Optimum Power Allocation for Sensor Networks That Perform Object Classification

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***Abstract*—In this publication, the power allocation problem for a distributed sensor network is formulated as a signomial program, and analytically solved by a Lagrangian setup. Typical examples of such networks are active radar systems with multiple nodes whose aim is to detect and classify target objects. As it is common for sensors with weak power-supplies, constraints by sum and individual power limitations are imposed. For each sensor node, an amplify-and-forward strategy for the reflected and received echo is proposed. This per-node information is transmitted over a communication channel and combined at a fusion center. The fusion center carries out the final decision about the type of the target object by a best linear unbiased estimator and a subsequent distance classification. In contrast to approaches in the literature, which combine discrete local decisions into a single global one, the approach in the current paper offers many advantages, ranging from the simplicity of its implementation to the achievement of an optimal solution in closed-form and design of the sensor network.**

***Index Terms*—Analytical power allocation, energy-efficient optimization, distributed target classification, network resource management, information fusion.**

I. Introduction

IN THIS publication, we investigate the power allocation problem in distributed sensor networks that are used for active radar applications. The potential application of our approach is radar sensing, where an unknown target object is observed for classification. Especially in large sensor networks for space and extreme environment, where power consumption is a crucial requirement, the demand for energy-aware design and operation becomes more important than ever. Thus, the present work aims at providing a theoretical insight into the optimal strategy for power allocation in active sensor networks. In particular, we consider a sensor network where each sensor node (SN) individually and independently emits a radar signal and receives the reflected echo from a jointly observed target object. Each observation sample serves as a classification feature for classifying the type of the present target object. Since local observations at each SN are noisy and thus unreliable, they are combined into a single reliable quantity at a remotely located fusion center to increase the overall system performance. In the classification process, the absence, the presence, and the type of the present target object are distinguished. The fusion center uses the best linear unbiased estimator in order to accurately estimate the reflection coefficient of the present target object, where each object is assumed to be uniquely characterized by its own reflection coefficient. Additionally, we assume that all SNs have only limited power available for sensing and communication. This setup is illustrated in Figure 1, whose technical components will be specified in detail later.

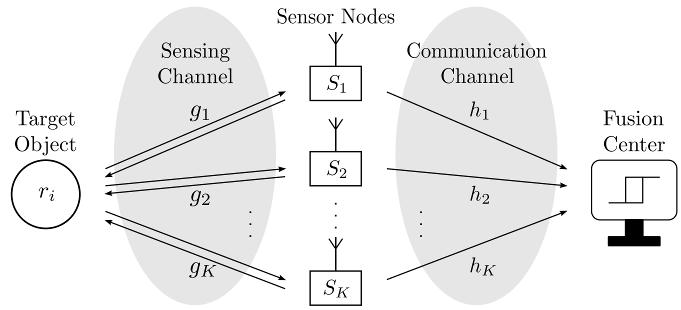

Fig. 1. Abstract representation of the distributed sensor network.

The research on distributed detection was originated from the attempt to combine signals of different radar devices [2]. Currently, distributed detection is rather discussed in the context of wireless sensor networks, where the sensor units may also be radar nodes [3]–[5]. In [6], the power allocation problem for distributed wireless sensor networks, which perform object detection and classification, is only treated for ultra-wide bandwidth (UWB) technology. Other applications, which require or benefit from detection and classification capabilities, are localization and tracking [7] or through-wall surveillance [8]. In [9], an approximate solution of the power allocation problem is proposed, which allows for an analytical treatment of an output power-range limitation per sensor node. The optimal power allocation in passive radar systems, instead of active systems, is investigated in [10]. The main difficulty for optimizing the power consumption is associated with finding a closed-form equation for the overall classification probability. As an example, for the Bayesian hypothesis test criterion the overall classification probability cannot be analytically evaluated [11]. This limits the usability of this criterion for solving the power allocation problem. Bounds, such as the Bhattacharyya bound [12], are also difficult to use for optimizing multidimensional problems. Hence, the best power allocation scheme is still an open problem in order to improve the overall classification probability.

In the present work, we analytically optimize the power allocation for the region of high signal-to-noise ratio (SNR)

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and present a closed-form solution for a network of amplify-and-forward SNs. Based on a simple system model, we apply a linear fusion rule and utilize the average deviation between the estimated and the actual reflection coefficient as a metric for defining the objective function. This approach is the key idea in the present work which enables the analytical optimization of the power allocation in closed-form. Since the power consumption of the entire network may be limited in various aspects, three different cases of power constraints are discussed and compared with each other. First, we demonstrate that all considered constraints lead to signomial optimization problems which are in general quite hard to solve. Then, all signomial problems are consecutively solved by a Lagrangian setup and each leads to explicit policies for the optimal method of power allocation. These are the main contributions of the present work which extend our previous investigations that started in [1].

The present paper is organized as follows. We start with a detailed description of the underlying technical system in the next section. Subsequently, the power allocation problem is specified and analytically solved. The achieved results are then discussed and carefully compared with each other.

A. Mathematical Notations

Throughout this paper we denote the sets of natural, integer, real, and complex numbers by $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}$, and $\mathbb{C}$, respectively. The imaginary unit is denoted by j . Note that the set of natural numbers does not include the element zero. Moreover, $\mathbb{R}_+$ denotes the set of non-negative real numbers. Furthermore, we use the subset $\mathbb{F}_N \subseteq \mathbb{N}$ which is defined as $\mathbb{F}_N := \{1, \ldots, N\}$ for any given natural number N. We denote the absolute value of a real or complex-valued number z by $|z|$ while the expected value of a random variable v is denoted by $\mathcal{E}[v]$. Moreover, the notation $V^\star$ stands for the value of an optimization variable V where the optimum is attained.

II. Overview and Technical System Description

At any instance of time, a network of $K \in \mathbb{N}$ independent and spatially distributed SNs receives random observations. If a target object is present, then the received power at the SN S_k is a part of its own emitted power which is back-reflected from the jointly observed target object and is weighted by its reflection coefficient r_i. The object may be of I different types. It should be noted that sheer detection may be treated as the special case of $I = 2$ which corresponds to the decision '*some object is present*' versus '*there is no object*'. We assume that all different object types and their corresponding reflection coefficients are known by the network. Moreover, the received signal at each SN is weighted by the corresponding channel coefficient and disturbed by additive noise. It is obvious that the sensing channel is wireless. The sensing task and its corresponding communication task for a single classification process are performed in consecutive time slots. All SNs take samples from the disturbed received signal and amplify them without any additional data processing in each time slot. The amplified samples remain buffered in the SNs during the current time slot. Simultaneously in the same time slot, new radio waves are emitted by all SNs for the next observation and classification process. In addition, the buffered samples of the former classification process are communicated to the fusion center which is placed in a remote location. We assume that SNs have only limited sum-power available for sensing the object and communicating local observations to the fusion center. Furthermore, each SN may be limited in its transmission power-range due to transmission-power regulation standards or due to the functional range of its circuit elements. The sensing task as well as the communication to the fusion center are performed by using distinct waveforms (pulse shapes) for each SN so as to distinguish sensing and communication of different SNs. Each waveform has to be suitably chosen in order to suppress inter-user (inter-node) interference at other SNs and also at the fusion center. Furthermore, we assume that in the frequency domain each waveform is orthogonal to all other waveforms in order to calculate the sensing power of each SN independent from its communication power. Hence, the K received signals at the fusion center are uncorrelated and assumed to be conditionally independent. Each received signal at the fusion center is influenced by the corresponding channel coefficient and additive noise, as well. The communication channel between the SNs and the fusion center can either be wireless or wired. The disturbed received signals at the fusion center are weighted and combined together in order to obtain a single reliable observation $\tilde{r}$ of the actual reflection coefficient r_i. Note that we disregard time delays within all transmissions and assume synchronized data communication.

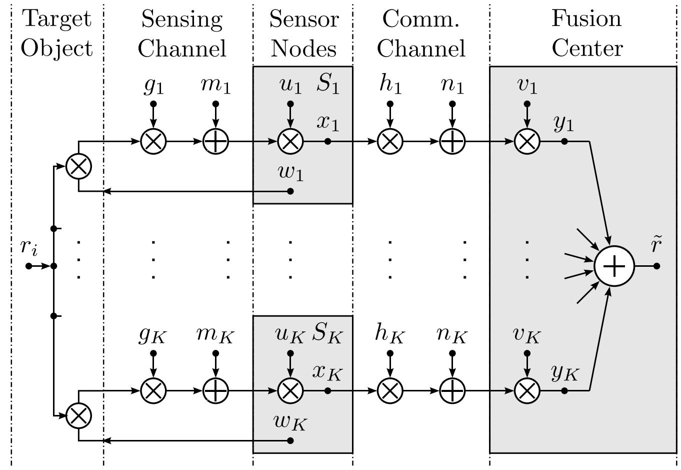

Fig. 2. System model of the distributed active sensor network.

