Power allocation through revenue-maximising pricing on a CDMA reverse link shared by energy-constrained and energy-sufficient heterogeneous data terminals

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***Abstract*—We perform market-oriented management of the reverse link of a CDMA cell populated by data terminals, each with its own data rate, channel gain, willingness to pay (wtp), and link-layer configuration, and with energy supplies that are limited for some, and inexhaustible for others. For both types, appropriate performance indices are specified. A terminal pays in proportion to its *fraction* of the total power at the receiver, which directly determines its signal-to-interference ratio (SIR), and hence its performance. Hence, a terminal can individually choose its optimal power fraction without knowing the choices made by others. The network sets for each terminal an individual price that drives the terminal to the "revenue per Watt" maximiser. An optimal link-layer configuration can be identified through this analysis. Distinguishing features of our model are: (i) the simultaneous consideration of both limited and unlimited energy supplies, (ii) the performance metrics utilised (one for each type of energy supply), (iii) the generality of our physical model, and (iv) our focus on the *fraction* of total power at the receiver allocated to a terminal.**

I. Introduction

For reverse-link CDMA power allocation, many useful decentralised algorithms patterned after games of strategy have been reported, among others, by [1], [2], [3], [4], [5], [6], among others.

Pricing is a critical tool to both generate revenues, and induce efficient resource use. Herein, we propose and analyse a technical-economic scheme for the management of the reverse link of a CDMA cell populated by data terminals (each with its own data rate, channel gain, willingness to pay (wtp), and possibly individual link-layer configuration). Some terminals are battery-powered while others have boundless energy, and we specify pertinent utility functions for each type. A terminal pays in proportion to its *fraction* of the total power at the receiver, which directly determines its signal-to-interference ratio (SIR), and hence its performance. Hence, under our scheme, a terminal can individually choose its optimal power fraction without knowing the choices made by others. By contrast, the literature typically makes a terminal pay in proportion to its (absolute) power level, which complicates substantially everyone's decision process because it makes a terminal's optimisation dependent upon the choices made by *all the others*.

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Below, we first discuss the underlying physical communication model, and specify the technical-economic rationale of the terminals. Then, we focus on the network, and arrive at our pricing scheme. Subsequently, we analyse for both types of terminal how each reacts under the proposed scheme. Then, we study how the network can set prices to maximise its revenue. Several numerical examples and figures are given throughout. In a final section, we summarise and discuss key contributions. This development overlaps with [7], but there we allocate power to maximise "social benefit", *not* network profit.

II. Generalities

A. Physical Model

N terminals *upload* data *to* a CDMA base station (BS). The index i identifies a terminal. z or x may be used as generic function arguments. E_i is the energy budget, if applicable P_i is the transmission power, $p_i := h_i P_i$ is the received power, and $\hat{P}_i$ the power constraint (if any). R_i is the data rate. The chip rate is assumed equal to the bandwidth, W. $G_i := W/R_i$ is the spreading gain. Information is sent in M_i-bit packets carrying $L_i < M_i$ information bits. The signal-to-interference ratio (SIR) at the receiver is defined as $\sigma_i = G_i \kappa_i$ with $\kappa_i = p_i/Y_i$ the carrier-to-interference ratio (CIR), and $Y_i = \sum_{j \neq i} p_j + p_0$, with p_0 the Gaussian noise. F_i is the packet-success rate function (PSRF) giving the probability of correct reception of a data packet as a function of the SIR at the receiver. For some technical reasons, $f_i(x) := F_i(x) - F_i(0)$ replaces F_i [8]. Its graph is assumed to have the S-shape shown in fig 1. Our analysis does *not* rely on any specific PSRF.

B. Terminals' rationale

Two categories of data terminals are of interest: energy-constrained (battery powered), and throughput-driven terminals, referred as e-terminals and t-terminals, below. An e-terminal focuses on the total number of information bits transferred with its total energy budget. The t-terminal focuses on information bits transferred over a reference time period (such as the time unit). The model below works for both categories.

