A Computationally Efficient Block Transmission Scheme Based on Approximated Cholesky Factors

C. Vincent Sinn
Telecommunications Laboratory
University of Sydney, Australia
cvsinn@ee.usyd.edu.au

Daniel Bielefeld
Theoretical Information Technology
RWTH Aachen University, Germany
bielefeld@ti.rwth-aachen.de

Jürgen Götze
Information Processing Laboratory
University of Dortmund, Germany
juergen.goetze@uni-dortmund.de

***Abstract*— Least square and minimum mean square error data estimation based on rectangular and circulant channel submatrices is compared in terms of estimation quality, storage requirements and the number of multiplications. The circulant case is applied in OFDM based transmission systems. When applying an approximated Cholesky decomposition, the computation of the estimator in the rectangular case may require even less computations than in the circulant case, while yielding better bit error rates in a time dispersive wireless transmission scenario. The considered approximations may improve the condition of the estimator and thus may even yield lower bit error rates than the exact estimator. The actual computation of the estimates is usually more efficient in the circulant case than in the rectangular.**

I. INTRODUCTION

In high speed wireless communications the transmitted data symbols will usually suffer from severe inter symbol interference caused by the time dispersive channel. Since the data symbols are assumed to be transmitted very fast, only a very short time period is available in real time applications to remove this interference. Processing power is also limited, therefore there is a need to perform the interference removal very efficiently. Block transmission systems relying on Orthogonal Frequency Domain Multiplexing (OFDM) are seen to fulfill this requirement and are therefore used in many high speed wireless communication standards [1], [3], [4]. If we describe the OFDM based block transmissions by a linear data model, different concept are found that contribute to OFDM's low complexity in equalizing the wireless channel: Data blocking is used to create small circulant structured submatrices, least square estimates of the transmitted data are computed blockwise on the basis of the small submatrices, and the eigenvalue decomposition (EVD) of the circulant submatrices is used [6], [8], [9].

It is known that the estimation of the transmitted data based on rectangular structured submatrices shows significant advantages in terms of the quality of the estimates and the guaranteed existence of the estimator [5], [6]. However, it is seen to be far more computational demanding in terms of number of multiplications and storage requirements. Here we suggest an implementation of the data estimator on the basis of rectangular submatrices by using approximated Cholesky decompositions. By using the approximations, the required number of multiplications to compute the estimator and the required storage may be even smaller than in the circulant case. However, the estimation of the data itself requires more multiplications in the rectangular case than in the circulant case, if the size of one data block is only a few times longer than the channel vector, which is usually the case in the above cited WLAN standards. We also find that the considered approximations may have a positive effect on the estimation quality. They may improve the condition number of the exact Cholesky factor and thus feature lower noise enhancement, which might be reflected in lower bit error rates of a block transmission system applying the approximated Cholesky factors instead of the exact ones.

A system model that is used to discuss blockwise Least Square (LS) and Minimum Mean Square Error (MMSE) estimation on the basis of rectangular and circulant submatrices is presented in Section II. The number of multiplications and the storage requirements for computing the data estimates are compared for the two structures of submatrices. In the circulant case, the EVD is applied, in the rectangular case the Cholesky decomposition. The usage of the approximated Cholesky factors is discussed in Section III. Bit error rates of block transmission systems that apply the considered estimation methods are compared in Section IV. Conclusions are drawn in Section V.

II. SYSTEM MODEL

A data vector $\boldsymbol{d} \in \mathbb{C}^{JB}$ is to be transmitted over a time dispersive wireless channel that is described by a channel vector $\boldsymbol{h} \in \mathbb{C}^{L}$. On the channel, which is assumed to be time invariant during the transmission of the data vector $\boldsymbol{d}$, a noise vector $\boldsymbol{n} \in \mathbb{C}^{JB+L-1}$, which is obtained by sampling a white Gaussian noise process with power σ^2, is additively superimposed. The received vector $\boldsymbol{x} \in \mathbb{C}^{JB+L-1}$ can then be computed according to

$$\boldsymbol{x} = \boldsymbol{H}\boldsymbol{d} + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{H} \in \mathbb{C}^{(JB+L-1)\times(JB)}$ denotes the channel convolution matrix. The data vector $\boldsymbol{d}$ consists of J data blocks $\boldsymbol{d}^{(j)} \in \mathbb{C}^{B}$ that are arranged amongst one another. The receiver shall compute linear estimates $\hat{\boldsymbol{d}}$ of the transmitted data vector $\boldsymbol{d}$ by using perfect channel state information (knowledge of vector $\boldsymbol{h}$), an estimate of the power σ^2 and the received vector $\boldsymbol{x}$. Assuming uncorrelated data $\boldsymbol{d}$ and noise $\boldsymbol{n}$ and an autocorrelation of the data according to $E\{\boldsymbol{d}\boldsymbol{d}^H\} = \boldsymbol{I}$,

