Optimization of a CPSS-based Flexible Transportation System

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***Abstract*—This paper maximizes the expected profits of a cyber-physical social system (CPSS) for flexible passenger transportation. In the CPSS-based transportation system users can choose from an assortment of travel offers. When shared rides are proposed, the users have a great impact on the system performance, since every passenger of the proposed shared ride is able to choose another travel option or to reject it. In the worst case, the rejections result in a not profitable transportation of the passengers accepting the shared ride. To draw attention to this problem, a mixed integer problem considering the choices of the customers is formulated and applied to synthetic data. At last, a heuristic based on the idea of forming best pairs is proposed and evaluated.**

I. Introduction

Many transportation systems act as conventional cyber physical systems (CPS), since they do not consider human dynamics. For example, they assign only one travel option to each user and assume that the travel option is always accepted [1]. In particular, most mathematical problem formulations follow this assumption [2]. Some literature examples even assume that customers accept a frequent change of vehicles on their travel [3].

Following the ideas in [4] and [5] we aim at integrating humans substantially into the system. In this paper, we therefore optimize the assortments of travel offers in a flexible transportation system, by maximizing the expected system profits regarding the probability distribution of individual user choices.

In flexible transportation users can choose from an assortment of travel offers. For example, a single mode ride and a shared mode ride can be proposed by the system. Since the choice of each user has an impact on the routing of the vehicles, the system should consider the probability distribution of the user's travel mode decision. The example in Figure 1 shows that the consideration of the probability distribution of the decisions is crucial, since the decisions can heavily affect the performance of the system.

We assume that each edge traversal leads to costs of one and the price of a transport equals two times the shortest path costs from a requested pick up location to requested destination location. In the dotted red solution the rejection of offers is not considered. Since profit is maximized, the system decides to offer customer one and two a shared ride with a profit of $3 = 2 \cdot sp(s_1, d_1) + 2 \cdot sp(s_2, d_2) - sp(s_{f_1}, s_1) - sp(s_1, s_2) - sp(s_2, d_1) - sp(d_1, d_2)$, where $sp(v, u)$ provides the costs of the shortest path from node v to u. However, if we assume that customers reject the offers with probability 0.5, the expected profit of the system is $-0.5 = 3 \cdot 0.5^2 - 2 \cdot 0.5^2 - 3 \cdot 0.5^2$, since the system makes a profit of 3 when both accept, a loss of 2 when only customer one accepts and a loss of 3 when only customer two accepts.

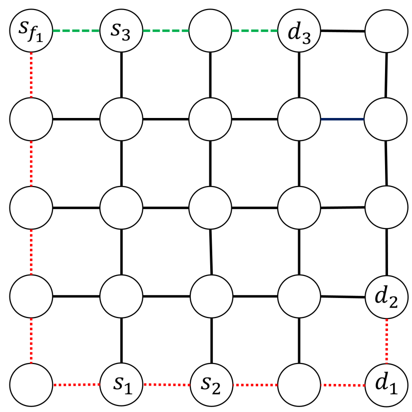

Fig. 1. In dotted red a shortest path route of vehicle f_1 starting at location s_{f_1} and servicing customer 1 and 2 with pick up location s_1 and s_2 and destination location d_1 and d_2, respectively. The shortest path route servicing customer 3 is depicted by the dashed green line.

If the probability distributions of the customer's decisions are considered the system would offer only customer 3 a single mode ride with an expected profit of $0.5 = 1 \cdot 0.5 = (2 \cdot sp(s_3, d_3) - sp(s_{f_1}, s_3) - sp(s_3, d_3)) \cdot 0.5$ instead (dashed green solution).

In contrast to solutions not considering the choices of customers, our solution generates shared mode rides with higher expected profits and therefore performs better in the long run. Surprisingly, there is no literature considering the rejection of shared rides to our knowledge. Most flexible transportation systems offer services from different companies like train and taxi companies [9] or independent travel modes like walk, bike, car, car-pool, and transit [10].

To the best of our knowledge, there is only one literature example that considers flexible transportation within one system [11]. The authors propose to maximize the expected utility, given the detailed preference of each customer. However, the part wise rejection of shared rides is not handled since each customer request is considered sequentially.

In contrast, we generate the assortments considering the choices of all customers simultaneously, which makes the application more practical, as users need time to choose a travel option.

In general, a change from pure system optimization to customer preference optimization can be found in transportation system literature. Instead of just minimizing costs or maximizing overall system profits, the objectives of the users are considered.