The utility function has the "quasi-linear" form ([9, Ch. 10]): $v_i B_i + y_i$ where (i) v_i is the terminal's "willingness to pay" (wtp), which equals the monetary value to the terminal of one information bit successfully transferred, and is a constant for a

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given terminal, (ii) B_i is the (average) number of information bits the terminal has successfully transferred within a period of interest, T (e.g., battery life, for an e-terminal), and (iii) y_i is the amount of money the terminal has left after payments. B_i will generally depend on some resource, z. When the terminal must pay $c_i(z)$, it chooses z to maximise benefit minus cost:

$$v_i B_i(z) - c_i(z) \tag{1}$$

III. The Network's General Approach

Many contributions follow [1] in proposing direct per-Watt pricing, which has a mayor disadvantage: a terminal's performance is not directly determined by its own power, but rather by the ratio of its power to the interfering power at the receiver, which is determined by the other terminals. A pricing scheme based on "power ratios" (CIR, SIR, etc) would be an improvement. But only certain such ratios are feasible.

A. SIR/CIR feasibility

When does a system of N equations like

$$\frac{p_i}{\sum_{\substack{j=1\\ j\neq i}}^{N} p_j + p_0} = \frac{\sigma_i}{G_i} := \kappa_i \tag{2}$$

has a non-negative solution, and what is it? This has been studied by [10], [11] and others.

Let

$$\pi_i := \frac{\kappa_i}{1+\kappa_i} \equiv \frac{\sigma_i}{G_i+\sigma_i} \tag{3}$$

The system has a unique solution given by:

$$p_i = p_0 \pi_i / (1-\bar{\pi}) \tag{4}$$

provided that,

$$\bar{\pi} := \sum_{j=1}^{N} \pi_j < 1 \tag{5}$$

Notice that $\kappa_i = \kappa(\pi_i)$, with

$$\kappa(z) := \frac{z}{1-z} \equiv \frac{1}{z^{-1}-1} \tag{6}$$

κ is strictly increasing: $\kappa'(z) = (1-z)^{-2} > 0$.

B. Power limits and the "ruling terminal"

With $\hat{P}_i$ denoting the highest transmission power level available to terminal i and $\hat{p}_i := h_i \hat{P}_i$, combining eq. (4) with the power constraint, one obtains

$$\frac{\pi_i}{1-\bar{\pi}} p_0 \leq \hat{p}_i \rightarrow \bar{\pi} \leq 1 - \frac{\pi_i}{\hat{p}_i / p_0} \tag{7}$$

While in principle, an inequality such as (7) is needed for each active terminal, only the "tightest" such constraint is necessary. If $\pi_k/\hat{p}_k \geq \pi_i/\hat{p}_i \quad \forall i \in \{1,\ldots,N\}$, we call terminal k the "ruling terminal", and notice that then the *only* feasibility condition is

$$\bar{\pi} \leq 1 - \frac{\pi_k}{\hat{p}_k / p_0} \tag{8}$$

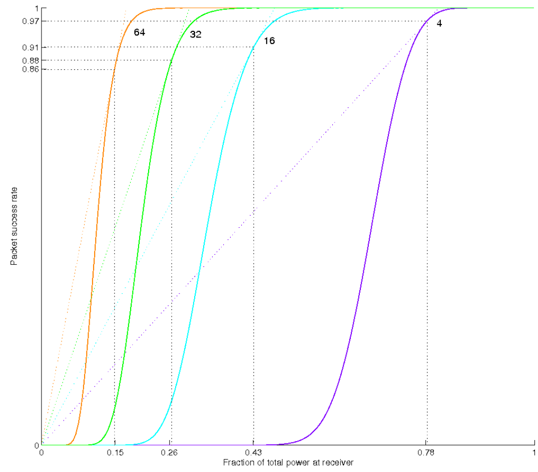


Figure 1. The composite function $f(G\kappa(z))$, with f the PSRF, G the spreading gain, and $\kappa(z) := z/(1-z)$, plays a key role. $f(x) = [1-\exp(-x/2)/2]^{80} - 2^{-80}$ is displayed.