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where $\boldsymbol{I}$ denotes an identity matrix, MMSE estimates of the transmitted data and the MMSE estimator matrix are obtained from

$$\hat{\boldsymbol{d}}_{\text{mmse}} = \left(\boldsymbol{H}^H\boldsymbol{H} + \sigma^2\boldsymbol{I}\right)^{-1}\boldsymbol{H}^H\boldsymbol{x}. \tag{2}$$

Setting the power σ^2 to zero in this equation yields the LS estimates of the transmitted data and also the LS estimator.

If we use this approach in high speed wireless communications we are confronted with the fact that only a very short period of time is available to compute the MMSE/LS estimator from the channel vector $\boldsymbol{h}$ and the noise power σ^2 and to compute the matrix vector product between the estimator and the received vector. This approach might be inappropriate. First, because processing power is limited, and the larger the involved matrices and the dimension L of the channel vector, the more processing power and storage is required to compute the MMSE/LS estimator and the data estimates. Second, the assumption of time invariant channels does not hold for arbitrarily long data vectors $\boldsymbol{d}$.

To reduce the required computational complexity it would be very helpful to reduce the size of the data vector $\boldsymbol{d}$, therefore reducing the size of the matrices that are involved in the computation of the estimates. If we divide the data vector $\boldsymbol{d}$ in equation (1) into many (lets say J) data blocks $\boldsymbol{d}^{(j)}$ of block size B and separate the data blocks by guard periods of sufficient length, we obtain several much smaller systems of equations that are independent of each other. The LS/MMSE estimation may then be performed on the basis of the small systems of equations. This concept is known as data blocking. It may reduce the required computational complexity enormously: We have to compute the LS/MMSE estimator only once for one small data block, which means we may compute the estimator on the basis of a small channel submatrix. The same estimator can then be used to compute all estimates of the J data blocks $\boldsymbol{d}^{(j)}$ that belong to the data vector $\boldsymbol{d}$ in a blockwise manner. Therefore, both the required computational complexity to compute the estimator and to compute the estimated symbols is reduced. Once the estimator is computed, we would need $B + L - 1$ complex multiplications for the computation of one estimated symbol instead of $JB + L - 1$.

If we use zero pads that serve as a guard period between the data blocks, we may decompose the channel matrix $\boldsymbol{H}$ of equation (1) into many small rectangular channel matrices $\bar{\boldsymbol{H}}_B$. We illustrate this for the case $J = B = L = 2$, where we have to use $L - 1 = 1$ zero pad:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} h_1 & & & & \\ h_2 & h_1 & & & \\ & h_2 & h_1 & & \\ & & h_2 & h_1 & \\ & & & h_2 & h_1 \\ & & & & h_2 \end{bmatrix} \begin{bmatrix} d_1^{(1)} \\ d_2^{(1)} \\ 0 \\ d_1^{(2)} \\ d_2^{(2)} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \end{bmatrix} \tag{3}$$

$$= \left[\begin{array}{cc|cc} h_1 & & & \\ h_2 & h_1 & & \\ & h_2 & & \\ \hline & & h_1 & \\ & & h_2 & h_1 \\ & & & h_2 \end{array}\right] \begin{bmatrix} d_1^{(1)} \\ d_2^{(1)} \\ d_1^{(2)} \\ d_2^{(2)} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \end{bmatrix}. \tag{4}$$