In the following subsections, we mathematically describe the underlying system model that is depicted in Figure 2. The continuous-time system is modeled by its discrete-time equivalent, where the sampling rate of the corresponding signals is equal to the target observation rate, for the sake of simplicity.

A. Target Object

We assume that all objects have the same size, shape and alignment, but different material and hence complex-valued reflection coefficients $r_i \in \mathbb{C}$, $i \in \mathbb{F}_I$. Thus, the reflection coefficients are the only recognition features in this work. The a-priori probability of occurrence for each object type is denoted by $\pi_i \in \mathbb{R}_+$, $i \in \mathbb{F}_I$, with $\sum_{i=1}^{I} \pi_i = 1$. The root mean squared value of the reflection coefficients is given as

$$r_{\text{rms}} := \sqrt{\sum_{i=1}^{I} \pi_i |r_i|^2}\ . \tag{1}$$

Furthermore, the actual target object is assumed to be static during consecutive observation steps.

B. Sensing Channel

Each propagation path of the sensing channel, from each SN to the object and again back to the same SN, is described by a corresponding random channel coefficient g_k. For the investigation of the power allocation problem, the concrete realization of the channel coefficients is needed and hence can be used for postprocessing of the received signals at each SN. We assume that all channel coefficients are complex-valued and static during each target observation step. Furthermore, the coherence time of all sensing channels is assumed to be much longer than the whole length of the classification process. Thus, the expected value and the quadratic mean of each coefficient during each observation step can be assumed to be equal to their instantaneous values, i.e., $\mathcal{E}[g_k] = g_k$ and $\mathcal{E}[|g_k|^2] = |g_k|^2$. In practice, it is often difficult to measure or estimate these coefficients. Thus, the results of the present work are applicable for scenarios where the channel coefficients can somehow be accurately estimated during each observation process or they are nearly deterministic and thus can be measured before starting the radar task.

Furthermore, the channel coefficients are assumed to be uncorrelated and jointly independent. Note that all channel coefficients include the radar cross section, the influence of the antenna, the impact of the filters, as well as all additional attenuation of the target signal.

At the input of each SN, the disturbance is modeled by the complex-valued additive white Gaussian noise (AWGN) m_k with zero mean and finite variance $M_0 := \mathcal{E}[|m_k|^2]$ for all k. Note that the channel coefficient and the noise on the same propagation path are also uncorrelated and jointly independent.

C. Sensor Nodes

We model each SN by an amplify-and-forward unit with extended capabilities, where both sensing and communication signals are transmitted simultaneously. The sensing signal w_k, without loss of generality, is assumed to be non-negative, real-valued and deterministic. The expected value of its instantaneous power is then described by

$$W_k := \mathcal{E}[|w_k|^2] = |w_k|^2, \quad k \in \mathbb{F}_K. \tag{2}$$

Note that the specific value of w_k is adjustable and will be determined later by the power allocation procedure.

The ratio of the communication signal to the received signal is described by the non-negative real-valued amplification factor u_k which is assumed to be constant over the whole bandwidth and power-range. Thus, the communication signal and the expected value of its instantaneous power are described by

$$x_k := (r_i g_k w_k + m_k) u_k, \quad k \in \mathbb{F}_K \tag{3}$$

and

$$X_k := \mathcal{E}[|x_k|^2] = (r_{\text{rms}}^2 |g_k|^2 W_k + M_0) u_k^2, \quad k \in \mathbb{F}_K, \tag{4}$$

respectively. The amplification factor is an adjustable parameter and will be determined later by the power allocation procedure, as well. Note that the instantaneous power fluctuates from observation to observation depending on the present target object.

In order to solve the power allocation problem and make a closed-form solution amenable, we assume that the noise power M_0 from (4) is negligible in comparison to $r_{\text{rms}}^2 |g_k|^2 W_k$. This pertains only for the region of high SNR. Thus, we only will consider the useful power

$$X_k \simeq r_{\text{rms}}^2 |g_k|^2 W_k u_k^2, \quad k \in \mathbb{F}_K, \tag{5}$$

instead of (4) in what follows.

If the received signal is negligible in comparison to the output signal and if the nodes have smart power components with low-power dissipation loss, then the average power consumption of each node is approximately equal to its average output power $W_k + X_k$. The addition of both transmission powers is justified because the corresponding signals are assumed to be separated by distinct waveforms. We also assume that the output power-range of each SN is limited by P_{max} and that the average power consumption of all SNs together is limited by the sum-power constraint P_{tot}. Hence, the constraints

$$W_k + X_k \leq P_{\text{max}} \Leftrightarrow \left(1 + r_{\text{rms}}^2 |g_k|^2 u_k^2\right) W_k \leq P_{\text{max}}, \quad k \in \mathbb{F}_K \tag{6}$$

and

$$\sum_{k=1}^{K} \underbrace{\underbrace{W_k}_{\text{Radar task}} + \underbrace{X_k}_{\text{Data communication}}}_{\text{Average transmission power of one sensor for a single observation}} \leq P_{\text{tot}}$$

$$\Leftrightarrow \quad \sum_{k=1}^{K} \left(1 + r_{\text{rms}}^2 |g_k|^2 u_k^2\right) W_k \leq P_{\text{tot}} \tag{7}$$

arise consequently. We remark that the described method can also be extended to individual output power-range constraints per SN.

Note that the sum-power constraint P_{tot} is a requirement to compare energy-efficient radar systems.

D. Communication Channel

Analogous to the sensing channel, each propagation path of the communication channel is described by a corresponding random channel coefficient h_k. But in contrast to the sensing channel, the concrete realization of all communication channel-coefficients is measurable by using pilot sequences at each SN. Accordingly, the channel coefficients can be used for postprocessing of received signals at the fusion center. We assume that all channel coefficients are complex-valued and static during each target observation step. Furthermore, the coherence time of all communication channels is also assumed to be much longer than the whole length of the classification process. Thus, the expected value and the quadratic mean of each channel coefficient can be assumed to be equal to their instantaneous values, i.e., $\mathcal{E}[h_k] = h_k$ and $\mathcal{E}[|h_k|^2] = |h_k|^2$. Furthermore, the channel coefficients are assumed to be uncorrelated and jointly independent. Note that all channel coefficients include the influence of the antenna, the impact of the filters, as well as all additional attenuation of the corresponding sensor signal.

At the input of the fusion center, the disturbance on each communication path is modeled by the complex-valued AWGN n_k with zero mean and finite variance $N_0 := \mathcal{E}[|n_k|^2]$ for all k. Note that the channel coefficient and the noise on the same propagation path are also uncorrelated and jointly independent.

E. Fusion Center

The fusion center combines all different local observations into a single reliable one by applying a linear combiner. Thus, the received signals are weighted with the complex-valued factors v_k and summed up to yield an estimate $\tilde{r}$ of the actual target signal r_i. In this way, we obtain

$$y_k := (x_k h_k + n_k) v_k, \quad k \in \mathbb{F}_K, \tag{8}$$

and hence,

$$\tilde{r} := \sum_{k=1}^{K} y_k = r_i \sum_{k=1}^{K} w_k g_k u_k h_k v_k + \sum_{k=1}^{K} (m_k u_k h_k + n_k) v_k. \tag{9}$$

Note that each weight can be written as $v_k = |v_k| \exp(\mathrm{j}\,\vartheta_k)$, $k \in \mathbb{F}_K$, where ϑ_k is a real-valued number which represents the phase of the corresponding weight.

Note that the fusion center can separate all input streams because the communication channel is either wired or the data communication is performed by distinct waveforms for each SN. Consequently, if the communication channel is wireless then a matched-filter bank is essential at the input of the fusion center to separate the data streams of different SNs. In addition, we do not consider inter-user (inter-node) interferences at the fusion center because of the distinct waveform choices.

In order to obtain a single reliable observation at the fusion center, the value $\tilde{r}$ should be a good estimate for the present reflection coefficient r_i. Thus, we optimize the sensing power W_k, the amplification factors u_k, and the weights v_k in order to minimize the average absolute deviation between $\tilde{r}$ and the true reflection coefficient r_i. This optimization procedure is elaborately explained in the next section. After determining W_k, u_k and v_k, the fusion center observes a disturbed version of the true reflection coefficient r_i at the input of its decision unit. Hence, by using the present system model, we are able to separate the power allocation problem from the classification problem and optimize both independently.

F. Remarks on the System Model

All described assumptions are necessary to obtain a framework suitable for analyzing the power allocation problem, without studying detection, classification and estimation problems in specific systems and their settings.

The accurate estimation of all channel coefficients is necessary for both the radar process and the power allocation. Sometimes it is not possible to estimate the transmission channels; consequently the channel coefficients g_k and h_k remain unknown. In such cases, the radar usually fails to perform its task.

Since the channel coefficients g_k are in practice difficult to estimate or to determine, our approach rather shows theoretical aspects of the power allocation than the practical realization and implementation. Hence, the presented results act as theoretical bounds and references for comparing real radar systems.

Moreover, since the coherence time of communication channels as well as sensing channels is assumed to be much longer than the whole length of the classification process, the proposed power allocation method is applicable only for scenarios with slow-fading channels.