C. Dividing "a pie" among the terminals

Notice that, with

$$\bar{p} := \sum_{i=1}^{N} p_i \tag{9}$$

and with $Y_i := \bar{p} - p_i + p_0$ (the interference experienced by terminal i), and $\kappa_i = p_i/Y_i$, π_i can be written as:

$$\frac{p_i/Y_i}{p_i/Y_i+1} \equiv \frac{p_i}{p_i+Y_i} \equiv \frac{p_i}{\bar{p}+p_0} \tag{10}$$

Thus, π_i is terminal i's fraction of the total power at the receiver, $\Pi = \sum_{i=1}^{N} p_i + p_0$.

In fact, "noise" can be viewed as "terminal 0", with $\pi_0 := p_0/(\bar{p}+p_0)$, and condition (5) re-written as $\sum_{j=0}^{N} \pi_j = 1$.

D. Power-fraction pricing

The network can set and announce the price c_i at which terminal i can "buy" π_i. For a given value of π_i, the terminal can obtain directly the corresponding CIR (equation (6)), and hence its performance. Thus, for the given c_i terminal i can choose optimally π_i, without knowing the choices made by the others.

IV. Terminal's Choice

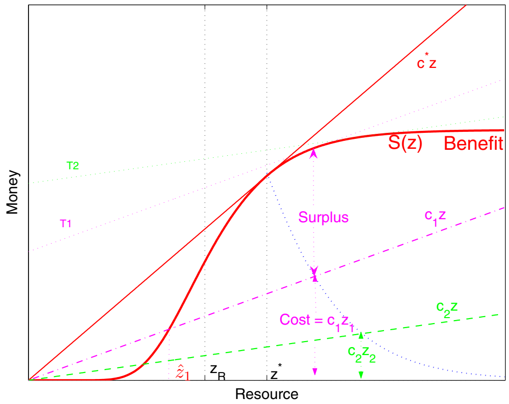
A. Choice by a throughput-driven terminal

The period of interest is some pre-specified time length, say a time unit. For a given π_i, the terminal's cost is $c_i\pi_i$, and (with $\kappa(z)$ given by eq. (6)) the average number of information bits transferred over a time unit is:

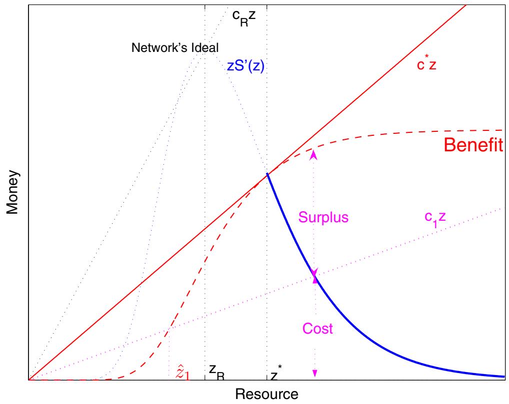
$$\frac{L_i}{M_i} R_i f_i(G_i \kappa(\pi_i)) \tag{11}$$

The terminal should choose the value of π_i that maximises benefit minus cost:

$$\frac{L_i}{M_i} R_i f_i(G_i \kappa(\pi_i)) - c_i \pi_i \tag{12}$$



(a) The terminal's side: With a power share of z, the terminal maximises benefits minus costs: $S(z)-cz$. The largest c for which the terminal choose to operate is c^*, the slope of the tangenu of S.



(b) The network's side: Revenue follows the graph $zS'(z)$. The network would prefer the price c_R but it is "too high" for the terminal. Thus, it settles for c^*.