If we use cyclic prefixing instead, add the columns that correspond to identical data symbols and discard the first $L-1$ rows we obtain independent circulant structured channel submatrices $\tilde{\boldsymbol{H}}_B$:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} h_1 & & & & & \\ h_2 & h_1 & & & & \\ & h_2 & h_1 & & & \\ & & h_2 & h_1 & & \\ & & & h_2 & h_1 & \\ & & & & h_2 & h_1 \\ & & & & & h_2 \end{bmatrix} \begin{bmatrix} d_2^{(1)} \\ d_1^{(1)} \\ d_2^{(1)} \\ d_2^{(2)} \\ d_1^{(2)} \\ d_2^{(2)} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \\ n_7 \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} x_2 \\ x_3 \\ x_5 \\ x_6 \end{bmatrix} = \left[\begin{array}{cc|cc} h_1 & h_2 & & \\ h_2 & h_1 & & \\ \hline & & h_1 & h_2 \\ & & h_2 & h_1 \end{array}\right] \begin{bmatrix} d_1^{(1)} \\ d_2^{(1)} \\ d_1^{(2)} \\ d_2^{(2)} \end{bmatrix} + \begin{bmatrix} n_2 \\ n_3 \\ n_5 \\ n_6 \end{bmatrix}. \tag{6}$$

The existence of the LS estimator in the rectangular case is guaranteed, whereas in the circulant case, the estimator might not exist. However, the MMSE estimator is guaranteed to exist in both cases.

The insertion of a cyclic prefix actually requires transmission energy, which is discarded at the receiver, whereas zero padding does not require extra transmission energy. Basically, different data blocking methods convert the large system of equation (1) into J small systems, whose system matrix is either rectangular (subscript R) or circulant (subscript C)[6]. The small systems are

$$\boldsymbol{x}_{\text{R}}^{(j)} = \bar{\boldsymbol{H}}_B\boldsymbol{d}_{\text{R}}^{(j)} + \boldsymbol{n}_{\text{R}}^{(j)}, \quad \boldsymbol{x}_{\text{C}}^{(j)} = \tilde{\boldsymbol{H}}_B\boldsymbol{d}_{\text{C}}^{(j)} + \boldsymbol{n}_{\text{C}}^{(j)}. \tag{7}$$

Data estimation may now be performed on the basis of these systems. The LS/MMSE estimators in the two cases are according to equation (2) given by

$$\boldsymbol{E}_{\text{R}} = \left(\bar{\boldsymbol{H}}_B^H\bar{\boldsymbol{H}}_B + \sigma_{\text{R}}^2\boldsymbol{I}\right)^{-1}\bar{\boldsymbol{H}}_B^H, \tag{8}$$

$$\boldsymbol{E}_{\text{C}} = \left(\tilde{\boldsymbol{H}}_B^H\tilde{\boldsymbol{H}}_B + \sigma_{\text{C}}^2\boldsymbol{I}\right)^{-1}\tilde{\boldsymbol{H}}_B^H. \tag{9}$$

Each of the J received data blocks need to be multiplied by one of those estimators. To keep the computational complexity of this operation as small as possible suitable matrix decompositions may be applied.

Here we use the Cholesky decomposition of correlation matrices and the EVD of circulant matrices [2]. The Cholesky decomposition of the correlation is given by

$$\left(\bar{\boldsymbol{H}}_B^H\bar{\boldsymbol{H}}_B + \sigma_{\text{R}}^2\boldsymbol{I}\right) = \boldsymbol{R}_B^H\boldsymbol{R}_B, \tag{10}$$

where $\boldsymbol{R}_B$ is the upper triangular Cholesky factor. The EVD is given by

$$\tilde{\boldsymbol{H}}_B = \boldsymbol{F}_B^H\boldsymbol{D}_B\boldsymbol{F}_B, \tag{11}$$

where $\boldsymbol{F}_B$ and $\boldsymbol{F}_B^H$ are the DFT and IDFT matrices of size $B \times B$. The diagonal matrix $\boldsymbol{D}_B$ contains the eigenvalues of $\tilde{\boldsymbol{H}}_B$. Applying these decompositions to the estimators in equations (8) and (9) leads to

$$\boldsymbol{E}_{\text{R}} = \boldsymbol{R}_B^{-1}\boldsymbol{R}_B^{-H}\bar{\boldsymbol{H}}_B^H, \tag{12}$$

$$\boldsymbol{E}_{\text{C}} = \boldsymbol{F}_B^H\underbrace{\left(\boldsymbol{D}_B^H\boldsymbol{D}_B + \sigma_{\text{C}}^2\boldsymbol{I}\right)^{-1}\boldsymbol{D}_B^H}_{D_{B,\text{mmse}}^{-1}}\boldsymbol{F}_B. \tag{13}$$