Note that only the linear fusion rule together with the proposed objective function enable optimizing the power allocation in closed-form. The optimization of power allocation in other cases is in general hardly amenable analytically.

Sensor nodes commonly have only one power amplifier and a single antenna. The antenna is usually connected to a circulator in order to separate the signal of the transmitter to the antenna from the signal of the antenna to the receiver, which is not depicted in Figure 2. The power amplifier is also shared for sensing and communication tasks, but not considered in this work.

In order to increase the available power-range at each SN, time-division multiple-access (TDMA) can be used to completely separate the sensing task from the communication task and perform each task in a different time slot.

The introduced system model describes a baseband communication system without considering time, phase and frequency synchronization problems.

In order to distinguish the current operating mode of each SN in what follows, we say a SN is *inactive* or *idle* if the allocated power is zero. We say a SN is *active* if the allocated power is positive. Finally, we say a SN is *saturated* if the limitation of its output power-range is equal to the allocated power, i.e., $P_{\max} = W_k + X_k$.

An overview of all notations that we will use hereinafter and are needed for the description of each observation process is depicted in Table I.

III. Power Allocation

In this section, we introduce the power optimization problem and consecutively present its analytical solutions for different power constraints. First, we investigate the case where only a sum-power constraint $P_{\text{tot}} \in \mathbb{R}_+$ for the cumulative sum of the expected power consumption of each SN is given. Afterwards, we present the analytical solution of the power allocation problem for the case where the average transmission power of each SN is limited by the output power-range limitation $P_{\max} \in \mathbb{R}_+$. Finally, we extend the power allocation problem to the case where both constraints simultaneously hold and present the corresponding optimal solution.

TABLE I
NOTATION OF SYMBOLS THAT ARE NEEDED FOR THE DESCRIPTION OF EACH OBSERVATION PROCESS

Notation	Description
K	number of all nodes;
$\mathbb{F}_K$	the index-set of K nodes;
$\tilde{K}$	number of all active nodes;
I	number of different reflection coefficients;
r_i, π_i	reflection coefficient of i^{th} target object and its probability of occurrence;
r_{rms}	root mean squared absolute value of reflection coefficients;
$\tilde{r}$	estimate of the actual reflection coefficient r_i;
g_k, h_k	complex-valued channel coefficients;
m_k, n_k	complex-valued AWGN with zero-mean;
M_0, N_0	variances of m_k and n_k;
u_k, v_k	non-negative amplification factors and complex-valued weights;
ϑ_k	phase of v_k;
ϕ_k	phase of the product $g_k h_k$;
w_k, x_k	sensing and communication signal of k^{th} sensor node;
W_k, X_k	sensing and communication power of k^{th} sensor node;
y_k	input signals of the combiner;
$P_{\max}$	output power-range constraint of each sensor node;
P_{tot}	sum-power constraint.

In general, the objective is to maximize the overall classification probability, however, a direct solution to the allocation problem does not exist, since no analytical expression for the overall classification probability is available. Instead, we minimize the average deviation between $\tilde{r}$ and r_i, in order to determine the power allocation. The motivation for this method is the separation of the power allocation problem from the object classification procedure, as described in the last section. The corresponding optimization problem is elaborately described in the next subsection.

A. The Optimization Problem

As mentioned in the last section, the value $\tilde{r}$ should be a good estimate for the actual reflection coefficient r_k of the present target object. In particular, we aim at finding estimators $\tilde{r}$ of minimum mean squared error in the class of unbiased estimators for each i.

The estimate $\tilde{r}$ is unbiased simultaneously for each i if $\mathcal{E}[\tilde{r} - r_i] = 0$, i.e., from equation (9) with (2) we obtain the identity

$$\sum_{k=1}^{K} \sqrt{W_k}\, g_k u_k h_k v_k = 1. \tag{10}$$

This identity is our first constraint in what follows. Note that the mean of the second sum in (9) vanishes since the noise is zero-mean. Furthermore, we do not consider the impact of both random variables g_k and h_k as well as their estimates in our calculations because the coherence time of both channels is assumed to be much longer than the target observation time. Note that equation (10) is complex-valued and may be separated as

$$\sum_{k=1}^{K} \sqrt{W_k}\, u_k |v_k g_k h_k| \cos(\vartheta_k + \phi_k) = 1 \tag{11}$$

and

$$\sum_{k=1}^{K} \sqrt{W_k}\, u_k |v_k g_k h_k| \sin(\vartheta_k + \phi_k) = 0\,, \tag{12}$$

where ϑ_k and ϕ_k are phases of v_k and $g_k h_k$, respectively.

The objective is to minimize the mean squared error $\mathcal{E}[|\tilde{r} - r_i|^2]$. By using equation (9) and the identity (10) we may write the objective function as

$$V := \mathcal{E}\big[|\tilde{r} - r_i|^2\big] = \sum_{k=1}^{K} |v_k|^2 \big(u_k^2 |h_k|^2 M_0 + N_0\big). \tag{13}$$

Note that (13) is only valid if m_k and n_k are white and jointly independent.

As mentioned in the last section, each SN has an output power-range limitation and the expected overall power consumption is also limited. Hence, the objective function is also subject to (6) and (7), which are our second and last constraints, respectively.

In summary, the optimization problem is to minimize the mean squared error in (13) with respect to u_k, v_k, and W_k, subject to constraints (6), (7), (11) and (12). Note that the optimization problem is a *signomial program*, which is a generalization of *geometric programming*, and is thus non-convex in general [13]. Furthermore, it is important to note that signomial programs cannot be always transformed into convex optimization problems [14]. Hence, we apply the general method of Lagrangian multiplier with equality constraints to solve all optimization problems in the present work, see [15, pp. 323–335]. In order to ensure the global optimality of our results, it is necessary to find all stationary points of the associated Lagrangian, by considering the corresponding derivatives, and show that the number of stationary points is equal to one so as to obtain sufficiency, as well. In a case, where the number of stationary points is greater than one, more effort is needed to ensure sufficiency, e.g., by checking the regularity condition of each stationary point [15, p. 325] and its Hessian.

B. Power Allocation Subject to the Sum-Power Constraint

In this case, the output power-range constraint per SN is assumed to be greater than the sum-power constraint and thus does not have any effect on the optimization problem, because the feasible set of the optimization problem is only limited by the sum-power constraint. This leads to the corresponding constrained Lagrange function (relaxation with respect to the range of W_k, u_k and $|v_k|$)

$$\begin{aligned} &L_1(W_k, u_k, v_k; \eta_1, \eta_2, \tau; \xi) \\ &\quad := \sum_{k=1}^{K} |v_k|^2 \big(u_k^2 |h_k|^2 M_0 + N_0\big) \\ &\qquad + \bigg(1 - \sum_{k=1}^{K} \sqrt{W_k} u_k |v_k g_k h_k| \cos(\vartheta_k + \phi_k)\bigg)\eta_1 \\ &\qquad - \bigg(\sum_{k=1}^{K} \sqrt{W_k} u_k |v_k g_k h_k| \sin(\vartheta_k + \phi_k)\bigg)\eta_2 \\ &\qquad + \bigg(P_{\text{tot}} - \xi - \sum_{k=1}^{K} W_k \left(1 + u_k^2\, r_{\text{rms}}^2\, |g_k|^2\right)\bigg)\tau, \end{aligned} \tag{14}$$

where η_1, η_2 and τ are Lagrange multipliers while ξ is a slack variable.

In order to satisfy (12), all phases $\vartheta_k + \phi_k$ have to be equal to $q_k\pi$, $q_k \in \mathbb{Z}$, for all $k \in \mathbb{F}_K$. If there were a better solution for $\vartheta_k + \phi_k$, then the first partial derivatives of L_1 with respect to ϑ_k would vanish at that solution, due to the continuity of trigonometric functions. But the first derivatives would lead to the equations $\eta_1 \sin(\vartheta_k + \phi_k) = \eta_2 \cos(\vartheta_k + \phi_k)$ which cannot simultaneously satisfy both equations (11) and (12) for all η_1 and η_2. Thus, $q_k\pi$ is the unique solution. Hence, we may consequently write a modified Lagrange function as

$$\begin{aligned}\tilde{L}_1(W_k, u_k, |v_k|, q_k; \eta_1, \tau; \xi)\\ := \sum_{k=1}^{K} |v_k|^2\big(u_k^2|h_k|^2 M_0 + N_0\big)\\ +\left(1 - \sum_{k=1}^{K} \sqrt{W_k} u_k |v_k g_k h_k| \cos(q_k\pi)\right)\eta_1\\ +\left(P_{\text{tot}} - \xi - \sum_{k=1}^{K} W_k\left(1 + u_k^2\, r_{\text{rms}}^2\, |g_k|^2\right)\right)\tau. \quad (15)\end{aligned}$$

