Efficient Power Allocation for OFDM with Imperfect Channel State Information

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***Abstract*—This paper studies the effect of the imperfect channel state information (CSI) on the performance of adaptive orthogonal frequency division multiplexing (OFDM) systems. To perform resource allocation, CSI must be fed back to the transmitter. Such feedback CSI is always imperfect due to the time-varying channel, the noisy channel estimation and the limited feedback. First, we analyze the imperfection of the feedback CSI from these three aspects. Then, we propose an efficient method to perform power allocation, where Jensen's inequality is used to approximate the objective of the power allocation in order to enhance the computational efficiency. Simulations show that performance loss due to the CSI imperfection and this approximation is very small for a proper frame length.**

I. Introduction

Compared to the single carrier transmission, OFDM has the simpler receiver structure to combat the effects of delay spread in frequency selective fading channels, specified in [1]. It has been applied in many present digital communication systems, such as digital audio and video broadcasting (DAB/DVB) [2], worldwide interoperability for microwave access (WiMAX) [3] and wireless local area networks (WLAN).

In multipath channels, different subcarriers in OFDM transmission generally have different channel gain-to-noise ratios (CNRs). To take advantage of this feature, water-filling [1] and other methods [4], [5] have been proposed to maximize the transmission rate or the system performance margin efficiently. These methods usually assume perfect CSI at the transmitter. However, only imperfect CSI is available in practice. The induced performance degradation has been studied in [6], [7]. In [8]–[10] the CSI imperfection has been partially analyzed.

In this paper, we thoroughly consider the CSI impairment due to the channel variation during the unavoidable delay, the noisy channel estimation and the limited feedback. The CSI error induced by the limited feedback is generally considered as complex Gaussian distributed. The Cramer-Rao lower bound (CRLB) is used to efficiently determine the variance of the channel estimation error. To keep the separability of subcarriers while allocating power, the correlation of channel coefficients of subcarriers is not taken into account. With slight performance loss by doing so, water-filling can still be used and the computational efficiency remains high.

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The paper is organized as follows. Section II presents the system model and assumptions. In Section III, the CSI imperfection is analyzed. Based on this analysis, the objective of the resource allocation is approximated to maximize the transmission rate in Section IV. Numerical results in Section V indicate that near-optimal solution can be achieved by our method and the frame length plays an important role for the system performance. Finally, this paper is concluded.

II. System Model And Assumptions

We consider an OFDM system with N subcarriers and a temporal correlation of the fading channel. The channel is composed of L multiple paths. Each path, indexed by l, is subject to independent Rayleigh fading with zero mean and variance $\sigma_{h_l}^2$. The data stream is divided into frames. Each of them consists of I OFDM symbols. At the receiver, time and frequency are perfectly synchronized. It is assumed that the channel remains invariant within the duration of one frame T, so that the same power and rate allocation scheme can be employed within a frame. This assumption has also been widely used in works on channel estimation.

By denoting the channel coefficient of path l by $h_{l,k}$ while transmitting the kth frame, the Clark's correlation function of the fading process on this path is defined as

$$E\{h_{l,k}^* h_{l,k+m}\} = \alpha_m \sigma_{h_l}^2, \tag{1}$$

where α_m is defined by the Bessel function of 0th order $J_0(2\pi f_D mT)$ [11], and f_D is the Doppler frequency. Note that k indicates the frame index but not the time index. In frequency domain, the kth received vector on the nth subcarrier is referred to as

$$\boldsymbol{Y}_{n,k} = H_{n,k}\boldsymbol{X}_{n,k} + \boldsymbol{\Omega}_{n,k}. \tag{2}$$

Each noise sample in $\boldsymbol{\Omega}_{\mathbf{n},\mathbf{k}}$ is independent complex Gaussian distributed with zero mean and variance σ_Ω^2, and $\boldsymbol{X}_{n,k}$ is the transmitted vector. To assure inter-symbol interference-free transmission, $N \geq L$ must hold. Since the L channel taps experience independent Rayleigh fading in time domain, the channel coefficients of the nth subcarrier in frequency domain are identical Rayleigh fading with variance $\sigma_H^2 = \sum_{l=0}^{L-1} \sigma_{h_l}^2 / N$. To allow for theoretical analysis, continuous rates are considered throughout this paper.