By using these estimators, we would require the following number of real multiplications to compute the estimates of one data block $\boldsymbol{d}^{(j)}$. In the circulant case, we have to perform a DFT, an IDFT and a multiplication by a diagonal matrix. By selecting B as a power of 2, FFT algorithms may be applied. They require $2B\ \mathrm{ld}(B/2)$ real multiplications per FFT and thus we obtain a total of $M_{\mathrm{C,EVD}} = 4B + 4B\mathrm{ld}(B/2)$ real multiplications. Note that the number is independent of the dimension L of the channel vector and depends solely on the block size B. In the rectangular case, we perform the multiplication by the inverse Cholesky factors by using back/forward substitutions. Since the Cholesky factor retains the band structure of the correlation matrix, we require less than $4LB$ real multiplications per substitution. The multiplication by the banded matrix $\bar{\boldsymbol{H}}_B^H$ requires $4LB$ real multiplications. So, we obtain a total of $M_{\mathrm{R,Chol}} < 12LB$. Here, the number depends on both the dimension L of the channel vector and the block size B. Without using the matrix decompositions, the matrix vector product of the estimator and a received data block would have required $M_{\mathrm{C}} = 4B^2$ and $M_{\mathrm{R}} = 4B(B + L - 1)$ real multiplications, respectively. We may compare the complexity of the rectangular and circulant cases by their ratio of necessary real multiplications:

$$M_{\mathrm{R,Chol}}/M_{\mathrm{C,EVD}} < 3L/(1 + \mathrm{ld}(B/2)). \tag{14}$$

Setting the block size $B = 64$ and $L = 17$ (parameters from WLAN standards [1], [3]), leads to $M_{\mathrm{R}}/M_{\mathrm{C}} < 8.5$. Smaller channel dimensions L or larger block sizes B would improve the ratio in favour of the rectangular system. However, for the considered parameters, the circulant system shows a lower number of multiplications. The advantages of the rectangular systems are higher estimation quality and the guaranteed existence of the LS estimator [6].

So far, we have considered the complexity for computing the estimates of one data block from the received symbols and the estimator. Additionally, we need to consider the computation of the estimator from the channel vector and the noise power, which will be addressed in the next section.

III. Approximated Cholesky Factors

In the circulant case, the computation of the estimator is particularly simple. We need to perform one DFT from the first column of $\tilde{\boldsymbol{H}}_B$ (denoted by $\tilde{\boldsymbol{H}}_B(:,1)$) to obtain the eigenvalues in $\boldsymbol{D}_B$. If the eigenvalues are arranged on a diagonal by using the function 'diag', we may describe the diagonal matrix $\boldsymbol{D}_B$ as

$$\boldsymbol{D}_B = \mathrm{diag}\left(\boldsymbol{F}_B \tilde{\boldsymbol{H}}_B(:,1)\right). \tag{15}$$

From this matrix and a noise power σ_{C}^2 we need to compute the diagonal matrix $\boldsymbol{D}_{B,\mathrm{mmse}}^{-1} = (\boldsymbol{D}_B^H \boldsymbol{D}_B + \sigma_{\mathrm{C}}^2 \boldsymbol{I})^{-1} \boldsymbol{D}_B^H$ and store the B values in it. The computation of $\boldsymbol{D}_B$ requires $2B\mathrm{ld}(B/2)$ real multiplications. For the computation of $\boldsymbol{D}_{B,\mathrm{mmse}}^{-1}$ further $4B$ real multiplications (and B inverse real values) are necessary. On total the computation of $\boldsymbol{D}_{B,\mathrm{mmse}}^{-1}$ requires $M_{\mathrm{invDmmse}} = 4B + 2B\mathrm{ld}(B/2)$ real multiplications.

In order to compute the Cholesky factor, we must first compute the values in the correlation matrix $\boldsymbol{C}_B = (\bar{\boldsymbol{H}}_B^H \bar{\boldsymbol{H}}_B + \sigma_{\mathrm{R}}^2 \boldsymbol{I})$. Since the correlation matrix shows a hermitian banded Toeplitz structure, we only have to compute the first row. This requires $2L(L+1)$ real multiplications. The band structure of the correlation matrix and the corresponding Cholesky factors are depicted in Figure 1. Unfortunately, the Toeplitz structure of the correlation matrix is not handed down to the Cholesky factors. The actual computation of the Cholesky factor from