At any stationary point of $\tilde{L}_1$ the first partial derivatives of $\tilde{L}_1$ with respect to W_k, u_k, $|v_k|$, η_1 and τ must vanish, if they exist. This leads to

$$\frac{\partial \tilde{L}_1}{\partial W_l} = -\frac{u_l |v_l h_l g_l| \cos(q_l\pi)}{2\sqrt{W_l}}\eta_1 - \big(1 + u_l^2 r_{\text{rms}}^2 |g_l|^2\big)\tau = 0, \quad l \in \mathbb{F}_K, \quad (16)$$

$$\frac{\partial \tilde{L}_1}{\partial |v_l|} = 2|v_l|\big(u_l^2|h_l|^2 M_0 + N_0\big) - \sqrt{W_l} u_l |h_l g_l| \cos(q_l\pi)\eta_1 = 0, \quad l \in \mathbb{F}_K, \quad (17)$$

$$\frac{\partial \tilde{L}_1}{\partial u_l} = 2|v_l|^2 u_l |h_l|^2 M_0 - \sqrt{W_l}|v_l h_l g_l| \cos(q_l\pi)\eta_1 - 2W_l u_l r_{\text{rms}}^2 |g_l|^2 \tau = 0, \quad l \in \mathbb{F}_K, \quad (18)$$

$$\frac{\partial \tilde{L}_1}{\partial \eta_1} = 1 - \sum_{k=1}^{K} \sqrt{W_k} u_k |v_k g_k h_k| \cos(q_k\pi) = 0 \quad (19)$$

and

$$\frac{\partial \tilde{L}_1}{\partial \tau} = P_{\text{tot}} - \xi - \sum_{k=1}^{K} W_k\left(1 + u_k^2\, r_{\text{rms}}^2\, |g_k|^2\right) = 0. \quad (20)$$

By multiplying (17) with $|v_l|$, summing up the outcome over all l, and using the identities (11) and (13), we obtain

$$\eta_1 = 2V \quad (21)$$

which is a positive real number due to definition of V. Because of the last relationship and according to (17), the value of $\cos(q_l\pi)$ must be a positive number and hence each q_l must be an even integer number. Thus, we can choose $q_l^\star = 0$ for all $l \in \mathbb{F}_K$ and conclude

$$\vartheta_l^\star = -\phi_l, \quad l \in \mathbb{F}_K. \quad (22)$$

This solution gives the identity $\cos(q_l^\star\pi) = 1$ which can be incorporated into (16), (17), (18) and (19).

Again by multiplying (17) with

$$\frac{1}{2}\,\frac{u_l|h_l g_l|\sqrt{W_l}}{u_l|h_l|^2 M_0 + N_0}, \quad (23)$$

summing up the outcome over all l, and using (11), (13) and (21), we obtain

$$V = \frac{\eta_1}{2} = \left[\sum_{k=1}^{K} \frac{u_k^2|h_k g_k|^2 W_k}{u_k^2|h_k|^2 M_0 + N_0}\right]^{-1}. \quad (24)$$

In turn, by incorporating (24) into (17), it yields

$$|v_l| = \frac{V u_l |h_l g_l| \sqrt{W_l}}{u_l^2|h_l|^2 M_0 + N_0} \quad (25)$$

for all $l \in \mathbb{F}_K$.

Note that for each feasible u_l and W_l, $l \in \mathbb{F}_K$, equation (25) describes a feasible value for each $|v_l|$. Since for each $u_l W_l > 0$ the relation $|v_l| > 0$ consequently follows, the feasible optimal values of each $|v_l| > 0$ are not on the boundary $|v_l| = 0$. Thus, finding optimal values for each u_l and W_l, $l \in \mathbb{F}_K$, leads to optimum values for each $|v_l|$, $l \in \mathbb{F}_K$, due to the convexity of (15) with respect to each $|v_l|$. Hence, finding a unique global optimum for u_l and W_l, $l \in \mathbb{F}_K$, yields the sufficient condition for the globally optimal solution of the minimization problem (15).

We replace each $|v_l|$ in (16) and (18) with (25) and thus we obtain two equations for τ as

$$\tau = \frac{-V^2}{1 + u_l^2 r_{\text{rms}}^2 |g_l|^2}\,\frac{u_l^2|h_l g_l|^2}{u_l^2|h_l|^2 M_0 + N_0} \quad (26)$$

and

$$\tau = \frac{-V^2}{r_{\text{rms}}^2}\,\frac{|h_l|^2 N_0}{\big(u_l^2|h_l|^2 M_0 + N_0\big)^2}. \quad (27)$$

Note that because of the negativity of τ, due to (26) or (27), and positivity of η_1 there exists a feasible subspace in which the optimization problem (15) is convex in both u_l and W_l, $l \in \mathbb{F}_K$, as well. Hence, the Lagrange function (15) is convex near the optimum/stationary point in each u_l, $|v_l|$ and W_l, but it seems not to be a jointly convex function, at all. Since the Lagrangian is separately convex in each direction, the stationary point cannot be a maximum. To be a saddle point is also not possible, because then there would at least exist one additional stationary point which is not the case here. Thus, the Lagrangian (15) must actually be a jointly convex function in the neighborhood of its stationary point. Furthermore, since the number of stationary points is equal to one, all equality (active) constraints are regular. Hence, the separate convexity together with the regularity condition is even a sufficient condition for global optimality in the present case.

For the sake of simplicity and in order to compare the results later on, we define new quantities as

$$\alpha_k := \frac{M_0}{r_{\text{rms}}^2|g_k|^2} \;\Rightarrow\; \alpha_k \in \mathbb{R}_+, \quad (28)$$

$$\beta_k := \frac{N_0}{|h_k|^2} \;\Rightarrow\; \beta_k \in \mathbb{R}_+, \quad (29)$$

$$c_k := \sqrt{\alpha_k} + \sqrt{\beta_k} \;\Rightarrow\; c_k \in \mathbb{R}_+, \quad (30)$$

and

$$\tilde{u}_k := M_0 u_k^2 \;\Leftrightarrow\; u_k = +\sqrt{\frac{\tilde{u}_k}{M_0}}. \tag{31}$$

By direct algebra from (26) and (27), we infer the optimal solution for each u_l as

$$\tilde{u}_l^\star = \sqrt{\alpha_l \beta_l} \quad \Leftrightarrow \quad u_l^\star = \sqrt{\frac{\sqrt{N_0}}{r_{\text{rms}}\sqrt{M_0}\,|h_l g_l|}}\,,\; l \in \mathbb{F}_K. \tag{32}$$

After replacing u_l in (27) with (32), it follows

$$\tau = -\Big(\frac{V\, r_{\text{rms}}}{c_l}\Big)^2. \tag{33}$$

Since τ is a constant and thus should be independent on any index l, only some of the SNs can be active. In order to identify active SNs, we re-index all SNs such that the inequality chain

$$c_k \le c_{k+1}, \quad k \in \mathbb{F}_{K-1}, \tag{34}$$

holds. Then by using (32), we may rewrite (20) and (24) as

$$1 = (P_{\text{tot}} - \xi)\left(\sum_{k=1}^{K} \frac{W_k c_k}{\sqrt{\alpha_k}}\right)^{-1} \tag{35}$$

and

$$V = \left(\frac{1}{r_{\text{rms}}^2} \sum_{k=1}^{K} \frac{W_k}{\sqrt{\alpha_k}\, c_k}\right)^{-1} \tag{36}$$

respectively. In turn, we incorporate (35) into (36) and infer

$$V = \left[\frac{P_{\text{tot}} - \xi}{r_{\text{rms}}^2 \sum\limits_{k=1}^{K} \frac{W_k c_k}{\sqrt{\alpha_k}}} \sum_{k=1}^{K} \frac{W_k c_k}{\sqrt{\alpha_k}} \frac{1}{c_k^2}\right]^{-1}. \tag{37}$$

It is obvious that (37) is strictly increasing with respect to ξ. Thus, the optimal value for the slack variable is zero, i.e., $\xi^\star = 0$. Moreover, the quadratic mean in (37) is less than its greatest quadratic element. Thus and because of (34), the inequality

$$V \ge \left(\frac{P_{\text{tot}}}{r_{\text{rms}}^2 c_1^2}\right)^{-1} \tag{38}$$

arises consequently. Hence, the optimal value for the objective is given as

$$V^\star = \frac{r_{\text{rms}}^2 c_1^2}{P_{\text{tot}}}. \tag{39}$$

If the value of c_1 is unique, i.e., $c_1 < c_k$ for all $k \in \mathbb{F}_K$ with $k > 1$, then only the first SN is active and consumes the whole available sum-power P_{tot}. All other SNs participate neither in the data communication, nor in the classification of the target object. Its corresponding transmission powers W_1 and X_1 result from (35) and (5) as

$$W_1^\star = \frac{\sqrt{\alpha_1} P_{\text{tot}}}{c_1} \quad \text{and } X_1^\star = \frac{\sqrt{\beta_1} P_{\text{tot}}}{c_1}. \tag{40}$$

In turn, the weight $|v_1|$ results from incorporating (32), (39) and (40) into (25). This leads to

$$|v_1^\star| = \sqrt{\frac{r_{\text{rms}}^2 c_1}{P_{\text{tot}}\,|h_1|^2 \sqrt{\beta_1}}}\,. \tag{41}$$

Note that the global optimality of the obtained results is trivially reasoned. Firstly, the Lagrangian (14) has only one stationary point as shown by the results (22), (32), (39), (40) and (41). Secondly, the Lagrange function (15) is jointly convex near the optimum point in u_l, $|v_l|$ and W_l as aforementioned.