In practice, reliable transmission always requires very low bit-error rates (BERs), like quasi-error-free in DVB corresponding to BER $\approx 10^{-10}$, see [2]. Thus, we assume that our system throughout this paper is operated in the high signal-to-noise ratio (SNR) range, where low BERs are guaranteed.

III. CSI Imperfection

Due to the channel variation during the unavoidable delay, the noisy channel estimation and the limited feedback, perfect channel knowledge is not reachable by the transmitter in practice. In this section, we analyze the imperfection of the feedback CSI by considering those three causes.

A. Time-Varying Channel

After the kth frame arrives the receiver, power and rate allocation can be effectively performed for the $(k+m)$th frame at the transmitter, where the delay m is caused by the distance between the transmitter and the receiver and by the calculating time for channel estimation and resource allocation.

With the correlation function (1), the channel coefficient of the nth subcarrier in frequency domain for the $(k+m)$th frame can be written as

$$H_{n,k+m} = \alpha_m H_{n,k} + \sqrt{1-\alpha_m^2} V_{n,k}, \tag{3}$$

where the channel variation $V_{n,k}$ and the channel coefficient $H_{n,k}$ are stochastically independent, and $V_{n,k}$ is complex Gaussian distributed with zero mean and variance

$$\sigma_V^2 = \sigma_H^2 = \frac{1}{N} \sum_{l=0}^{L-1} \sigma_{h_l}^2. \tag{4}$$

Due to the independency of multiple paths in time domain, the channel variations are identically distributed over subcarriers in frequency domain.

B. Noisy Channel Estimation

If the transmitted data is known or correctly decided by the receiver, the estimated channel coefficient of the nth subcarrier in frequency domain $\hat{H}_{n,k}$ can be derived with the least-squares channel estimation, shown as

$$\hat{H}_{n,k} = \frac{\boldsymbol{Y}_{n,k}^{\mathrm{H}} \boldsymbol{X}_{n,k}}{\boldsymbol{X}_{n,k}^{\mathrm{H}} \boldsymbol{X}_{n,k}} = H_{n,k} + \underbrace{\frac{\boldsymbol{\Omega}_{n,k}^{\mathrm{H}} \boldsymbol{X}_{n,k}}{\boldsymbol{X}_{n,k}^{\mathrm{H}} \boldsymbol{X}_{n,k}}}_{E_{n,k}}.$$

The channel estimation error $E_{n,k}$ is a zero-mean complex Gaussian random variable with variance

$$\sigma_{E_{n,k}}^2 = \frac{\sigma_\Omega^2}{I P_{n,k}}, \tag{5}$$

where $P_{n,k} = \boldsymbol{X}_{n,k}^{\mathrm{H}} \boldsymbol{X}_{n,k} / I$ is the average transmission power. It is also independently distributed across subcarriers.

After a simple transform, $H_{n,k}$ is written as

$$H_{n,k} = \hat{H}_{n,k} - E_{n,k}. \tag{6}$$

The variance $\sigma_{E_{n,k}}^2$ may be interpreted as the uncertainty of $H_{n,k}$ or as the reliability of $\hat{H}_{n,k}$. It is equal to the mean square error (MSE) of channel estimation $E\{|H_{n,k} - \hat{H}_{n,k}|^2\}$. The pair $(\hat{H}_{n,k}, \sigma_{E_{n,k}}^2)$ is called a soft channel estimate, see [12]. The channel coefficient can be treated as a complex Gaussian random variable, distributed as $\mathcal{CN}(\hat{H}_{n,k}, \sigma_{E_{n,k}}^2)$.

The CRLB for the MSE of the blind or semi-blind channel estimation on one subcarrier can be interpreted intuitively. It is derived as the MSE of the least-squares channel estimation given all data symbols, as shown by (5). It is very close to the CRLB at low BER. Thus, $\sigma_{E_{n,k}}^2$ can be expressed by (5) at the range of high SNR under the earlier assumption.

Since the channel time-variation is independent with the additive channel noise, by taking (6) to (3), the channel coefficient for the $(k+m)$th frame can be predicted as

$$H_{n,k+m} \sim \mathcal{CN}(\alpha_m \hat{H}_{n,k}, \alpha_m^2 \sigma_{E_{n,k}}^2 + (1-\alpha_m^2)\sigma_H^2). \tag{7}$$

The pair $(\alpha_m \hat{H}_{n,k}, \alpha_m^2 \sigma_{E_{n,k}}^2 + (1-\alpha_m^2)\sigma_H^2)$ is called a soft channel prediction of subcarrier n for the $(k+m)$th frame. The variance represents the reliability of this channel prediction.