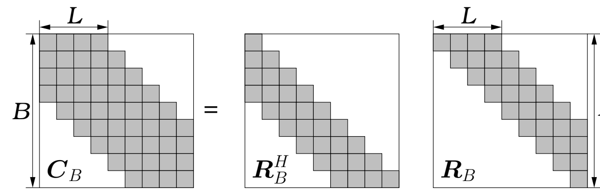

Fig. 1. Cholesky decomposition. The band structure of $\boldsymbol{C}_B$ is retained in the Cholesky factors. The Toeplitz structure is not.

the correlation matrix is performed by the Cholesky algorithm [2], [6]. It may compute the values in $\boldsymbol{R}_B$ row by row, starting with the first row. In each row, the diagonal element is computed first, followed by the non diagonal elements that are to the right of the diagonal element. We only have to compute the values in the Cholesky factor that are different from zero and we only have to perform the multiplications, where no zero value is involved. If we do so, $2(L-1)(B-L/2)$ real multiplications are required to compute the diagonal elements. The computation of the elements of one row without diagonal element require $2(L-2)(L-1)$ real multiplications. Since we may also save some operations in the computation of the first and last $L-1$ rows, we get an upper limit of the required number of real multiplications to compute the banded Cholesky factor from the channel vector and the noise power: $M_{\mathrm{bandchol}} < 2L(L+1)+2(L-1)(B-L/2)+2B(L-2)(L-1)$.

By now, the computation of the correlation matrix and the Cholesky factor seems to demand significantly more computations than the computation of the diagonal matrix $\boldsymbol{D}_{B,\mathrm{mmse}}^{-1}$ in the circulant case of the estimator. It also requires more storage space: A little less than $LB + L$ values need to be stored instead of B in the circulant case. For the example ($B = 64$, $L = 17$), this means the computation of the exact Cholesky factor requires about 37 times the number of multiplications required by the computation of $\boldsymbol{D}_{B,\mathrm{mmse}}^{-1}$ from the channel vector and the noise power. To reduce these computational requirements in the rectangular case, we consider the following approximations: The correlation matrix $\boldsymbol{C}_B$ shows a Toeplitz structure, which is not handed down to the Cholesky factor. However, the Cholesky factor of a correlation matrix that occurs in the context of joint detection of multi-user CDMA signals, shows an approximated Toeplitz structure [7]. It is observed there that the Cholesky factor draws with increasing number of computed rows much closer to a Toeplitz structure. This observation also holds for the correlation matrix $\boldsymbol{C}_B$ which we are dealing with here.

This observation might be exploited by computing only a

Matrix	Approx. number of real mult.
$D^{-1}_{B,\text{mmse}}$	896
Exact banded Cholesky factor	33108
Approx. Cholesky factor ($D = 17$)	9044
Approx. Cholesky factor ($D = 1$)	628

TABLE I

REAL MULTIPLICATIONS NECESSARY TO COMPUTE DIFFERENT MATRICES ($B = 64$, $L = 17$).

few rows, lets say D rows, of $\boldsymbol{R}_B$ and by copying the last computed row $(B-D)$ times down the diagonal. Then, an approximated Cholesky factor is obtained. If the parameter D is small enough, the required number of multiplications to compute the approximated Cholesky factor is enormously smaller than the computation of the exact Cholesky factor. It might even be smaller than the number of multiplications required to compute the values in $\boldsymbol{D}^{-1}_{B,\text{mmse}}$. The number of necessary real multiplications to compute the approximated Cholesky factors, the exact Cholesky factor and the matrix $\boldsymbol{D}^{-1}_{B,mmse}$ from the channel vector and the noise power are compared in Table I for $L = 17$ and $B = 64$. The larger the parameter D, the lower is the deviation between the approximated and the exact Cholesky factor. The approximation also lowers the required storage space since now $LD + L$ values need to be stored (the values in the first D rows of $\boldsymbol{R}_B$ and in the first row of $\bar{\boldsymbol{H}}^H_B$) instead of a little less than $LB + L$ (values in the whole Cholesky factor and in the first row of $\bar{\boldsymbol{H}}^H_B$). We may also force a complete Toeplitz structure into the Cholesky factor by copying the D^{th} row not only down the diagonal but also overwriting the first $D - 1$ rows. In this case, we only have to store $2L$ values to represent this approximation of the Cholesky factor and the banded Toeplitz matrix $\bar{\boldsymbol{H}}^H_B$. These are less values than we would store in the circulant case, since $2L$ is smaller than B in the case of $B = 64$ and $L = 17$.