Now, we consider the improbable case, which is investigated only for theoretical reasons. If the value of c_1 is not unique, i.e., the first $\tilde{K}$ SNs have the same value of $c_k = c_1$ for all $k \in \mathbb{F}_{\tilde{K}}$ with $1 < \tilde{K} \le K$, then the power allocation has no unique solution. The available sum-power can arbitrary be allocated among the first $\tilde{K}$ SNs providing that both relationships (5), (32) and (35) hold. From these equations and after allocating the available sum-power arbitrary to the sensing powers W_k, we obtain

$$X_k^\star = W_k^\star \sqrt{\frac{\beta_k}{\alpha_k}}, \quad k \in \mathbb{F}_{\tilde{K}}, \tag{42}$$

and

$$|v_k^\star| = \sqrt{\frac{W_k^\star\, r_{\text{rms}}^2 c_k^2}{P_{\text{tot}}^2\,|h_k|^2 \sqrt{\alpha_k \beta_k}}}, \quad k \in \mathbb{F}_{\tilde{K}}. \tag{43}$$

Note that in this case, the global optimality cannot be ensured, because the Lagrangian (14) has more than one stationary point and thus is not bijective. Nevertheless, we conjecture that all results (22), (32), (39), (42) and (43) describe also globally optimal points, since the global optimum point of the relaxed problem coincides with the real range of all variables.

Note that by using the above results, the corresponding fusion rule is simplified by discarding the influence of inactive SNs from the fusion rule. The fusion rule (9) becomes

$$\tilde{r} = \sum_{k=1}^{\tilde{K}} y_k, \quad i \in \mathbb{F}_I. \tag{44}$$

The equations (22), (32) and (39), either with (40) and (41), or with some arbitrary sensing powers and (42)–(43), are the solutions of the power allocation problem only subject to the sum-power constraint. They are hence the main contribution of the present subsection.

C. Interpretation of the Solution

In practice, the value of each c_k is in general unique such that only a single SN is active. The SN with the smallest c_k consumes the whole available sum-power P_{tot}, because the combination of its sensing and communication channel is the best compared to other SNs. Hence, the information reliability of each SN is determined by the value of the corresponding c_k which can be interpreted as *interference-power*. All other SNs do not get any transmission power, since their information reliability is too poor to be considered for data fusion. They can be discarded from the fusion rule such that the observation of the target object is less interfered by noise and consequently results in a better data communication.

Note that $\tilde{r}$ is an unbiased estimator for each r_i due to constraint (10). By similar methods we can also minimize the mean squared error without restricting ourself to unbiased estimators. Obviously, the optimal value of V will then be smaller than that in (39).

D. Power Allocation Subject to Individual Power Constraints

In the current case, the sum-power constraint is assumed to be much greater than the output power-range constraint and thus does not have any effect on the optimization problem, because the feasible set of the optimization problem is only limited by the output power-range constraints. This leads to the corresponding constrained Lagrange function (relaxation with respect to the range of W_k, u_k and $|v_k|$)

$$\begin{aligned}L_2(W_k,u_k,v_k;\eta_1,\eta_2,\lambda_k;\varrho_k)\\ := \sum_{k=1}^{K}|v_k|^2\big(u_k^2|h_k|^2M_0+N_0\big)\\ +\Big(1-\sum_{k=1}^{K}\sqrt{W_k}u_k|v_kg_kh_k|\cos(\vartheta_k+\phi_k)\Big)\eta_1\\ -\Big(\sum_{k=1}^{K}\sqrt{W_k}u_k|v_kg_kh_k|\sin(\vartheta_k+\phi_k)\Big)\eta_2\\ +\sum_{k=1}^{K}\Big(P_{\max}-\varrho_k-W_k\left(1+u_k^2\,r_{\text{rms}}^2\,|g_k|^2\right)\Big)\lambda_k,\end{aligned} \tag{45}$$

where λ_k are new Lagrange multipliers while ϱ_k are new slack variables.

Since the behavior of L_2 is identical to that of L_1 with respect to $|v_k|$ and ϑ_k, we obtain the same results for the phases as given in (22). Hence, we may modify L_2 as

$$\begin{aligned}\tilde{L}_2(W_k,u_k,|v_k|;\eta_1,\lambda_k;\varrho_k)\\ := \sum_{k=1}^{K}|v_k|^2\big(u_k^2|h_k|^2M_0+N_0\big)\\ +\Big(1-\sum_{k=1}^{K}\sqrt{W_k}u_k|v_kg_kh_k|\Big)\eta_1\\ +\sum_{k=1}^{K}\Big(P_{\max}-\varrho_k-W_k\left(1+u_k^2\,r_{\text{rms}}^2\,|g_k|^2\right)\Big)\lambda_k.\end{aligned} \tag{46}$$

Note that since the equality $\sin(\vartheta_k^\star+\phi_k)=0$ holds due to (22), the constraint (12) is discarded in (46).

At any stationary point of $\tilde{L}_2$ the first partial derivatives of $\tilde{L}_2$ with respect to W_k, u_k, $|v_k|$, η_1 and λ_k must vanish, if they exist. This leads to

$$\frac{\partial\tilde{L}_2}{\partial W_l} = -\frac{u_l|v_lh_lg_l|}{2\sqrt{W_l}}\eta_1-\big(1+u_l^2r_{\text{rms}}^2|g_l|^2\big)\lambda_l=0,\ l\in\mathbb{F}_K, \tag{47}$$

$$\frac{\partial\tilde{L}_2}{\partial|v_l|} = 2|v_l|\big(u_l^2|h_l|^2M_0+N_0\big)-\sqrt{W_l}u_l|h_lg_l|\eta_1=0,\quad l\in\mathbb{F}_K, \tag{48}$$

$$\begin{aligned}\frac{\partial\tilde{L}_2}{\partial u_l} &= 2|v_l|^2u_l|h_l|^2M_0-\sqrt{W_l}|v_lh_lg_l|\eta_1\\ &\quad -2W_lu_lr_{\text{rms}}^2|g_l|^2\lambda_l=0,\ l\in\mathbb{F}_K,\end{aligned} \tag{49}$$

$$\frac{\partial\tilde{L}_2}{\partial\eta_1} = 1-\sum_{k=1}^{K}\sqrt{W_k}u_k|v_kg_kh_k|=0 \tag{50}$$

and

$$\frac{\partial\tilde{L}_2}{\partial\lambda_l}=P_{\max}-\varrho_l-W_l\left(1+u_l^2\,r_{\text{rms}}^2\,|g_l|^2\right)=0,\quad l\in\mathbb{F}_K. \tag{51}$$

By similar procedure as described in Subsection III-B, we obtain the same results as given in (24), (25) and (32), because the equations (47)–(50) and (16)–(19) are pairwise the same except of the difference between τ and λ_l. Incorporating (32) and (51) into (24), and using the same definitions as in (28)–(31), lead to

$$V=\left[\sum_{k=1}^{K}\frac{P_{\max}-\varrho_k}{r_{\text{rms}}^2c_k^2}\right]^{-1}, \tag{52}$$

which is obviously strictly increasing with respect to each ϱ_k. Thus, the optimal value for each slack variable is zero, i.e., $\varrho_k^\star=0$ for all $k\in\mathbb{F}_K$. Hence, we infer

$$V^\star=\left[\frac{P_{\max}}{r_{\text{rms}}^2}\sum_{k=1}^{K}\frac{1}{c_k^2}\right]^{-1}, \tag{53}$$

and by considering the relationships (5), (32) and (51), it follows

$$W_l^\star=\frac{P_{\max}\sqrt{\alpha_l}}{c_l}\quad\text{and}\quad X_l^\star=\frac{P_{\max}\sqrt{\beta_l}}{c_l} \tag{54}$$

for all $l\in\mathbb{F}_K$. In turn, the weights $|v_l|$ result from incorporating (32), (53) and (54) into (25). This leads to

$$|v_l^\star|=\frac{1}{\sum\limits_{k=1}^{K}c_k^{-2}}\sqrt{\frac{r_{\text{rms}}^2}{P_{\max}\,c_l^3|h_l|^2\sqrt{\beta_l}}},\ l\in\mathbb{F}_K. \tag{55}$$

As mentioned in Subsection III-B, the global optimality of the obtained results is trivially reasoned. Firstly, the Lagrangian (45) has only one stationary point as shown by the results (22), (32), (53), (54) and (55). Secondly, the Lagrange function (45) is jointly convex near the optimum point as discussed in Subsection III-B.

Note that by using the above results, the corresponding fusion rule cannot be simplified, since all SNs are active and they cannot thus be discarded from the fusion rule.