The authors of [6] maximize the overall user satisfaction considering attributes like driving time and time window violation. To each attribute a weight is assigned to reflect its importance. In [7], machine learning is used to capture the preferences considering travel cost, travel time, number of co-passengers, user's seat, working status and user's demographic information.

The authors of [8] address co-passenger preferences in taxi sharing. Their system continuously updates a ride sharing social network and assigns greater detours to customers that know each other.

The remainder of this paper is organized as follows. After describing the CPSS for flexible transportation in Section II, the assortment generation problem is modeled as a mathematical optimization problem in Section III. For this problem a heuristic based on the formation of best pairs is proposed in Section IV. In Section V the solutions are applied to synthetic data and compared to each other. In particular, the solution not considering the rejection of offers is evaluated regarding the expected profits. At last, the paper is concluded and further research ideas are given.

II. CPSS description

Our CPSS consists of customers requesting a transport (social component), a fleet of vehicles transporting the customers (physical component) and a fleet management optimizing the vehicle customer assignments and vehicle routes (cyber component). The social component, which represents the humans using the system, is significant in flexible transportation, since the customers can choose from an assortment of travel offers after optimization.

In more detail, customers request a transport by sending their pick up location, destination location and pick up time to the system. The system then calculates an assortment of travel options for each customer, considering their preferences and the probability distribution of their decision. The customer preferences are either set by the customer or learned by the system. For example, reinforcement learning can be used to determine the preferences and the probability distribution, as the assortment and the decision of a customer can be used as training data, while the system is in operation. Each customer then chooses a travel option from the assortment, which can be seen as a booking. At last, the fleet management sends the optimized routes to the vehicles. In Figure 2, the system components and the interactions are shown.

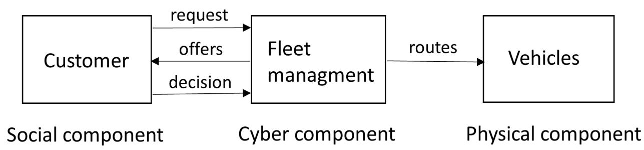

Fig. 2. System components and interactions.

III. Problem formulation

We first formulate the mathematical problem of offering every customer at most one shared ride and at most one single mode ride maximizing the system profit, without considering the rejection of offers.

Let $\mathcal{F} = \{f_1, \dots, f_m\}$ be the set containing the fleet of homogeneous vehicles with k available seats and $\mathcal{C} = \{1, \dots, n\}$ the customer set, each requesting a transport $r_i = (s_i, d_i, t_i)$ with pick up location s_i, destination location d_i and pick up time t_i, $i = 1, \dots, n$. The problem can be extended to time windows, however cycles have to be prevented then. When the time windows are smaller than the duration of the trips, the formulation contains no cycles. Otherwise a preprocessing procedure, that breaks cycles, or sub-tour elimination constraints has to be added [12].

The time feasible shared rides are collected in the set $\mathcal{S} = \{S \mid |S| \leq k,\ S \in \mathcal{P}(\mathcal{C}), S \text{ is time feasible}\}$, where $\mathcal{P}(\mathcal{C})$ is the power set containing all subsets of $\mathcal{C}$. An offer S in the offer set $\mathcal{S}$ is time feasible, if all customers $i \in S$ can be picked up at their pick up time and serviced by only one vehicle. For each $S \in \mathcal{S}$ we determine the cost minimal pick up and drop off sequence $P_S^* = (v_1^*, \dots, v_{2\cdot|S|}^*)$ with $v_j^* \in \{s_i, d_i \mid i \in S\}$. Let $sp(v, u)$ provide the cost of the shortest path from node v to u, then the cost of a feasible sequence $P = (v_1, \dots, v_l)$ is given by $\overline{sp}(P) = \sum_{i=1}^{l-1} sp(v_i, v_{i+1})$. Since the number of customers in an offer S is bounded by the capacity k of the vehicles, the sequence with minimal cost can be calculated by evaluating all feasible sequences in reasonable time. For every cost minimal sequences $P_S^* = (s_S^*, \dots, d_S^*)$, we create a request $r_S = (s_S^*, d_S^*, t_S^*)$, where s_S^* is the location of the first pick up, d_S^* the location of the last drop off and t_S^* the pick up time of the customer picked up first in the cost minimal sequence.