To keep the separability of subcarriers while performing resource allocation, we do not consider the combination of channel estimates of different subcarriers, different from [9]. Later, we will show the benefits from doing so.

C. Limited CSI Feedback

The additive channel noise has almost identical characteristics over time. The temporal correlation of the fading channel (1) can be known by the transmitter by locating the mobile receiver, channel sounding or other methods. Thus, the reliability of the channel prediction can be available at the transmitter, and the receiver only needs to feed the estimated channel coefficients back. Before that, they must be quantized, i.e., represented by a limited number of bits, denoted by B.

As above mentioned, the estimated channel coefficient $\hat{H}_{n,k}$ is constant for one channel realization. If we take the random process of the channel model into account, the estimated channel coefficient can be viewed as a complex Gaussian distributed random variable with zero mean and variance $\sigma_H^2 + \sigma_{E_{n,k}}^2$. It is the sum of two independent random variables: one is the actual channel coefficient determined by the environment, distributed as $\mathcal{CN}(0, \sigma_H^2)$; the other is the channel estimation error induced by the additive channel noise.

The quantized channel estimate of an arbitrary subcarrier n is referred to as

$$\tilde{H}_{n,k} = \hat{H}_{n,k} - Z_{n,k}, \tag{8}$$

where $Z_{n,k}$ represents the quantization error. With the earlier analysis and the distortion theory from [13], the quantization error can be generally expressed as a complex Gaussian distributed random variable with zero mean and variance

$$\sigma_{Z_{n,k}}^2 = (\sigma_H^2 + \sigma_{E_{n,k}}^2) 2^{-2B}. \tag{9}$$

Consequently, $\tilde{H}_{n,k}$ and $Z_{n,k}$ are stochastically independent.

After quantization, by taking (6) and (8) to (3), the channel coefficient of subcarrier n for the $(k+m)$th frame can be expressed by

$$H_{n,k+m}=\alpha_m \tilde{H}_{n,k} + \underbrace{\alpha_m(Z_{n,k}-E_{n,k})+\sqrt{1-\alpha_m^2}V_{n,k}}_{\eta_{n,k}}. \quad (10)$$

From the previous analysis, the channel estimation error $E_{n,k}$, the quantization error $Z_{n,k}$ and the channel time-variation $V_{n,k}$ can be treated as stochastically independent. Therefore, the integrated CSI error $\eta_{n,k}$ is complex Gaussian distributed with zero mean and variance

$$\sigma_{\eta_{n,k}}^2 = \sigma_H^2(1-\alpha_m^2+\alpha_m^2 2^{-2B}) + \frac{\alpha_m^2\sigma_\Omega^2}{IP_{n,k}}(1+2^{-2B}). \quad (11)$$

This soft prediction $(\alpha_m \tilde{H}_{n,k}, \sigma_{\eta_{n,k}}^2)$ will be used to maximize the expected throughput in the following.

IV. Throughput Maximization With Imperfect CSI

In practical adaptive OFDM systems, only imperfect CSI is available at the transmitter. The impairment to CSI, caused by the channel time-variation, the noisy channel estimation and the limited feedback, is analyzed in the previous section. In the following, we will use such imperfect channel knowledge to efficiently maximize the expected transmission rate given the fixed total transmission power.

A. Effective Channel Gain-to-Noise Ratio

Channel estimation must also be performed for the $(k+m)$th frame. With (6), the estimated channel coefficient for the $(k+m)$th frame can be acquired as

$$\hat{H}_{n,k+m} = H_{n,k+m} + E_{n,k+m}. \quad (12)$$

It can be viewed as the prediction of the estimated channel coefficient for the $(k+m)$th frame based on the channel estimate for frame m. It is also complex Gaussian distributed with zero mean, explained by taking (10) to (12). Due to the independency of the additive channel noise over time, the reliability of this prediction is $\sigma_{\eta_{n,k}}^2 + \sigma_{E_{n,k+m}}^2$. Then, the pair $(\alpha\tilde{H}_{n,k}, \sigma_{\eta_{n,k}}^2 + \sigma_{E_{n,k+m}}^2)$ is the soft prediction of the estimated channel coefficient, unlike previous works, e.g., [9].