The equations (22), (32), (53), (54) and (55) are the optimal solution of the power allocation problem only subject to the output power-range constraint per SN. They are hence the main contribution of the present subsection.

E. Comparison of Solutions

In the last described power allocation case, all SNs are active in contrast to the case described in Subsection III-B where only a single SN is active. Their transmission power is equal to the output power-range constraint $P_{\max}$, according to (54). In order to compare both methods from Subsection III-B and III-D, we have to look at the values in (39) and (53). If the elements of the increasing sequence $(c_1, c_2, \ldots, c_K)$ are such conditioned that the inequality

$$\frac{P_{\text{max}}}{P_{\text{tot}}} < \frac{1}{c_1^2 \sum_{k=1}^{K} c_k^{-2}} \tag{56}$$

holds, then the power allocation method in Subsection III-B achieves a better solution than that in III-D. If the reverse inequality in (56) holds, then the method in Subsection III-D should be favored over III-B. But, the above comparison is not really fair, because the amount of the sum-power for the case described in Subsection III-D is equal to KP_{max} which should be compared to P_{tot}. Hence, we have to regard the inequality

$$\frac{1}{K}\sum_{k=1}^{K} c_k^{-2} < c_1^{-2} \tag{57}$$

for a fair decision, instead of (56), in order to assess both power allocation methods and choose the corresponding application. Interestingly, the inequality in (57) always holds, because the quadratic mean of any sequence is always less than or equal to the greatest element of the same sequence. Hence, the power allocation method from Subsection III-B is always better than that in III-D as long as in both cases the same sum-power is consumed. This means that a powerful single radar is better than a distributed multiple-radar system, if the single radar is established at the right position. On the other hand, if the right position is unknown, which is the common case, then a distributed multiple-radar system with more sum-power is needed to achieve the same or better performance than a single radar system. This is the case if the inequality

$$KP_{\text{max}} \geq \frac{P_{\text{tot}} c_1^{-2}}{\frac{1}{K}\sum_{k=1}^{K} c_k^{-2}} \tag{58}$$

holds, where the left hand side is equal to the sum-power consumption of the distributed multiple-radar system.

F. Power Allocation Subject to Both Types of Constraints

In the current subsection, we consider the optimization problem from Subsection III-A subject to all constraints, i.e., a sum-power constraint as well as an output power-range constraint per SN. Two of three different cases can be singled out and reduced to preceding instances.

First, if $KP_{\text{max}} < P_{\text{tot}}$, then the sum-power constraint is irrelevant, because the feasible set is only limited by the output power-range constraints. Hence, the power allocation problem reduces to the one described in Subsection III-D with results given in (22), (32), (53), (54) and (55). The only difference is that a part of the available sum-power remains unallocated and cannot be used.

Secondly, if $P_{\text{tot}} \leq P_{\text{max}}$, then the output power-range constraint is irrelevant, because the feasible set is only limited by the sum-power constraint. Hence, the power allocation problem is equal to the one described in Subsection III-B. The corresponding results are (22), (32) and (39), either with (40) and (41), or with some arbitrary sensing powers and (42)–(43).

The case of $P_{\text{max}} < P_{\text{tot}} \leq KP_{\text{max}}$ is the most challenging one. The amount of the available sum-power is possibly inadequate to supply each SN with a power equal to P_{max}. Besides, it is not possible to allocate the available sum-power only to a single SN since $P_{\text{max}} < P_{\text{tot}}$. Hence, it will be shown that for the optimal solution only a subset of $\tilde{K} \leq K$, $\tilde{K} > 1$, SNs are active.

Similar to the procedures in the previous subsections, we consider the corresponding constrained Lagrange function (relaxation with respect to the range of W_k, u_k and $|v_k|$)

$$\begin{aligned} &L_3(W_k, u_k, v_k; \eta_1, \eta_2, \tau, \lambda_k; \xi, \varrho_k) \\ &:= \sum_{k=1}^{K} |v_k|^2\big(u_k^2|h_k|^2 M_0 + N_0\big) \\ &\quad + \Big(1 - \sum_{k=1}^{K} \sqrt{W_k} u_k |v_k g_k h_k| \cos(\vartheta_k + \phi_k)\Big)\eta_1 \\ &\quad - \Big(\sum_{k=1}^{K} \sqrt{W_k} u_k |v_k g_k h_k| \sin(\vartheta_k + \phi_k)\Big)\eta_2 \\ &\quad + \Big(P_{\text{tot}} - \xi - \sum_{k=1}^{K} W_k\left(1 + u_k^2 r_{\text{rms}}^2 |g_k|^2\right)\Big)\tau \\ &\quad + \sum_{k=1}^{K}\Big(P_{\text{max}} - \varrho_k - W_k\left(1 + u_k^2 r_{\text{rms}}^2 |g_k|^2\right)\Big)\lambda_k. \end{aligned} \tag{59}$$

Since the behavior of L_3 is identical to that of L_1 and L_2 with respect to $|v_k|$ and ϑ_k, we obtain the same results for the phases as given in (22). Hence, we may modify L_3 as

$$\begin{aligned} &\tilde{L}_3(W_k, u_k, |v_k|; \eta_1, \tau, \lambda_k; \xi, \varrho_k) \\ &:= \sum_{k=1}^{K} |v_k|^2\big(u_k^2|h_k|^2 M_0 + N_0\big) \\ &\quad + \Big(1 - \sum_{k=1}^{K} \sqrt{W_k} u_k |v_k g_k h_k|\Big)\eta_1 \\ &\quad + \Big(P_{\text{tot}} - \xi - \sum_{k=1}^{K} W_k\left(1 + u_k^2 r_{\text{rms}}^2 |g_k|^2\right)\Big)\tau \\ &\quad + \sum_{k=1}^{K}\Big(P_{\text{max}} - \varrho_k - W_k\left(1 + u_k^2 r_{\text{rms}}^2 |g_k|^2\right)\Big)\lambda_k. \end{aligned} \tag{60}$$

At any stationary point of $\tilde{L}_3$ the first partial derivatives of $\tilde{L}_3$ with respect to W_k, u_k, $|v_k|$, η_1, τ and λ_k must vanish, if they exist. This leads to

$$\begin{aligned} \frac{\partial \tilde{L}_3}{\partial W_l} &= -\frac{u_l |v_l h_l g_l|}{2\sqrt{W_l}}\eta_1 \\ &\quad - \big(1 + u_l^2 r_{\text{rms}}^2 |g_l|^2\big)(\tau + \lambda_l) = 0\,,\ l \in \mathbb{F}_K, \end{aligned} \tag{61}$$

$$\begin{aligned} \frac{\partial \tilde{L}_3}{\partial |v_l|} &= 2|v_l|\big(u_l^2|h_l|^2 M_0 + N_0\big) - \sqrt{W_l} u_l |h_l g_l| \eta_1 = 0, \\ &\qquad l \in \mathbb{F}_K, \end{aligned} \tag{62}$$

$$\begin{aligned} \frac{\partial \tilde{L}_3}{\partial u_l} &= 2|v_l|^2 u_l |h_l|^2 M_0 - \sqrt{W_l} |v_l h_l g_l| \eta_1 \\ &\quad - 2W_l u_l r_{\text{rms}}^2 |g_l|^2 (\tau + \lambda_l) = 0\,,\ l \in \mathbb{F}_K, \end{aligned} \tag{63}$$

$$\frac{\partial \tilde{L}_3}{\partial \eta_1} = 1 - \sum_{k=1}^{K} \sqrt{W_k} u_k |v_k g_k h_k| = 0, \tag{64}$$

$$\frac{\partial \tilde{L}_3}{\partial \tau} = P_{\text{tot}} - \xi - \sum_{k=1}^{K} W_k \left(1 + u_k^2 r_{\text{rms}}^2 |g_k|^2\right) = 0 \tag{65}$$

and

$$\frac{\partial \tilde{L}_3}{\partial \lambda_l} = P_{\max} - \varrho_l - W_l \left(1 + u_l^2 r_{\text{rms}}^2 |g_l|^2\right) = 0, \ l \in \mathbb{F}_K. \tag{66}$$

By the same method as described in Subsection III-B, we obtain the same results as given in (24), (25) and (32), because the equations (61)–(64) and (16)–(19) are pairwise the same except of the difference between τ and $\tau + \lambda_l$. Incorporating (32) and (66) into (24) and (65), and using the same definitions as in (28)–(31), lead to

$$V^{-1} = \sum_{k=1}^{K} \frac{P_{\max} - \varrho_k}{r_{\text{rms}}^2 c_k^2}, \tag{67}$$

and

$$P_{\text{tot}} - \xi = \sum_{k=1}^{K} (P_{\max} - \varrho_k). \tag{68}$$

As one can see, the minimization of the signomial program in (59) is reduced to the maximization of (67) subject to (68) and $0 \leq \varrho_k \leq P_{\max} < P_{\text{tot}}$ with respect to each ϱ_k, $k \in \mathbb{F}_K$. Since the new maximization problem is a *linear program* of special structure, it is amenable to an optimal solution via majorization theory, see [16, p. 133, Proposition H.2.c]. For any admissible $\varrho_1, \varrho_2, \ldots, \varrho_K$, every sequence of the form

$$\big(\underbrace{P_{\max} - \varrho_1, P_{\max} - \varrho_2, \ldots, P_{\max} - \varrho_K}_{K \text{ elements}}\big) \tag{69}$$

is majorized by the sequence

$$\big(\underbrace{\underbrace{P_{\max}, P_{\max}, \ldots, P_{\max}}_{(\tilde{K}-1) \text{ elements}}, P_{\text{tot}} - (\tilde{K}-1) P_{\max}, 0, 0, \ldots, 0}_{K \text{ elements}}\big). \tag{70}$$