In order to formulate the integer problem based on network flows like in [12], we define decision variables $y_{f,S}$ with $f \in \mathcal{F}$ and $S \in \mathcal{S}$ that represent the ride of vehicle f from its starting location to offer S and decision variables $x_{S',S}$ that represent the ride from offer $S' \in \mathcal{S}$ to offer $S \in \mathcal{S}$. Further, we define the functions

$$p : \mathcal{S} \to \mathbb{Z}^+, c : (\mathcal{F} \cup \mathcal{S}) \times \mathcal{S} \to \mathbb{Z}^+, \tau : (\mathcal{F} \cup \mathcal{S}) \times \mathcal{S} \to \mathbb{Z}^+,$$

where p contains the profit of an offer $S \in \mathcal{S}$, which is equal to the sum of prices for each customer $i \in S$ minus the transportation costs of the shared ride. The cost and duration of a ride from a vehicle's starting location or offer destination location to the starting location of the offer serviced next, is assigned by c and τ, respectively. With these notations, following IP can be used to calculate the optimal profit, without considering the rejection of offers.

$$\max \sum_{f\in\mathcal{F},S\in\mathcal{S}} (p(S)-c(f,S))\cdot y_{f,S} + \sum_{S,S'\in\mathcal{S}} (p(S)-c(S',S))\cdot x_{S',S} \tag{1}$$

$$\text{s.t.} \sum_{S\in\mathcal{S}} y_{f,S} \leq 1 \quad \forall\, f\in\mathcal{F} \tag{2}$$

$$\sum_{S\in\mathcal{S}} x_{S',S} \leq \sum_{f\in\mathcal{F}} y_{f,S'} + \sum_{S\in\mathcal{S}} x_{S,S'} \quad \forall\, S'\in\mathcal{S} \tag{3}$$

$$\sum_{f\in\mathcal{F}} y_{f,\{i\}} + \sum_{S\in\mathcal{S}} x_{S,\{i\}} \leq 1 \quad \forall\, i\in\mathcal{C} \tag{4}$$

$$\sum_{S:i\in S,|S|>1}\sum_{f\in\mathcal{F}} y_{f,S} + \sum_{S:i\in S,|S|>1}\sum_{S'\in\mathcal{S}} x_{S',S} \leq 1 \quad \forall\, i\in\mathcal{C} \tag{5}$$

$$\tau(S',S)\cdot x_{S',S} - \sum_{S\in\mathcal{S}} t_S\cdot(1-x_{S',S}) \leq t_S - t_{S'} \quad \forall\, S,S'\in\mathcal{S} \tag{6}$$

$$\tau(f,S)\cdot y_{f,S} \leq t_S \quad \forall\, S\in\mathcal{S}, \forall\, f\in\mathcal{F} \tag{7}$$

$$y_{f,S}\in\{0,1\} \;\forall\, f\in\mathcal{F},\; \forall\, S\in\mathcal{S} \tag{8}$$

$$x_{S',S}\in\{0,1\} \;\forall\, S',S\in\mathcal{S} \tag{9}$$

As we subtract the cost to drive to an offer from the profits of each offer serviced, the objective function maximizes the overall system profit. Feasible vehicle routes are ensured by allowing each vehicle to start with at most one offer only (2) and the flow conservation constraints in (3). The constraints in (4) and (5) bound the number of proposed single mode respectively shared mode rides by one for each customer. In order to guarantee the requested pick up times the constraints (6) and (7) are added. At last the decision variables are constrained to be binary.
In order to consider the choices of the customers, the objective function is adapted as follows:

$$\max \sum_{f\in\mathcal{F},S\in\mathcal{S}}\sum_{T\in\mathcal{P}(S)} (p(T)-c(f,T))\cdot y_{f,S}\cdot Pr(T|S) + \sum_{S'\in\mathcal{S},S\in\mathcal{S}}\sum_{T\in\mathcal{P}(S)}\sum_{T'\in\mathcal{P}(S')} (p(T)-c(T',T))\cdot x_{S',S} \cdot Pr(T|S)\cdot Pr(T'|S'),$$

where $Pr(T|S) = \prod_{i\in T} Pr(i,S) \cdot \prod_{i\in S\setminus T}(1-Pr(i,S))$ and $Pr(i,S)$ equal to the probability that customer i chooses offer S. Note, that not only the profit is reduced, because of rejections. Since the last drop off location and the first pick up location of offers may change after rejections, the expected costs to drive to an offer have to be adapted, too. Unfortunately, the formulation does not consider the expected riding costs to drive to an offer, when all customers of the offer, planned to be serviced before, reject the ride. However, we calculate the actual expected costs of the optimized solutions in our simulation afterwards.