Figure 2. Pricing involves both a terminal's side and a network's side. Once the network understands how the terminal reacts to a price level (subfig. 2(a)), it can set the price that maximises its revenue(subfig. 2(b)).

f_i is the (slightly modified) packet-success-rate function, whose graph as a function of its argument, the SIR, is an S-curve. Let

$$g_i(z;G_i) := f_i(G_i\kappa(z)) \tag{13}$$

g_i is a composite function of f_i and κ, with independent variable z and parameter G_i (denoted simply as $g_i(z)$ when no confusion is caused). It turns out that the graph of g_i (as a function of z) inherits the S-shape of f_i (see figure 1). Thus, we need to understand the maximisation of an expression of the form $S(z)-cz$, where S is some S-curve.

Figure 2(a) illustrates the solution procedure in terms of the power fraction z. First, if the line cz lies entirely above S, except at the origin, the terminal should choose $z=0$ (decline to operate), since its cost would exceed its benefit for any positive z. Otherwise, the maximising choice is a point at which the derivative of the S-curve equals c. This derivative is "bell-shaped" (similar to the curve $zS'(z)$ shown in fig. 2(b)). Therefore, if c is sufficiently large, no value of z can satisfy $S'(z)=c$. Otherwise, two values of z satisfy $S'(z)=c$, and the maximiser is the largest of the two, that is, the one to the right of the inflexion point of S, where the second derivative $S''(z)$ is negative. The largest value of c for which the problem of maximising $S(z)-cz$ has a positive solution is denoted as c^*, and as shown in fig. 2, is obtained as the slope of the tangenu (the unique tangent line of S that goes through the origin). The "genu" is the tangency point, which plays an important role below. It is easy to see that replacing S with a multiple of S will change c^*. On the other hand, basic analytical geometry tells us that z^* must satisfy $S(z^*)=z^*S'(z^*)$, which immediately implies that replacing S with a multiple of S has no effect on the value of z^*. c^* and z^* are related by $c^*=S'(z^*)=S(z^*)/z^*$.

B. Choice by an energy-limited terminal

For an energy-limited terminal, the natural period of interest is battery life.

For π_i the corresponding transmission power is

$$P_i = p_i/h_i \equiv \pi_i(\bar{p}+p_0)/h_i \tag{14}$$

With energy E_i, battery life is

$$T_i = E_i/P_i \equiv \frac{E_ih_i}{\pi_i(\bar{p}+p_0)} \tag{15}$$

The terminal's benefit is v_iB_i, with B_i the total (average) number of (energy-earned) information bits over the period T_i:

$$B_i(\pi_i) = \frac{L_i}{M_i}R_if_i(G_i\kappa(\pi_i))T_i \tag{16}$$

In view of equation (15), the terminal's total cost over the period of interest is given by

$$c_i\pi_iT_i \equiv c_i\frac{E_ih_i}{\bar{p}+p_0} \tag{17}$$

Notice that π_i drops out of the total cost expression.

The terminal chooses π_i to maximise utility (total benefit minus total cost):

$$\frac{E_ih_i}{\bar{p}+p_0}\left(\frac{L_i}{M_i}v_iR_i\frac{f_i(G_i\kappa(\pi_i))}{\pi_i}-c_i\right) \tag{18}$$

Notice that, from the standpoint of the terminal, c_i is a known constant that has been announced by the network. Thus, the positive utility-maximising choice is the same that maximises the expression:

$$\frac{f_i(G_i\kappa(z))}{z} \equiv \frac{g_i(z;G_i)}{z} \tag{19}$$

("benefit per Watt"). For the sake of technical completeness, below we let $g_i(z)/z|_{z=0} := g_i'(0)$ [8].

As previously indicated, the graph of g_i as a function of z retains the S-shape of f_i (recall eq. (13) and see fig. 1). Thus, the expression $g_i(z)/z$ is the familiar ratio of an S-curve to its argument (see [12]). The ratio is "single-peaked", as shown by figure 3. Its maximiser is unique, and found at the genu

