Power Allocation and Node Clustering for Distributed Detection in IR-UWB Sensor Networks

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***Abstract*— In wireless sensor networks, the limited energy of the nodes should be utilized in such a way that the performance measure of the sensing application is optimized. In this paper, power-aware design of IR-UWB sensor networks for distributed signal detection is discussed. The design approach consists of two parts which aim at minimizing the global probability of detection error. First, an application-specific node clustering algorithm is performed. Based on the generated topology a resource allocation scheme adapted to distributed detection is carried out. It is based on both, information from the network topology and individual sensor detection quality. Numerical results indicate significant performance gains for sensor networks with realistic resource constraints.**

I. Introduction

The initial task of many applications of wireless sensor networks is the detection of targets in a region of interest. In distributed detection, the sensor nodes process their observations locally and make preliminary decisions about the state of the monitored environment. The local decisions are transmitted to a fusion center where they are combined to obtain a final detection result with high reliability. In practice, the detection performance of wireless sensor networks is influenced by resource constraints like limited available energy or a restricted maximum transmission range. Hence, resource allocation and networking algorithms should be adapted to the detection application [1] in order to optimally design the sensor system.

In the parallel fusion network, where all sensor nodes transmit their local decisions directly to a fusion center, the maximum area that can be covered is limited by the maximum transmission range of a node. The covered area can be extended by a clustering of the network into a tree structure resulting in hierarchical transmission of node decisions. This requires algorithms that perform the clustering. In the literature, several algorithms with different optimization objectives and different complexity have been suggested. The authors of [2] consider a TDMA scheme. In [3] two algorithms are presented, which are combined with an impulse radio ultra-wideband (IR-UWB) specific multiple access scheme with non-orthogonal channels. Usually, clustering algorithms are not adapted to specific applications. The asymptotic detection behavior of a tree network for distributed detection has been analyzed in [4]. However, for realistic numbers of sensor nodes, these asymptotic results provide only limited information.

In this paper, we present an algorithm which considers the interdependency between energy consumption and the overall detection performance by including the individual sensor detection performance in the process of cluster head election and cluster formation. Based on the generated topology, we furthermore suggest an application-specific assignment of transmission power levels that depends both on individual sensor qualities as well as the generated topology. It aims at minimizing the global probability of detection error given a budget of transmission power. As enabling technology for wireless sensor networks, we consider IR-UWB transceivers. Due to the possibility of power control, IR-UWB transceivers are well suited to adapt networking algorithms to specific applications. Compared to our preceding work [5], where the reduction of transmission energy for a given detection performance is analyzed, we ask the complementary question of how much the probability of detection error can be decreased given a fixed total power budget. Furthermore, we conduct a direct comparison of the detection performance of the parallel and the tree network with and without limitations of the transmission range, which reveals which topology is advantageous in which parameter range. Moreover, a trade-off between the power budget and the number of nodes is discussed.

The paper is organized as follows. The considered system model and the IR-UWB transceivers are introduced in Section II. In Section III, the problem of distributed detection in tree networks is stated. The approach for node clustering is described in Section IV and the application specific resource allocation strategy based on this topology is introduced in Section V. Finally, numerical results and conclusions are presented in Section VI.

II. System model

We consider a network with a set $\mathcal{N} = \{S_1, \ldots, S_N\}$ of transceiver nodes. The nodes of the non-empty subset $\mathcal{M} \subset \mathcal{N}$ are the cluster heads. Each remaining leaf node of the set $\mathcal{L} = \mathcal{N} \setminus \mathcal{M}$ is associated to exactly one cluster head by the mapping $c\colon \mathcal{L} \to \mathcal{M}$. The set of nodes that transmit to the same cluster head S_m is denoted by $\mathcal{C}_m$.