IV. A BEST-PAIR HEURISTIC GENERATING ATTRACTIVE SHARED RIDE OFFERS

In the mixed integer program formulations we generate all feasible shared mode offers, which does not scale with the number of requests and is therefore impractical in real application. In more detail, the formulation has $\mathcal{O}(m\cdot\binom{n}{k}+\binom{n}{k}^2)$ decision variables and constraints, if the number of available seats k is smaller than $\frac{n}{2}$, since in general all $\sum_{i=1}^{k}\binom{n}{i}$ shared mode offers $S\in\mathcal{P}(\mathcal{C})$ with cardinality smaller than k are considered. For this reason, we propose a heuristic generating only $\mathcal{O}(n)$ attractive shared ride offers, thus obtaining only $\mathcal{O}(m\cdot n+n^2)$ constraints and decision variables, which makes the formulation efficient.
Since our heuristic iteratively forms best pairs considering the breakage of shared rides, we call it expected best-pair heuristic. The application of a similar heuristic to the platooning problem can be found in [13].
For each pair of requests with summed cardinality smaller than k we calculate the expected cost savings of sharing a ride. The pair with highest expected cost savings is then merged to one request, added to the offer set $\mathcal{S}$ and considered in further best pair iterations. Since the number of request to merge decreases by one after each merging process, $\mathcal{O}(n)$ shared mode offers are generated. The following function is used to calculate the expected cost savings:

$$s:\mathcal{S}\times\mathcal{S}\to\mathbb{Z}^+$$

$$\begin{aligned}(S,T)\to &\sum_{U\in\mathcal{P}(S\cup T)}\Big(\Big(\sum_{i\in U} sp(s_i,d_i)\Big)-\overline{sp}(P_U^*)\Big)\cdot Pr(U|(S\cup T))\\ &-\sum_{U\in\mathcal{P}(S)}\Big(\Big(\sum_{i\in U} sp(s_i,d_i)\Big)-\overline{sp}(P_U^*)\Big)\cdot Pr(U|S)\\ &-\sum_{U\in\mathcal{P}(T)}\Big(\Big(\sum_{i\in U} sp(s_i,d_i)\Big)-\overline{sp}(P_U^*)\Big)\cdot Pr(U|T),\end{aligned}$$

where $\overline{sp}(P_U^*)$ with $U\in\mathcal{S}$ is equal to the minimal cost of the time feasible pick up and drop off sequence.
In the scenario of Figure 1 with edge costs equal to one and rejection probability of 0.5, request one and two form the first best pair, since

$$\begin{aligned}&s(\{1\},\{2\}) = 0.5 = (6-4)\cdot 0.5^2-(3-3)\cdot 0.5^2-0\cdot 0.5^2\\ &> s(\{1\},\{3\}) = -1.75 = (5-12)\cdot 0.5^2+0\cdot 0.5^2+0\cdot 0.5^2\\ &> s(\{2\},\{3\}) = -2 = (5-13)\cdot 0.5^2+0\cdot 0.5^2+0\cdot 0.5^2\end{aligned}$$

No further requests are merged, since:

$$\begin{aligned}0 > s(\{1,2\},\{3\}) = -2 = (8-12)\cdot 0.5^3+(5-12)\cdot 0.5^3\\ +(5-12)\cdot 0.5^3+(6-4)\cdot 0.5^3-(6-4)\cdot 0.5^2.\end{aligned}$$

		Price coefficient					
		1.1	1.2	1.3	1.4	1.5	1.6
$\frac{\text{supply}}{\text{demand}}$	$\frac{1}{5}$	(1.83,1.51,1.05)	(2.89,2.52,2.29)	(4.31,3.78,3.78)	(5.85,5.01,5.28)	(7.57,6.38,7.1)	(9.77,8.27,9.36)
	$\frac{1}{2}$	(2.55,2.28,1.58)	(4.49,4.17,3.74)	(6.5,5.84,5.64)	(9.15,8.34,8.37)	(11.95,11.13,11.24)	(14.09,12.95,13.44)
	1	(3.63,3.35,2.92)	(5.92,5.60,5.1)	(8.95,8.43,8.14)	(12.16,11.5,11.15)	(15.49,14.39,14.39)	(19.39,18.19,18.41)
	2	(4.78,4.53,4)	(7.75,7.44,6.91)	(11.55,11.1,10.6)	(15.39,14.70,14.4)	(19.68,18.6,18.61)	(23.76,22.6,22.57)