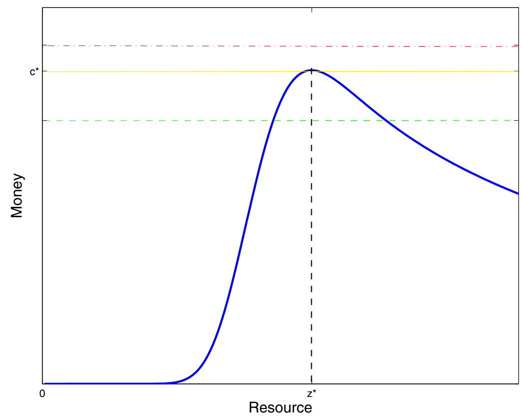


Figure 3. Revenue-maximising power-share pricing for an energy-constrained data terminal on the reverse link. For any $c \leq c^*$ the maximising choice remains z^*(occurring at the genu of $f_i(G_i\kappa(z))$ shown in fig. 1). For $c > c^*$, $z = 0$ is optimal. The network chooses c^*.

of the S-curve (tangency point with a ray emanating from the origin) (see fig. 1). If the tangency occurs beyond the feasible range the maximiser is the highest reachable value of z.

The terminal should choose the value of z that maximises $g_i(z)/z$, unless, of course, that choice results in a cost greater than the terminal's benefit (in which case the terminal should choose $\pi_i^* = 0$, i.e., decline to operate).

V. Optimal single-terminal pricing

A. Pricing for a throughput-driven terminal

From the previous discussion, as the price c grows, the terminal chooses smaller values of x. Hence, it is not obvious from the network's point of view what is the "best" c. To determine this", the network needs to understand how its revenue varies as a function of c.

First, let us ignore the terminal's power constraint. As discussed above, and illustrated by fig. 2(b), for a given $c_k \leq c^*$, the terminal chooses a value z_k that satisfies $S'(z_k) = c_k$. Then, the resulting network's revenue is $c_k z_k \equiv z_k S'(z_k)$. Thus, the network's revenue follows the curve $zS'(z)$. The curve $zS'(z)$ has a "bell shape" with a single peak at z_R. In principle, the network would like to drive the terminal to choose z_R, but at this level the terminal's cost exceeds its benefit. The highest price for which the terminal will operate is c^* for which it chooses z^*. For any $z > z^*$, $zS'(z) < z^*S'(z^*)$ as shown in fig. 2(b). Thus, the best the network can do is to set $c = c^*$, and receive revenue equals to $c^*z^* \equiv z^*S'(z^*) = S(z^*) =$

$$\frac{L}{M}\nu R f(G\kappa(z^*)) \tag{20}$$

Under certain conditions, the terminal may be unable to reach z^*. This may be result from power-emission limits, or because there are many terminals, and the "slice" left for this terminal is less than z^*. The network has then 2 choices: (i) to maintain $c = c^*$ in which case the terminal simply refuses to operate, because, as shown by fig. 2(b), $z < z^* \rightarrow c^*z > S(z)$; or (ii) to lower the price, so that the network's revenue equals the terminal's benefit when it operates at its highest reachable z. For example, suppose that $\hat{z}_1$ (shown in fig. 2(b)) is the highest z that the terminal can reach. The network can set $c = c_1$. Without limitations, when $c = c_1$ the terminal would choose z_1 (to the right of z^*). But now the terminal chooses $\hat{z}_1$ which is the only reachable z for which $c_1 z$ does not exceed the terminal's benefit. If there is only one terminal, lowering the price is best for the network. With many terminals, the network decision is less clear.

B. Pricing for an energy-constrained terminal

According to subsection IV-B, the terminal will either decline to operate, or operate at the point that maximises "benefit per Watt" (the maximiser of $g_i(z)/z$, (see eq. (19)). The specific value of c_i plays a role indirectly, because it can make the cost exceed the benefit. It makes sense for the network to set the highest value of c_i that is acceptable to the terminal (see eq. (18)). That is:

$$c_i^* = \frac{L_i}{M_i}\nu_i R_i \frac{f_i(G_i\kappa(z^*))}{z^*} \tag{21}$$

At such level, the terminal's benefit equals its cost. The total revenue provided by this terminal during the life of its battery equals (see eq. (17)) :

$$\frac{E_i h_i}{\bar{p} + p_0} c_i^* \tag{22}$$

What the network receives from this terminal per time unit equals:

$$c_i^* z^* = \frac{L_i}{M_i}\nu_i R_i f_i(G_i\kappa(z^*)) \tag{23}$$

VI. Serving many terminals

We assume that the network can set an individual price per terminal, and *in principle* treat each terminal independently, following section V. However, the sum of the π_i^* may violate (8). From all the sets of terminals that satisfy (8), the network needs to choose the "best" set — the well-known "knapsack problem".