With *perfect* CSI, the CNR of subcarrier n for the $(k+m)$th frame is

$$\frac{|H_{n,k+m}|^2}{\sigma_\Omega^2}. \quad (13)$$

With (12), the received vector through subcarrier n in the $(k+m)$th frame can be expressed by

$$\boldsymbol{Y}_{n,k+m} = \hat{H}_{n,k+m}\boldsymbol{X}_{n,k+m} + E_{n,k+m}\boldsymbol{X}_{n,k+m} + \boldsymbol{\Omega}_{n,k+m}.$$

With *imperfect* CSI, the effective CNR of the nth subcarrier for the $(k+m)$th frame is derived as

$$\frac{|\hat{H}_{n,k+m}|^2}{P_{n,k+m}\sigma_{E_{n,k+m}}^2 + \sigma_\Omega^2} = \frac{|\hat{H}_{n,k+m}|^2}{\sigma_\Omega^2(1+\frac{1}{I})}, \quad (14)$$

which includes the CSI imperfection due to the channel variation during the feedback delay, the limited feedback, twice noisy channel estimations for the kth and $(k+m)$th frames, and the additive channel noise. In [14], the allocated power on subcarriers for the $(k+m)$th frame is included in the effective noise power. In this work, (14) shows that the allocated power can be excluded from the effective noise power, which is only related to the frame length.

B. Maximization for Expected Sum Rate

For the $(k+m)$th frame, given the constraint on the total transmission power, expressed by $\sum_{n=1}^N P_{n,k+m} \le P$, the primal objective of the rate-adaptive problem is to maximize the instantaneous per-frame data transmission rate, shown as

$$\max_{P_{n,k+m}} \sum_{n=1}^N \log_2(1+\frac{P_{n,k+m}|H_{n,k+m}|^2}{\sigma_\Omega^2}),$$

while CSI is perfectly known by the transmitter.

However, according to the earlier analysis, only the soft prediction of the estimated channel coefficient is available at the transmitter including the prediction of the estimated channel coefficient and its reliability. The aim of the rate-adaptive problem is changed to maximize the expected transmission rate, formulated as

$$C = \max_{P_{n,k+m}} E\left\{\sum_{n=1}^N \log_2\left(1+\frac{P_{n,k+m}|\hat{H}_{n,k+m}|^2}{\sigma_\Omega^2(1+\frac{1}{I})}\right)\right\}. \quad (15)$$

Since the frequency response on each subcarrier $|\hat{H}_{n,k+m}|$ is Rice distributed, it is very hard to analytically solve the above problem, so the following approximation is employed.

By using Jensen's inequality, an upper bound to (15) is given as

$$\begin{aligned} C &\le C_1 \\ &= \max_{P_{n,k+m}} \sum_{n=1}^N \log_2\left(1+\frac{P_{n,k+m}E\{|\hat{H}_{n,k+m}|^2\}}{\sigma_\Omega^2(1+\frac{1}{I})}\right) \\ &= \max_{P_{n,k+m}} \sum_{n=1}^N \log_2\left(1+\frac{P_{n,k+m}(\alpha_m^2|\tilde{H}_{n,k}|^2+\sigma_{\eta_{n,k}}^2)+\frac{\sigma_\Omega^2}{I}}{\sigma_\Omega^2(1+\frac{1}{I})}\right), \end{aligned}$$

see [13], [15]. The above equation shows that C_1 is still positive even if the allocated power on each subcarrier is zero. To solve this problem, the objective can be further written as

$$\begin{aligned} C_2 &= C_1 - N\log_2(1+\frac{1}{I+1}) \\ &= \max_{P_{n,k+m}} \sum_{n=1}^N \log_2\left(1+\frac{P_{n,k+m}(\alpha_m^2|\tilde{H}_{n,k}|^2+\sigma_{\eta_{n,k}}^2)}{\sigma_\Omega^2(1+\frac{2}{I})}\right). \end{aligned}$$

Then, water-filling with linear complexity of $\mathcal{O}(N)$ [1] can be employed to determine the power allocation.