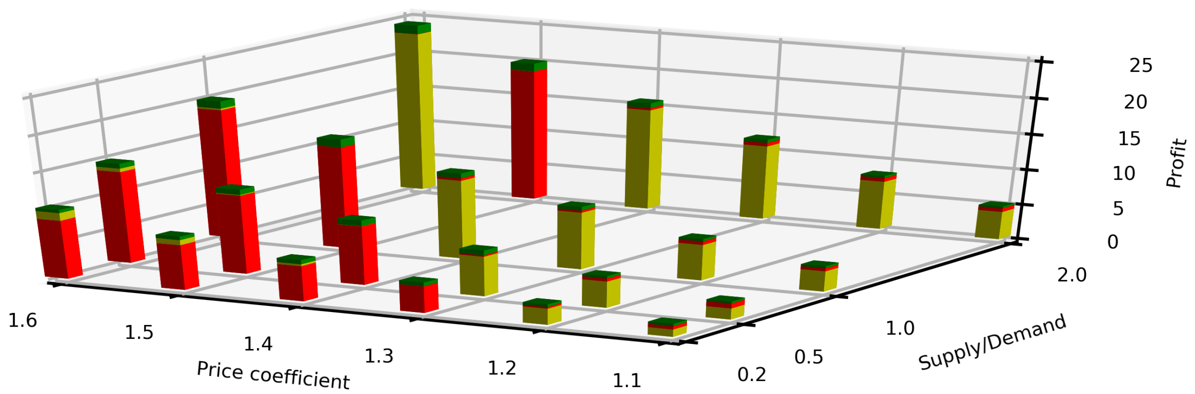

Fig. 3. Table and graph with average expected profits of the optimal solution (green), expected best-pair heuristic (red) and the optimal solution not considering the choices (yellow) for different travel prices and supply/demand ratios. The simulation was run 100 times for each case.

V. Simulation and Results

As the aim of the paper is to show how the expected profits of a CPSS for flexible passenger transportation can be improved, we simulate the problem with ten requests on a simple routing graph depicted in Figure 1 with time and cost of one for each edge traversal. The pick up, destination and vehicle starting locations are chosen uniformly on the routing graph and the pick up time is chosen uniformly in [0,20]. Further, we assume that each customer chooses one offer with a probability of 0.5 and the capacity of a vehicle is 4. In Figure 3 the average expected profits of the optimal solution, the expected best pair heuristic and the optimal solution not considering the choices of the customers is depicted for different transportation prices and supply/demand ratios. The prices are differentiated by multiplying the shortest path costs from the pick up location to the destination location with different coefficients. Figure 3 indicates that in the case of low prices the consideration of customer choices yields to much higher profits. When demand is higher than supply and prices are low, the solution considering the choices has an average profit increase of at least 24% compared to the optimal solution not considering the choices. In the case of $\frac{\text{supply}}{\text{demand}} = \frac{1}{5}$ and price coefficient=1.1 the profit increase is even greater than 74%. One explanation of such a high profit increase is that lower prices near the driving costs make the system more vulnerable to the breakage of shared rides. Further, only few single mode rides are proposed. When supply is greater than the demand, the system proposes many single mode rides. For this reason the profit increase gets smaller. In particular, the table shows that the algorithm without considering the rejection of offers is near optimal, when demand is lower than the supply and prices are high.

Overall the consideration of the choices of customers is imperative, since transportation systems compete with each other and have to offer small prices to be attractive. Further, they try to have a supply near the demand to reduce investment costs.

In the case of low prices the expected best pair heuristic outperforms the algorithm that does not consider the choices of customers. When the price coefficient is higher than 1.4 the heuristic performs worse. However, the heuristic computes a solution fast, since the offer set is small.

Since the accuracy of the expected best pair heuristic is good, when prices are low and supply equals demand, we expect good results in practical applications. In the case of high prices the application of a heuristic not considering the choices is satisfactory.

VI. Conclusion and Outlook

We introduced a new CPSS-based flexible transportation problem and solved it by a MIP formulation. For practical applications a heuristic solving the problem in reasonable time with good accuracy in the case of low prices, was proposed. The simulation shows that the consideration of user choices has great impact on the system performance. Since transportation companies try to adapt supply to demand and have to choose prices near the costs as they compete with each other, we expect that the consideration of choices has also great impact on the performance of flexible transportation services in real application.

In future, we want to formulate the corresponding online problem and to develop re-optimizing methods. We expect that the consideration of customer choices is also important in the online case allowing re-optimization, since the offered assortments with travel time proposals shrink the re-optimization possibilities.

Possibly, machine learning can be used to learn the probability distribution of the customer's decisions, when the flexible transportation system is in operation. Like the authors in [11], we believe that the probability distribution depends also on the actual characteristics of offered rides and individual preferences.

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