A. Which terminals to serve?

1) The (fractional) knapsack problem: There is a finite set of items, each characterised by a "weight" and a "value". One seeks the combination of items that maximises the sum of the values, without exceeding a total weight constraint. The problem is in general NP-hard [13]. However, if one can include in the knapsack any desired fraction of any item, the problem admits a very simple and intuitive solution. Items are sorted by their "value to weight" ratio, and whole items are inserted in order. When no space is left for another whole item, the pertinent fraction of the next item is added to completely fill the knapsack [14]. In our problem, serving "a fraction" of a terminal is to admit it with a lowered π_i than it wants. However, the analogy is imperfect, because the "value" of the terminal is not linear with its "slice", π_i. Thus, the fractional knapsack solution yields a suboptimal choice in our case (which we neglect below).

2) "Benefit per Watt" priority: A terminal's "weight" should be (a function of) its service "slice", π_i, which is itself proportional to the terminal's received "Wattage" (eq. (4)). The obvious "value" measure (from the network's viewpoint) is revenue contribution, but over which period (time unit or battery life)?. The time unit is a natural choice for t-terminals. It turns out that it makes sense for the network to consider, for value-to-weight purposes, an e-terminal's revenue per second contribution. By doing so, the network measures both categories of terminal with the same yard stick. Furthermore, an e-terminal whose battery charge runs out is likely replaced by a new terminal which (statistically) has similar properties to the departing one. Thus, the network may as well focus on revenue per second.

Equations (20) and (23) are equivalent. Thus, the value/weight ratio for terminal i, while operating with a power fraction of $\pi_i > 0$ and paying the network an amount that equals the terminal's benefit, can be expressed as $v_i\hat{R}_i/\pi_i$, with $g_i(z) = f_i(G_i\kappa(z))$ as before, and

$$\hat{R}_i := \frac{L_i}{M_i} g_i(\pi_i) R_i \tag{24}$$

Given the preceding pricing analysis, at the operating point, π_i should either be (i) z^*, the value at the genu of g_i, or, if such value is "too high" for some reason, (ii) the highest reachable z.

3) Optimal physical layer configuration: Notice that $(L_i/M_i)(W/G_i)g_i(z^*;G_i)/z^*$ is determined by the physical-layer configuration (modulation, coding, data rate). If several such configurations are available, one should choose the one that offers the largest:

$$\frac{L_i}{M_i}\frac{f_i(G_i\kappa(z^*))}{G_i z^*} \tag{25}$$

because it leads to greater "benefit per Watt" when the terminal operates optimally. Thus, two terminals that have a common spreading gain (or data rate), should have a common PSRF, f_i.

B. Power limited cell

The key is subsection III-B. If $\mathcal{A}$ are the indices of a certain set of terminals, they can occupy the cell each with a power fraction π_i, if the total fractional "slice" allocated to them satisfies a condition similar to (8): with $k \in \mathcal{A}$, and $\pi_k/\bar{p}_k \geq \pi_i/\hat{p}_i \quad \forall i \in \mathcal{A}$,

$$\sum_{i\in\mathcal{A}} \pi_i \leq 1 - \frac{\pi_k}{\hat{p}_k/p_0} \tag{26}$$

Terminal k is the "ruling terminal" of the set $\mathcal{A}$. Notice that π_k appears on the left side of constraint (26). Thus, with terminal k active, the total fractional "slice" left for possible companions is

$$1 - \pi_k - \frac{\pi_k}{\hat{p}_k/p_0} \tag{27}$$

Hence, if $\pi_k(1 + p_0/\hat{p}_k) \geq 1$, then *no* other terminal can join k. And if $\pi_k \geq \hat{p}_k/p_0$ not even terminal k (alone in the cell) can reach π_k; $\bar{z}_k = \hat{p}_k/(\hat{p}_k + p_0)$ would be the best it could do.