As transmission scheme of the nodes we assume IR-UWB [6]. In each frame of length T_f one ultra short pulse with shape $w(t)$ is transmitted resulting in an ultra-wide occupancy

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of the frequency spectrum. Data bits are assumed to be coded by binary pulse position modulation (PPM) with modulation index α. Multiple access to the channel is realized by pseudo random time hopping codes c_i. Inside a frame, the pulse is delayed by an integer multiple of the chip length T_c given by the hopping code. The resulting transmitted signal $s_j(t)$ of sensor S_j then reads as

$$s_j(t) = A_j \sum_{i=-\infty}^{\infty} w(t - iT_f - c_i^{(j)} T_c - \alpha d^{(j)}_{\lfloor i/N_j \rfloor}). \quad (1)$$

Here $d^{(j)}$ are the data bits of node S_j, which are transmitted by a number of N_j subsequent equally modulated impulses of amplitude A_j. The signal to interference and noise ratio (SINR) builds the basis for the design of our power-aware algorithms. For one link between S_j and its receiver S_{m_j} it can be written as

$$\mathrm{SINR}_j = \frac{g_{jm_j} p_j}{\varsigma^2 \sum_{k \neq j} g_{km_j} p_k + \frac{\eta_{m_j}}{T_f}}, \quad (2)$$

where ς^2 is a parameter depending on the correlation properties of the employed impulse form, g_{jm_j} is the path gain of the link between S_j and its receiver S_{m_j} and η_{m_j}/T_f is an additional noise term.

III. Distributed Detection in Tree Networks

The problem of distributed detection in tree networks can be stated as follows. We consider a binary hypothesis testing problem with hypotheses H_0, H_1 indicating the state of the observed environment and associated prior probabilities $\pi_0 = P(H_0)$, $\pi_1 = P(H_1)$. In order to detect the true state of nature, the network of sensors $S_1, \ldots, S_N$ collects an array of random observations $(X_1, \ldots, X_N)' \in \mathcal{X}_1 \times \cdots \times \mathcal{X}_N$. The random observations $X_1, \ldots, X_N$ are assumed to be conditionally independent across sensors given the underlying hypothesis and distributed according to

$$H_0\colon X_j \sim \mathcal{N}(0, \sigma_j^2), \quad H_1\colon X_j \sim \mathcal{N}(\mu_j, \sigma_j^2), \quad (3)$$

$S_j \in \mathcal{N}$. The variance σ_j^2 describes Gaussian background noise and the mean μ_j indicates the deterministic signal component under hypothesis H_1 at sensor S_j. The local observation signal-to-noise ratio (SNR) at sensor S_j is given by

$$\mathrm{SNR}_j = 10 \log_{10} \left(\frac{\mu_j^2}{\sigma_j^2} \right) \quad [\mathrm{dB}]. \quad (4)$$

A. Leaf node decision rules

The leaf nodes S_j of set $\mathcal{L}$ process their respective observations X_j independently by forming local decisions

$$U_j = \delta_j(X_j), \quad S_j \in \mathcal{L}. \quad (5)$$

In the case of binary quantization, the leaf node decision rules are mappings $\delta_j\colon \mathcal{X}_j \to \{0,1\}$. Sensor decision rules leading to optimal configurations are monotone likelihood ratio quantizers provided that the observations are conditionally independent across sensors [7]. Thus, for the leaf nodes $S_j \in \mathcal{L}$, we consider decision rules δ_j that can be parameterized by real-valued quantization thresholds θ_j. In this way, each local decision U_j of a leaf node $S_j \in \mathcal{L}$ is characterized by the following local false alarm and miss probabilities

$$P_{f_j} = P(U_j = 1|H_0) = P(L_j > \theta_j|H_0), \quad (6)$$
$$P_{m_j} = P(U_j = 0|H_1) = P(L_j \leq \theta_j|H_1), \quad (7)$$

where L_j is the local log-likelihood ratio of observation X_j.