If the combination of channel estimates over subcarriers is considered, the effective noise on one subcarrier would be additionally related to the transmission power allocated on other subcarriers. Then, water-filling could not be used. The computational efficiency of channel estimation and resource allocation would be degraded. It follows that the feedback delay would be extended and the performance of resource allocation would decrease due to the time-varying channel. Furthermore, even though the performance of the channel estimation and resource allocation would be improved by doing so, this improvement is not necessary while the frame length I is large. For example, in WiMAX [3], a $5\,\text{ms}$ frame contains $I = 48$ OFDM symbols. This means that the combination of channel estimates across OFDM symbols within a frame has $16.8\,\text{dB}$ gain for channel estimation and already makes the channel estimation error very small. Further improvement would not benefit the adaptive transmission too much. However, if zero power is allocated on one subcarrier in frame k, pilot symbols or the combination over subcarriers has to be used in order to obtain the channel prediction of this subcarrier for the $(k+m)$th frame.

V. Simulation Results

In this section, the impact of CSI impairment on adaptive OFDM systems is presented by simulations. In the simulation, we use the system parameters of WiMAX from [3]. The OFDM system is composed of 256 subcarriers. The duration of one OFDM symbol is $103\,\mu s$. The carrier frequency is $2.4\,\text{GHz}$. Since the system performance must increase in the number of bits representing feedback CSI, we set $B = 4$ to make the impact of limited feedback small. The frequency selective channel is modeled as consisting of $N/8$ independent Rayleigh distributed paths with an exponential attenuation profile and $\sigma_H^2 = 1$. The noise power on each subcarrier is $0\,\text{dB}$. For convenience, the transmission power is uniformly allocated over subcarriers for frame k. The channel estimates are provided by the least-squares channel estimation. With the channel estimation for frame k, we perform resource allocation for the $(k+m)$th frame with $m = 1$ at minimum. This will not impair the generality of our simulation.

Fig. 1 gives the rate achievement with imperfect CSI in comparison to the one with perfect CSI and the one using uniform power allocation across subcarriers. We choose three values for the frame length $I = 1, I = 10$ and $I = 100$. When the terminal moves at the walking speed $v = 5\,\text{km/h}$, the rate achievement with imperfect CSI and $I = 10$ is larger than the other two with $I = 1$ and $I = 100$. It approaches to the ideal rate achievement as the transmission power increases.

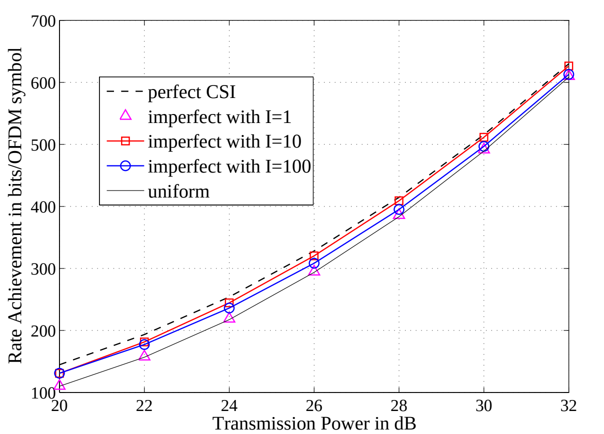

Fig. 1. Achieved rate vs. transmission power with $v = 5$ km/h.

When a frame contains 100 OFDM symbols, the channel estimation error is very small and the channel variation during the feedback delay is the main cause of performance degradation. When only one OFDM symbol is in each frame, the adaptive resource allocation does not have any benefit compared to the uniform power allocation and the channel estimation error plays a primary role for performance degradation. Moreover, the rate achievement reduces significantly, while the terminal moves at the driving speed $v = 40\,\text{km/h}$. If the terminal moves at the driving speed higher than $60\,\text{km/h}$, uniform power allocation is sufficient. Visual results of these two cases are not shown here due to the paper limit.

VI. Conclusion

In this paper, we have analyzed the CSI impairment caused by the channel variation during the unavoidable delay, the noisy channel estimation and the limited feedback. For the computational efficiency, we have considered only the combination of channel estimates over OFDM symbols and used the CRLB as the variance of channel estimation error. With this approach, water-filling can be used to efficiently solve our resource allocation problem approximated by Jensen's inequality. Our simulations have shown that the system performance with imperfect CSI decreases as the velocity of the terminal increases, and is not a monotonic function of the frame length.

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