Table I
KEY PARAMETERS FOR EACH TERMINAL.

i	v_i	G_i	π_i^*	$g(\pi_i^*)$	$\hat{R}_i$	r_i	$\hat{p}_i$
1	3,5	32	0,26	0,88	3,52	46,6	1
2	4,0	64	0,15	0,86	1,72	46,3	5
3	3,1	32	0,26	0,88	3,52	41,3	4
4	3,5	64	0,15	0,86	1,72	40,5	3
5	3,7	128	0,08	0,85	0,85	39,7	1
6	3,5	128	0,08	0,85	0,85	37,6	4
7	3,0	64	0,15	0,86	1,72	34,7	4

1) A terminal's "best subjects": Let $J(1)$, $J(2)$, ..., $J(N)$ be indices such that

$$\frac{\pi_{J(1)}}{\hat{p}_{J(1)}} \geq \frac{\pi_{J(2)}}{\hat{p}_{J(2)}} \geq \cdots \geq \frac{\pi_{J(N)}}{\hat{p}_{J(N)}}$$

Thus, when all terminals are active, terminal $J(1)$ is the ruling terminal. When $J(1)$ is *not* active, terminal $J(2)$ becomes ruling, and so on. Evidently, terminal $J(N)$ "rules", *only* when no one else is active.

The network is interested in identifying the "best subjects" of terminal $J(m)$ $(m < N)$, defined as the set of terminals that produces the most revenue (while satisfying the pertinent feasibility condition) when $J(m)$ is the ruling terminal, that is, with terminals $J(1)$, ..., $J(m-1)$ *not* active. One can identify this set, by applying the knapsack solution described in subsection VI-A1, with "knapsack capacity" given by eq. (27), that is, $1 - \pi_{J(m)}(1 + p_0/\hat{p}_{J(m)})$.

Let $\mathcal{A}_{J(m)}$ be the set that contains (the indices of) terminal $J(m)$ and its "best subjects" (with $\mathcal{A}_{J(N)} := \{J(N)\}$). The idea is to find $\mathcal{A}_{J(1)}$ and compute and store the combined revenue that $J(1)$ together with its "best subjects" produce. Subsequently, find $\mathcal{A}_{J(2)}$, and compute and store the combined revenue produced by the terminals in $\mathcal{A}_{J(2)}$. Then, proceed analogously with respect to $J(3)$, $J(4)$, and so on. Finally, from the previously obtained sets, choose the one that produces the most revenue, overall.

2) Numerical illustrations: Table I provides the key parameters for 7 terminals. Power limits are given as multiples of p_0. The common PSRF is that of fig. 1, and units are such that $(L_i/M_i)R_i = 128/G_i$. r_i stands for "value to weight" ratio. The service SIR's $(G_i\kappa(\pi_i^*))$ are 11,5, 11,1 and 10,9 for spreading gains 32, 64, and 128 respectively.

Table II applies the solution procedure of subsection VI-B1 to these terminals. The first column has the indices of the terminals sorted in order of descending value/weight ratio (r_i in table I). The top row has the "slice" that can be allocated to all terminals when the terminal whose index is directly below "rules" ($d_j := \pi_j^*/\hat{p}_j$). The second row has the terminals' indices sorted in descending order, from left to right, by ε_j (terminals 7, 2, and 6 are not shown for reasons explained below). Thus, terminal 1 has the greatest "value to weight" ratio, but coincidentally, it also has the highest $\pi_j^*/\hat{p}_j$ ratio (it is in the "worst situation"), which implies that terminal 1 "rules" when all are active. With terminal 1 absent, terminal 5 "rules"; and with 1 & 5 off, terminal 3 "rules", and so on.