B. Transmission of local decisions

Each leaf node $S_j \in \mathcal{L}$ transmits its decisions U_j to its associated cluster head $S_{m_j} \in \mathcal{M}$ and each cluster head $S_m \in \mathcal{M}$ transmits its decisions U_m to the fusion center. Due to noisy channels, the received decisions $\widetilde{U}_j$ and $\widetilde{U}_m$ are potentially corrupted. We model the communication channels $C_1, \ldots, C_N$ of both the leaf nodes and the cluster heads by binary symmetric channels with bit-error probabilities $\varepsilon_1, \ldots, \varepsilon_N$, i.e.

$$\varepsilon_j = P(\widetilde{U}_j = 1|U_j = 0) = P(\widetilde{U}_j = 0|U_j = 1) \quad (8)$$

for $S_j \in \mathcal{N}$. The modified error probabilities $\widetilde{P}_{f_j} = P(\widetilde{U}_j = 1|H_0)$ and $\widetilde{P}_{m_j} = P(\widetilde{U}_j = 0|H_1)$ are given as

$$\begin{aligned} \widetilde{P}_{f_j} &= P_{f_j} + \varepsilon_j(1 - 2P_{f_j}), \\ \widetilde{P}_{m_j} &= P_{m_j} + \varepsilon_j(1 - 2P_{m_j}). \end{aligned} \quad (9)$$

Based on the modified error probabilities (9), we define the weight of sensor S_j as

$$\widetilde{\lambda}_j = \log \left(\frac{(1 - \widetilde{P}_{f_j})(1 - \widetilde{P}_{m_j})}{\widetilde{P}_{f_j} \widetilde{P}_{m_j}} \right), \quad S_j \in \mathcal{N}. \quad (10)$$

C. Cluster head decision rules

Each cluster head $S_m \in \mathcal{M}$ processes its observation X_m with respect to the received local decisions from the leaf nodes. E.g., if the cluster head S_m receives the subset $\widetilde{U}_1, \ldots, \widetilde{U}_k$ of local decisions from the leaf nodes of its cluster $\mathcal{C}_m$, it makes a decision

$$U_m = \delta_m(X_m, \widetilde{U}_1, \ldots, \widetilde{U}_k), \quad (11)$$

where the cluster head decision rule δ_m is a mapping $\delta_m\colon \mathcal{X}_m \times \{0,1\}^k \to \{0,1\}$.

In optimal configurations, the cluster heads perform binary quantization of their local log-likelihood ratios, where the applied decision thresholds depend on the values of the received decisions from the leaf nodes [8].

D. Optimal channel-aware fusion rule

At the fusion center, the received decisions $\widetilde{U}_m$ from the cluster heads $S_m \in \mathcal{M}$ are combined to yield the final decision U_0, where the fusion rule δ_0 is a Boolean function $\delta_0\colon \{0,1\}^{|\mathcal{M}|} \to \{0,1\}$. The sensor network detection performance is measured in terms of the global probability of error $P_e = \pi_0 P_f + \pi_1 P_m$, which is a weighted sum of the global probability of false alarm $P_f = P(U_0 = 1|H_0)$ and

Algorithm 1 Algorithm for node clustering

Initialize:

$d_{ij} \leftarrow \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}; \quad S_i, S_j \in \mathcal{N}$

$(\mathcal{M} = \mathcal{H}) \leftarrow \emptyset;$

$(\mathcal{D} = \mathcal{L}) \leftarrow \mathcal{N};$

$\mathcal{T}_j \leftarrow \{S_i \in \mathcal{N} | d_{ji} \leq d_{\text{tr}}\}; \quad S_j \in \mathcal{N}$

$\mu(S_j) \leftarrow ((\frac{1}{|\mathcal{T}_j|} \sum_{S_i \in \mathcal{T}_j} d_{ji}) \cdot d_j^{\text{FC}})^{-1} \cdot (\text{SNR}_j); \; S_j \in \mathcal{N}$

while $\mathcal{D} \neq \emptyset$ **do**

  $S_k = \arg\max_{S_j \in \mathcal{D}} \mu(S_j);$

  $\mathcal{M} \leftarrow \mathcal{M} \cup S_k;$

  $\mathcal{L} \leftarrow \mathcal{L} \setminus S_k;$

  $\mathcal{H} \leftarrow \{S_j \in \mathcal{D} | d_{kj} < d_{\text{tr}}\};$

  $\mathcal{D} \leftarrow \mathcal{D} \setminus \mathcal{H};$

end while

$\mathcal{C}_m \leftarrow \{S_j \in \mathcal{L} | d_{jm} < d_{jn}, \; S_n \in \mathcal{M} \setminus S_m\}; \quad S_m \in \mathcal{M}$

the corresponding global probability of miss $P_m = P(U_0 = 0|H_1)$.