IV. SIMULATIONS

In this section, we compare the bit error rates as function of E_B/N_0 values of different block transmission systems that apply the presented methods. The reference system is based on the data estimation by applying the EVD of circulant submatrices. This system is very similar to the well known OFDM based transmission systems with the main difference that the IFFT (the multi carrier modulation) is performed at the receiver. Therefore, it shows exactly the same overall complexity in computing the data estimates as OFDM based block transmissions. The circulant submatrices are created by cyclic prefixing at the transmitter side and by discarding some received symbols, see example in equations (5) and (6). The EVD of the LS/MMSE estimator, which is computed on the basis of perfect channel state information, is then used to compute the estimated data. Rectangular submatrices are created by zero padding at the transmitter. The Cholesky decomposition is used to compute the LS/MMSE estimates of the transmitted data on the basis of the rectangular submatrix. We use both the exact Cholesky factor and the approximations of it. The first approximation (App1) uses a Cholesky factor, which is computed up to the D^{th} row, which is then copied down the diagonal. In addition, the second approximation (App2) overwrites the first $D-1$ rows of the Cholesky factor with the D^{th} row to obtain a complete Toeplitz structure. The corresponding transmission systems are depicted in Figure 2.

In our simulations we use BPSK modulated data, a block size of $B = 64$ and a channel length of $L = 17$. The $L = 17$ channel taps are separated by the symbol duration and their amplitudes are Rayleigh distributed. The mean powers of the Rayleigh taps are -4, 0, -1, -3, -2, -3, -15, -7, -6, -8, -9, -10, -3, -15, -7, -6, and -8 dB, respectiveley. In Figure 3, we consider the bit error rates of transmissions using LS estimation on the basis of either circulant or rectangular submatrices. The case 'REC-LS-CHOL' uses the exact Cholesky factor in the rectangular case. It clearly outperforms "CIR-LS-EVD", which computes the LS estimates on the basis of the circulant system. The large difference between these two bit error rate curves reflect that the LS estimator might not exist in the circulant case and that even when it exists, it averages a larger condition number than the one based on rectangular submatrices. We then use an approximated Cholesky factor (case 'REC-LS-APP1-D=1') with approximation depth $D = 1$. This means only the first row of the Cholesky factor is computed, which requires particularly few computations, and is then used to build a Toeplitz structured approximated Cholesky factor. This method shows up to about 13 dB a lower simulated bit error rate than using the exact Cholesky factor. The approximated Cholesky factor shows an improved condition number compared to the exact Cholesky factor. Therefore, it causes lower noise enhancement and can finally result in lower bit error rates than the exact method. In contrast, for higher E_B/N_0 values, the influence of noise enhancement is diminished and the approximation leads to an error floor, which means that for large E_B/N_0 values the exact method results in a lower bit error rate than the approximated one. Increasing the approximation depth to $D = 17$ (case 'REC-LS-CHOL-APP1-D=17'), shows the lowest BER in the range of ratios E_B/N_0 considered. By using the second approximation, i.e. building a full Toeplitz matrix on the basis of the D^{th} computed row does not result in an improved condition of the Cholesky factor. This approximation shows the highest floor of the simulated BER, see case 'LS-CHOL-APP2-D=17'.

From this, we see that the approximations may have two effects. On one hand, they only compute the approximated solution of a system of equations, on the other hand, the approximation may reduce the influence of noise enhancement. Therefore, approximations may also result in improved quality of the estimates. This also gives a design rule of an approximation of receiver algorithms in general: The approximation should reduce the condition number of the respective system matrix.

In contrast to the LS estimator, the existence of the MMSE estimator on the basis of the circulant matrix is guaranteed. The MMSE estimator also averages a lower condition number. This results in a significantly improved bit error rate of the case 'CIR-MMSE-EVD' in Figure 4 in comparison to case

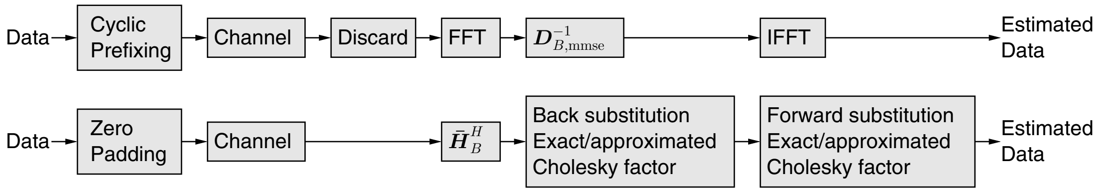

Fig. 2. Block transmission systems using zero padding and LS/MMSE estimation on the basis of either circulant or rectangular submatrices. We also apply the EVD and the exact/approximated Cholesky decomposition.