The last two columns have respectively π_i and $\hat{v}_i := v_i \hat{R}_i$ (the individual revenue contribution).

Table II
A SET OF THE TERMINALS IS CHOSEN FOR SERVICE.

	0,74	0,92	0,93	0,95	$\overleftarrow{1-d_j}$	
i	**1**	**5**	**3**	**4**	$\downarrow \pi_i$	$\downarrow \hat{v}_i$
1	**[1]**	**[0]**	**[0]**	**[0]**	0,26	12,3
2	1	1	1	1	0,15	6,9
3	1	1	**[1]**	**[0]**	0,26	10,6
4	0,40	1	1	**[1]**	0,15	6,0
5	0	**[1]**	**[0]**	**[0]**	0,08	3,1
6	0	1	1	1	0,08	2,9
7	0	1	1	1	0,15	5,1
	0,74	0,87	0,79	0,52	$\overleftarrow{\sum \pi_i}$	
	30,1	**35,0**	31,9	21,0	$\overleftarrow{\sum \hat{v}_i}$	

A "1" (resp. "0") in the position (i,j), (row, column), means that when terminal j "rules", terminal i is (resp. is *not*) among the "best subjects" of j. A number between 0 and 1 at such position indicates that when j "rules", terminal i is "fractionally served". The brackets denote mandatory (from the algorithm viewpoint) inclusions or exclusions.

Thus, when terminal 1 "rules", terminal 2 and 3 are "fully" served, and terminal 4 is also served but with less than half of its desired "slice". In this case, the sum of the slices is 0,74 and the total revenue brought by these terminals is 30,1. When terminal 5 "rules" (next column), terminal 1 is turned off by construction, and each of the other terminals can be "fully" served. The sum of the slices is 0,87 and the total revenue is 35,0. At this point the algorithm can be stopped. The group "ruled" by terminal 5 is chosen, because it produces more revenue than terminal 1 and its best subjects. Thus, even though terminal 1 itself produces more revenue (in absolute and relative terms) than any other terminal acting alone, it is the only terminal left out because of its "bad situation". The final two columns (3 and 4 rules, respectively) are provided as illustration, but not needed.

VII. Discussion

We have focused on the reverse link of a CDMA network, populated by data terminals, each with its own data rate, channel gain, willingness to pay (wtp), and link-layer configuration. Some have limited energy, but others not, and we have specified appropriate performance metrics for both types. We have proposed and analysed a technical-economic scheme based on a terminal's share of the total power at the receiver: π_i. π_i immediately determines the carrier-to-interference ratio, $\kappa_i = \pi_i/(1-\pi_i)$, which directly leads to the SIR, $\sigma_i = G_i \kappa_i$. Thus, given a price on π_i, the terminal can individually make an optimal choice irrespective of choices made by others, which is a key advantage of our proposal.

The network ultimately chooses for each terminal an individual price that forces it to operate where "revenue per Watt" is highest. Of course, the sum of the "slices" ordered by the terminals may exceed resource availability. Then, the network follows a "knapsack" approach ([13]) to find, among all sets of terminals that satisfy the constraint, the revenue maximiser. Thus, our proposal simplifies the terminal's choice at the expense of (reasonably) complicating the network's. This is a favourable trade-off, given the energy and/or computational limitations of a terminal.

In [7], a planner maximises "social benefit" with a common price. The first terminals to become active are those with the "steepest" tangenu, which is precisely the criterion used by the revenue-maximising network. At the common planner's price, each active terminal pays less that it would under network's pricing, but it also "consumes" more, which tends to reduce the number of terminals that can be served.

Numerical evaluations have been performed, but excluded because of space constraints. A "game" in which each terminal chooses its power to maximise its own performance without a direct cost is used as a base-line for performance. A "player" with unlimited energy will evidently always set its power to the maximal level, but energy-conservation can be induced through a limit on total energy spent. The pricing scheme always outperforms the game, and the performance gap grows with the number of terminals in the system, and also tends to increase with "social inequality".

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