If the sequence of the elements c_k is ordered in a strictly ascending manner, the optimum achieving point of the objective is unique and described by (70). Hence, the scalar product of the sequence $(c_1^{-2}, c_2^{-2}, \ldots, c_K^{-2})$ with (70) describes the optimum value of (67) and thus yields the unique solution of the optimization problem. This means that only the first $\tilde{K}$ SNs are active, where each of which receives $P_{\max} = W_k + X_k$ for its data transmission, with the exception of the last one. The last SN consumes the remaining part of the available sum-power which is given by

$$P_{\text{remain}} \coloneqq P_{\text{tot}} - (\tilde{K} - 1) P_{\max}, \tag{71}$$

where the relationship $0 < P_{\text{remain}} \leq P_{\max}$ holds. The last $K - \tilde{K}$ SNs remain inactive. Note that the slack variable ξ vanishes due to complementary slackness, i.e., $\xi^\star = 0$. In summary, we obtain

$$W_k^\star = \frac{P_{\max} \sqrt{\alpha_k}}{c_k} \quad \text{and} \quad X_k^\star = \frac{P_{\max} \sqrt{\beta_k}}{c_k} \tag{72}$$

for the first $\tilde{K} - 1$ nodes, and

$$W_{\tilde{K}}^\star = \frac{P_{\text{remain}} \sqrt{\alpha_{\tilde{K}}}}{c_{\tilde{K}}} \quad \text{and} \quad X_{\tilde{K}}^\star = \frac{P_{\text{remain}} \sqrt{\beta_{\tilde{K}}}}{c_{\tilde{K}}} \tag{73}$$

for the last active SN. The number $\tilde{K}$ of active SNs results from $0 < P_{\text{remain}} \leq P_{\max}$, that must be fulfilled for the last SN, and is given by the smallest integer number for which the inequality

$$\tilde{K} \geq \frac{P_{\text{tot}}}{P_{\max}} \tag{74}$$

holds. The remaining optimum values result from incorporating (32), (72) and (73) into (67) and (25). This leads to

$$V^\star = \left[\frac{P_{\text{tot}}}{r_{\text{rms}}^2 c_{\tilde{K}}^2} + \frac{P_{\max}}{r_{\text{rms}}^2} \sum_{k=1}^{\tilde{K}} (c_k^{-2} - c_{\tilde{K}}^{-2}) \right]^{-1}, \tag{75}$$

$$|v_l^\star| = \frac{1}{P_{\text{tot}} c_{\tilde{K}}^{-2} + P_{\max} \sum_{k=1}^{\tilde{K}} (c_k^{-2} - c_{\tilde{K}}^{-2})} \sqrt{\frac{r_{\text{rms}}^2 P_{\max}}{c_l^3 |h_l|^2 \sqrt{\beta_l}}} \tag{76}$$

for all $l \in \mathbb{F}_{\tilde{K}-1}$, and

$$|v_{\tilde{K}}^\star| = \frac{\sqrt{P_{\text{tot}} - (\tilde{K}-1) P_{\max}}}{P_{\text{tot}} c_{\tilde{K}}^{-2} + P_{\max} \sum_{k=1}^{\tilde{K}} (c_k^{-2} - c_{\tilde{K}}^{-2})} \sqrt{\frac{r_{\text{rms}}^2}{c_{\tilde{K}}^3 |h_{\tilde{K}}|^2 \sqrt{\beta_{\tilde{K}}}}}. \tag{77}$$

Note that because of the same argumentation as in Subsections III-B and III-D, the global optimality of the obtained results is ensured.

Now, we consider the improbable case, which is investigated only for theoretical reasons. If the sequence of the elements c_k is ordered in an ascending (not necessarily strictly) manner and in addition the relationship $c_{k_1} = c_{k_1+1} = \cdots = c_{k_2}$ is satisfied for some $k_1 < k_2$ with $1 \leq k_1 \leq \tilde{K} \leq k_2 \leq K$, then the optimum achieving point of the objective is not unique and (69) is majorized by any sequence of the form

$$\big(\underbrace{\underbrace{P_{\max}, P_{\max}, \ldots, P_{\max}}_{(k_1-1) \text{ elements}}, \omega_{k_1}, \omega_{k_1+1}, \ldots, \omega_{k_2}, 0, 0, \ldots, 0}_{K \text{ elements}}\big), \tag{78}$$

where the subsequence $(\omega_{k_1}, \omega_{k_1+1}, \ldots, \omega_{k_2})$ is a decreasing (not necessarily strictly) sequence of non-negative real numbers not greater than $P_{\max}$ with a cumulative sum of $P_{\text{tot}} - (k_1 - 1) P_{\max}$. Hence, the scalar product of the sequence $(c_1^{-2}, c_2^{-2}, \ldots, c_K^{-2})$ with any sequence of the form (78) describes an optimum value of (67) and thus yields a possible solution of the optimization problem. This means that the first $\tilde{K}$ SNs are active while the SNs with an index greater than $\tilde{K}$ and less than $k_2 + 1$ may be active. Moreover, the SNs with an index greater than k_2 remain inactive. The first $k_1 - 1$ SNs receive $P_{\max}$ for their data transmission while all other active nodes consume at most $P_{\max}$. Note that like in the previous case, the slack variable ξ vanishes due to complementary slackness, i.e., $\xi^\star = 0$. In summary, we obtain

$$W_k^\star = \frac{P_{\max} \sqrt{\alpha_k}}{c_k} \quad \text{and} \quad X_k^\star = \frac{P_{\max} \sqrt{\beta_k}}{c_k} \tag{79}$$

for the first $k_1 - 1$ nodes, and

$$W_k^\star = \frac{\omega_k \sqrt{\alpha_k}}{c_k} \quad \text{and} \quad X_k^\star = \frac{\omega_k \sqrt{\beta_k}}{c_k} \tag{80}$$

for the remaining active SNs. The number of active SNs depends on the desired power allocation among the remaining SNs and may be determined by the user or any other control instance. The remaining optimum values result from incorporating (32), (79) and (80) into (67) and (25). This leads to

$$V^\star = \left[\frac{P_{\text{tot}}}{r_{\text{rms}}^2 c_{k_1}^2} + \frac{P_{\max}}{r_{\text{rms}}^2} \sum_{k=1}^{k_1} (c_k^{-2} - c_{k_1}^{-2}) \right]^{-1}, \tag{81}$$

$$|v_l^\star| = \frac{1}{P_{\text{tot}}\, c_{k_1}^{-2} + P_{\max} \sum\limits_{k=1}^{k_1} (c_k^{-2} - c_{k_1}^{-2})} \sqrt{\frac{r_{\text{rms}}^2 P_{\max}}{c_l^3 |h_l|^2 \sqrt{\beta_l}}} \tag{82}$$

for all $l \in \mathbb{F}_{k_1 - 1}$, and

$$|v_l^\star| = \frac{1}{P_{\text{tot}}\, c_{k_1}^{-2} + P_{\max} \sum\limits_{k=1}^{k_1} (c_k^{-2} - c_{k_1}^{-2})} \sqrt{\frac{r_{\text{rms}}^2 \omega_l}{c_l^3 |h_l|^2 \sqrt{\beta_l}}} \tag{83}$$

for all remaining active SNs. Note that because of the same argumentation as in Subsection III-B, the global optimality of the obtained results cannot be ensured. Nevertheless, we conjecture that all results (22), (32) and (79)–(83) describe also globally optimal points.

Note that in the considered case, the fusion rule may be more complicated than in (44), since more SNs are active in general. On the other hand, the fusion rule may be less complicated than that from Subsection III-D, because not all SNs are possibly active.

In summary, equations (22), (32), (72)–(77) and (79)–(83) are the solution to the power allocation problem subject to both types of constraints. They are hence the main contribution of the present subsection.

G. Discussion of Solutions

We make the following observation on the solutions of the power allocation problem by comparing results from Subsections III-B and III-D with III-F.