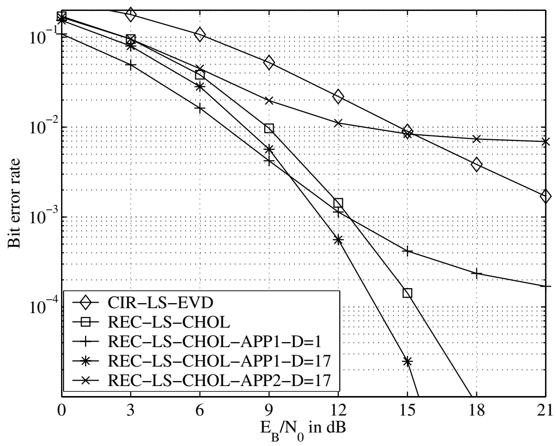

Fig. 3. Comparison of simulated bit error rates of block transmissions applying least square estimation based on rectangular (REC) or alternatively circulant (CIR) subsystems. The data estimates in the rectangular case are computed by using the Cholesky decomposition (CHOL) or alternatively by using the approximations of the Cholesky factors (APP1, APP2), which are described in Section III. The row of the Cholesky factor which is used in the approximations is either set to $D = 1$ or $D = 17$.

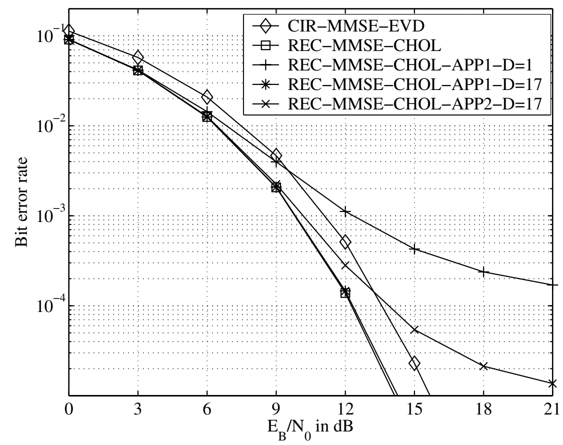

Fig. 4. Comparison of simulated BERs of block transmissions applying minimum mean square estimation based on rectangular (REC) or alternatively circulant (CIR) subsystems. The data estimates in the rectangular case are computed by using the Cholesky decomposition (CHOL) or alternatively by using the approximations of the Cholesky factors (APP1, APP2), which are described in Section III. The row of the Cholesky factor which is used in the approximations is either set to $D = 1$ or $D = 17$.

'CIR-LS-EVD' in Figure 3. However, a good estimate of the noise power σ^2 is required, which is not the case in the LS estimation. Cases 'REC-MMSE-CHOL-APP1-D=1' and 'REC-MMSE-CHOL-APP2-D=17' show a lower bit error rate than 'CIR-MMSE-EVD' up to about 9 and 13 dB, respectively. In the considered range, the first approximation 'REC-MMSE-APP1-D=17' shows a similar bit error rate than the exact Cholesky factor 'REC-MMSE-CHOL.'

V. CONCLUSIONS

The usage of the EVD for computing the LS/MMSE estimates in the circulant case may result in lower number of multiplications and lower storage requirements for computing the estimates and the estimator than using the Cholesky decomposition in the rectangular case. However, if the considered approximations of the Cholesky factors are used, storage requirements and the number of multiplications required to compute the estimator in the rectangular case may even become lower than in the circulant case. The computation of the data estimates is usually more efficient in the circulant case (assuming the block size does not significantly exceed several times the channel length).

The considered approximations may have the effect of improving the condition of the estimator in the rectangular case. This is reflected in a bit error rate that is up to a specific ratio E_{B}/N_0 even lower if the approximated instead of the exact Cholesky factor is used. One may think of designing approximations that lower beside the storage requirements and number of multiplications also the condition number. Then, the approximation may even improve bit error rates in noisy environments.

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