signals, Systems and Computers (ASILOMAR)*, Nov 2012, pp. 45–49.

[3] A. S. E. Everett and A. Sabharwal, "Passive Self-Interference Suppression for Full-Duplex Infrastructure Nodes," *IEEE Transactions on Wireless Communications*, vol. 13, pp. 680–694, 2014.

[4] M. S. Sim, M. Chung, D. Kim, J. Chung, D. K. Kim, and C. Chae, "Nonlinear Self-Interference Cancellation for Full-Duplex Radios: From Link- and System-Level Performance Perspectives," *CoRR*, vol. abs/1607.01912, 2016. [Online]. Available: http://arxiv.org/abs/1607.01912

[5] A. Sabharwal, P. Schniter, D. Guo, D. W. Bliss, S. Rangarajan, and R. Wichman, "In-Band Full-Duplex Wireless: Challenges and Opportunities," *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 9, pp. 1637–1652, Sept 2014.

[6] D. Bharadia and S. Katti, "Full Duplex MIMO Radios," in *11th USENIX Symposium on Networked Systems Design and Implementation (NSDI 14)*, 2014, pp. 359–372.

[7] J. Liu, J. Dai, J. Wang, X. Yin, Z. Jiang, and J. Wang, "Achievable rates for full-duplex massive mimo systems with low-resolution adcs/dacs under imperfect csi environment," *EURASIP Journal on Wireless Communications and Networking*, vol. 2018, no. 1, p. 222, Sep 2018.

[8] N. Li, Y. Li, M. Peng, and W. Wang, "Resource allocation in multi-carrier full-duplex amplify-and-forward relaying networks," in *2016 IEEE 83rd Vehicular Technology Conference (VTC Spring)*, May 2016, pp. 1–5.

[9] Y. Sun, D. W. K. Ng, Z. Ding, and R. Schober, "Optimal joint power and subcarrier allocation for full-duplex multicarrier non-orthogonal multiple access systems," *IEEE Transactions on Communications*, vol. 65, no. 3, pp. 1077–1091, March 2017.

[10] A. C. Cirik, K. Rikkinen, Y. Rong, and T. Ratnarajah, "A subcarrier and power allocation algorithm for ofdma full-duplex systems," in *2015 European Conference on Networks and Communications (EuCNC)*, June 2015, pp. 11–15.

[11] O. Taghizadeh, V. Radhakrishnan, A. C. Cirik, R. Mathar, and L. Lampe, "Hardware impairments aware transceiver design for bidirectional full-duplex mimo ofdm systems," *IEEE Transactions on Vehicular Technology*, vol. 67, no. 8, pp. 7450–7464, Aug 2018.

[12] V. Radhakrishnan, O. Taghizadeh, and R. Mathar, "Linear transceiver design for multi-carrier full-duplex mimo decode and forward relaying," in *WSA 2018; 22nd International ITG Workshop on Smart Antennas*, March 2018, pp. 1–6.

[13] S. Akbar, Y. Deng, A. Nallanathan, M. Elkashlan, and G. K. Karagiannidis, "Massive multiuser mimo in heterogeneous cellular networks with full duplex small cells," *IEEE Transactions on Communications*, vol. 65, no. 11, pp. 4704–4719, Nov 2017.

[14] A. Shojaeifard, K. Wong, M. D. Renzo, G. Zheng, K. A. Hamdi, and J. Tang, "Massive mimo-enabled full-duplex cellular networks," *IEEE Transactions on Communications*, vol. 65, no. 11, pp. 4734–4750, Nov 2017.

[15] L. Chen, F. R. Yu, H. Ji, B. Rong, and V. C. M. Leung, "Power allocation in small cell networks with full-duplex self-backhauls and massive mimo," *Wireless Networks*, vol. 24, no. 4, pp. 1083–1098, May 2018. [Online]. Available: https://doi.org/10.1007/s11276-016-1381-1

[16] W. Xie, X. Xia, Y. Xu, K. Xu, and Y. Wang, "Massive mimo full-duplex relaying with hardware impairments," *Journal of Communications and Networks*, vol. 19, no. 4, pp. 351–362, August 2017.

[17] X. Xia, D. Zhang, K. Xu, W. Ma, and Y. Xu, "Hardware impairments aware transceiver for full-duplex massive mimo relaying," *IEEE Transactions on Signal Processing*, vol. 63, no. 24, pp. 6565–6580, Dec 2015.

[18] W. Dinkelbach, "On nonlinear fractional programming," *Management Science*, vol. 13, no. 7, pp. 492–498, 1967. [Online]. Available: http://www.jstor.org/stable/2627691

[19] J. Wang and D. P. Palomar, "Worst-case robust mimo transmission with imperfect channel knowledge," *Trans. Sig. Proc.*, vol. 57, no. 8, pp. 3086–3100, Aug. 2009. [Online]. Available: http://dx.doi.org/10.1109/TSP.2009.2021451

[20] A. C. Cirik, Y. Rong, and Y. Hua, "Achievable rates of full-duplex mimo radios in fast fading channels with imperfect channel estimation," *IEEE Transactions on Signal Processing*, vol. 62, no. 15, pp. 3874–3886, Aug 2014.

[21] B. P. Day, A. R. Margetts, D. W. Bliss, and P. Schniter, "Full-Duplex MIMO Relaying: Achievable Rates Under Limited Dynamic Range," in *IEEE Journal on Selected Areas in Communications*, 2012.

[22] B. P. Day, D. W. Bliss, A. R. Margetts, and P. Schniter, "Full-duplex bidirectional MIMO: Achievable rates under limited dynamic range," in *2011 Conference Record of the Forty Fifth Asilomar Conference on Signals, Systems and Computers (ASILOMAR)*, Nov 2011, pp. 1386–1390.

[23] W. Namgoong, "Modeling and analysis of nonlinearities and mismatches in ac-coupled direct-conversion receiver," *IEEE Transactions on Wireless Communications*, vol. 4, no. 1, pp. 163–173, Jan 2005.

[24] G. Santella and F. Mazzenga, "A hybrid analytical-simulation procedure for performance evaluation in m-qam-ofdm schemes in presence of nonlinear distortions," *IEEE Transactions on Vehicular Technology*, vol. 47, no. 1, pp. 142–151, Feb 1998.

[25] H. Suzuki, T. V. A. Tran, I. B. Collings, G. Daniels, and M. Hedley, "Transmitter noise effect on the performance of a mimo-ofdm hardware implementation achieving improved coverage," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 6, pp. 867–876, August 2008.

[26] M. Duarte, C. Dick, and A. Sabharwal, "Experiment-Driven Characterization of Full-Duplex Wireless Systems," *IEEE Transactions on Wireless Communications*, vol. 11, no. 12, pp. 4296–4307, December 2012.

[27] B. R. Marks and G. P. Wright, "Technical note—a general inner approximation algorithm for nonconvex mathematical programs," *Operations Research*, vol. 26, no. 4, pp. 681–683, 1978. [Online]. Available: https://doi.org/10.1287/opre.26.4.681

[28] S. Boyd and L. Vandenberghe, *Convex Optimization*. New York, NY, USA: Cambridge University Press, 2004.

[29] A. Zappone and E. Jorswieck, *Energy Efficiency in Wireless Networks via Fractional Programming Theory*. Hanover, MA, USA: Now Publishers Inc., 2015.