In practice, the value of each c_k is in general unique such that the inequality chain $c_k < c_{k+1}$ for all $k \in \mathbb{F}_{K-1}$ holds. In this case, the optimal value of the objective (75) is decreasing with respect to both P_{tot} and $P_{\max}$. If $P_{\max}$ is fixed and P_{tot} varies in the range $P_{\max} \le P_{\text{tot}} \le K P_{\max}$, then the optimal value of the objective (75) is decreasing with respect to P_{tot} because the SNR of the whole sensor network is increasing with P_{tot}. The best situation is achieved only when all SNs are active, i.e., $P_{\text{tot}} = K P_{\max}$. In contrast, if P_{tot} is fixed and $P_{\max}$ varies in the range $\frac{1}{K} P_{\text{tot}} \le P_{\max} \le P_{\text{tot}}$, then the optimal value of the objective (75) is decreasing with respect to $P_{\max}$ because the capability of each SN is increasing with $P_{\max}$. The best situation is achieved only when a single SN is active, i.e., $P_{\max} = P_{\text{tot}}$.

In a practical application, the value of $P_{\max}$ is fixed and P_{tot} can suitably be adjusted within the extended range $0 < P_{\text{tot}} \le K P_{\max}$. In order to save energy, the value of P_{tot} should be as less as possible, which means that a single SN or only a few SNs are active. On the other hand, to accurately estimate additional quantities such as position, velocity, acceleration, angle of movement, and other important properties and parameters of the target object, more than few SNs are needed to be active. Hence, if the number $\tilde{K}$ of active SNs is satisfactory to accurately estimate all important parameters of the target, then the best energy-aware value of P_{tot} is equal to $\tilde{K} P_{\max}$. In turn, the value of $P_{\max}$ should be large enough to achieve a desired classification or detection probability. With this setup, all three system parameters $\tilde{K}$, $P_{\max}$ and P_{tot} are optimally determined for an energy-aware system design.

Note that all solutions from Subsections III-B, III-D, and III-F are different to the well-known *water-filling* solution, see [17]. The difference to the water-filling solution emerges from the fact that the information flow over each effective path, consisting of a single SN, its sensing channel, the modest signal processing of the same SN, and its communication channel followed by the associated weight in the fusion center, is adjustable due to the power optimization. Thus, on the one hand, the diversity of each effective path is not predetermined such that the water-filling solution cannot hold in its general form. On the other hand, the diversity of best effective paths is amplified in comparison to the diversity of poorest effective paths because of the optimal solution to the power allocation.

IV. CLASSIFICATION

As we have seen in the last section, we are able to optimize w_k, u_k and v_k such that the estimate $\tilde{r}$ is unbiased for each i. The input of the decision unit is hence a noisy version of the true reflection coefficient r_i, where the noise is complex-valued and additive white Gaussian with zero-mean. This can easily be seen by considering (9) together with the identity (10), which leads to

$$\tilde{r} = r_i + \sum_k (m_k u_k^\star h_k + n_k) v_k^\star, \quad i \in \mathbb{F}_I, \tag{84}$$

where the sum only ranges over all active SNs. If we consider the definition (13) and use one of the results from (39), (53), (75) or (81), then the corresponding covariance matrix of $\tilde{r}$ for each i is given as

$$\frac{V^\star}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{85}$$

Hence, the conditional probability density of $\tilde{r}$, given object i, is obtained as

$$f_i(r) = \frac{1}{\pi V^\star} \exp\left(-\frac{|r - r_i|^2}{V^\star} \right), \quad r \in \mathbb{C}, \quad i \in \mathbb{F}_I. \tag{86}$$

Due to the simple form of the conditional densities and identical covariance matrices for all i, we may use a distance classifier (nearest-neighbor algorithm) for the global classification rule. Distance classifiers are easily implementable, because in the present case we deal with linear discriminant functions. Furthermore, they yield a high classification performance [11] because they coincide with the Bayes classifier. Their average probability of correct classification can be calculated by

$$\int\limits_{r\in\mathbb{C}} \max_{i\in\mathbb{F}_I}\big(\pi_i f_i(r)\big)\,\mathrm{d}r, \tag{87}$$

for a single observation. Integral (87) is analytically extremely hard to evaluate, if at all possible. However, accurate numerical solutions are attainable. Note that the case of no prior information, i.e., $\pi_i = \frac{1}{I}$ for all $i \in \mathbb{F}_I$, is also included in the proposed classification method. Furthermore, classification methods based on other optimal criterions, for example multi-hypotheses Neyman-Pearson test, are conveniently applicable. At this point, we again emphasize that the proposed methods are suitable for detection and classification, and less appropriate for localization and tracking.

If the reflection coefficients are placed very close to each other in the complex plane, then the outcome of (87) can sometimes be unsatisfactory. In such cases, the performance of target classification can be improved by increasing the number of observations, provided that the whole network with its parameters and the target object are static during all observations.

Note that the outcome of the above integral must finally be averaged over the position of the target object as well as the realization of all channel coefficients g_k and h_k.

V. Conclusion

The main contribution of the present work is to present an optimal solution to the power allocation problem in distributed active multiple-radar systems subject to different power constraints. We have introduced a system model, a linear fusion rule and a simple objective function, which enable us to analytically solve the power allocation problem for the region of high SNR. Three different cases of power constraints have been investigated. For a limitation of transmission power per sensor node and a sum-power limitation as well as their combination, we have analytically obtained optimal solutions in closed-form. Furthermore, all proposed solutions are valid for AWGN channels as well as for frequency-flat slow-fading channels, provided that channel state information is available at each receiver. The proposed methods also support selecting the right number of sensor nodes which transmit information more reliably than all other ones. This selection method allows us to decrease the number of active sensor nodes. It subsequently increases the classification performance while the computational complexity is simultaneously decreased. Furthermore, all proposed methods enable the application of simple distance classifiers which are easy to implement and achieve high classification performance.

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References

[1] G. Alirezaei and R. Mathar, "Optimum power allocation for sensor networks that perform object classification," in *Proc. Austral. Telecommun. Netw. Appl. Conf. (ATNAC)*, Christchurch, New Zealand, Nov. 2013, pp. 1–6.

[2] R. Srinivasan, "Distributed radar detection theory," *IEE Proc. F Commun., Radar Signal Process.*, vol. 133, no. 1, pp. 55–60, Feb. 1986.

[3] A. L. Hume and C. J. Baker, "Netted radar sensing," in *Proc. IEEE Radar Conf.*, May 2001, pp. 23–26.

[4] L. Pescosolido, S. Barbarossa, and G. Scutari, "Radar sensor networks with distributed detection capabilities," in *Proc. IEEE Radar Conf.*, May 2008, pp. 1–6.

[5] Y. Yang, R. S. Blum, and B. M. Sadler, "A distributed and energy-efficient framework for Neyman-Pearson detection of fluctuating signals in large-scale sensor networks," *IEEE J. Sel. Areas Commun.*, vol. 28, no. 7, pp. 1149–1158, Sep. 2010.

[6] G. Alirezaei and R. Mathar, "Channel capacity related power allocation for distributed sensor networks with application in object classification," in *Proc. Int. Conf. Comput., Netw. Commun. (ICNC)*, Jan. 2013, pp. 502–507.

[7] S. Gezici *et al.*, "Localization via ultra-wideband radios: A look at positioning aspects for future sensor networks," *IEEE Signal Process. Mag.*, vol. 22, no. 4, pp. 70–84, Jul. 2005.

[8] C. Debes, J. Riedler, A. M. Zoubir, and M. G. Amin, "Adaptive target detection with application to through-the-wall radar imaging," *IEEE Trans. Signal Process.*, vol. 58, no. 11, pp. 5572–5583, Nov. 2010.

[9] G. Alirezaei and R. Mathar, "Power allocation for power-limited sensor networks with application in object classification," in *Proc. Global Inf. Infrastruct. Symp. (GIIS)*, Choroní, Venezuela, Dec. 2012, pp. 1–5.

[10] G. Alirezaei, M. Reyer, and R. Mathar, "Optimum power allocation in sensor networks for passive radar applications," *IEEE Trans. Wireless Commun.*, vol. 13, no. 6, pp. 3222–3231, Jun. 2014.

[11] R. O. Duda, P. E. Hart, and D. G. Stork, *Pattern Classification*, 2nd ed. New York, NY, USA: Wiley, 2000.

[12] A. Lapidoth, *A Foundation in Digital Communication*. Cambridge, U.K.: Cambridge Univ. Press, 2009.

[13] M. Chiang, *Geometric Programming for Communication Systems*. Princeton, NJ, USA: NOW, 2005.

[14] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.

[15] D. G. Luenberger and Y. Ye, *Linear and Nonlinear Programming*, 3rd ed. New York, NY, USA: Springer, 2008.

[16] A. W. Marshall, I. Olkin, and B. C. Arnold, *Inequalities: Theory of Majorization and Its Applications*, 2nd ed. New York, NY, USA: Springer, 2011.

[17] S. Stańczak, M. Wiczanowski, and H. Boche, *Fundamentals of Resource Allocation in Wireless Networks: Theory and Algorithms*, 2nd ed. Berlin, Germany: Springer-Verlag, 2008.

Gholamreza Alirezaei, photograph and biography not available at the time of publication.

Rudolf Mathar, photograph and biography not available at the time of publication.