The optimal channel-aware fusion rule can be implemented by a linear threshold test

$$\sum_{S_m \in \mathcal{M}} \widetilde{\lambda}_j \widetilde{U}_j \underset{U_0 = 0}{\overset{U_0 = 1}{\gtrless}} \vartheta \tag{12}$$

with decision threshold

$$\vartheta = \log \left(\frac{\pi_0}{\pi_1} \prod_{S_m \in \mathcal{M}} \frac{1 - \widetilde{P}_{f_j}}{\widetilde{P}_{m_j}} \right). \tag{13}$$

IV. Node Clustering

In this section, we present an application-specific algorithm that performs the clustering of the sensor network into a tree structure as considered in the previous section. The clustering is based on an application-specific metric $\mu(S_j)$, given by

$$\mu(S_j) = \left(\left(\frac{1}{|\mathcal{T}_j|} \sum_{i \in \mathcal{T}_j} d_{ji} \right) \cdot d_j^{\text{FC}} \right)^{-1} \cdot \text{SNR}_j, \quad S_j \in \mathcal{N}, \tag{14}$$

where d_{ji} is the distance between nodes S_j and S_i and d_j^{FC} denotes the distance of S_j to the fusion center. Set $\mathcal{T}_j$ includes all nodes inside the maximum transmission range d_{tr} of node S_j. The metric aims to minimize the necessary transmission power by privileging cluster configurations with low distances between transmitters and receivers. Furthermore, nodes with a high local observation SNR are favored to become cluster head, because it was observed in [9], that it is advantageous for hierarchical detection networks to order sensors from least reliable to most reliable detection quality.

A formal description of the algorithm is given in Algorithm 1. It starts with an initialization of the already introduced sets $\mathcal{M}$, $\mathcal{L}$ and $\mathcal{N}$. Initially, set $\mathcal{H}$ is empty, and $\mathcal{D}$ contains all nodes.

In the main loop of the algorithm the element of $\mathcal{D}$ with maximum metric is chosen as cluster head. Afterwards, all neighboring nodes inside the maximum transmission range d_{tr} of the new cluster head being in $\mathcal{D}$ are deleted from $\mathcal{D}$. If $\mathcal{D}$ is nonempty the process returns to the beginning of the loop and the next cluster head is selected. After the loop is finished and the cluster heads have been selected, all leaf nodes associate themselves to the spatially nearest cluster head.

V. Opportunistic Resource Allocation

A. Determination of target SINRs

In the following we propose an opportunistic resource allocation strategy, which can be conducted after the topology is generated by the algorithm of the previous section. It is based on an application-specific choice of the target SINRs γ_j. The objective is to minimize the global probability of detection error P_e given a budget of total transmission power. Fig. 1 shows the effective sensor weight $\widetilde{\lambda}$ according to (10) dependent on the SINR γ for different initial sensor weights λ. It can be observed that for high values of γ the effective sensor quality approaches the initial sensor quality. In this case, increasing γ does not result in an improved effective sensor quality. The value of γ from which on the effective sensor quality $\widetilde{\lambda}$ is not further improved significantly, increases with the initial sensor quality λ. It is therefore advantageous to assign higher values of SINR to sensors with high initial quality than to ones with low initial quality. We employ a sensitivity analysis of the effective sensor weight and assign the SINR for which the slope of the effective sensor weight $\widetilde{\lambda}$ with respect to γ falls under a predetermined threshold ϱ. Fig. 2 illustrates this procedure.

To account for signal attenuation in the determination of target values for the SINR, we also consider the path gain between the transmitter and its receiver. Links with a low path gain are favored by using the path gain g_{jm_j} as a weighting factor, normalized by the maximum path gain $g_{\max}$. Eventually, we determine the designated target SINR γ_j of sensor S_j according to

$$\gamma_j = \left(\frac{g_{jm_j}}{g_{\max}} \right) \cdot \left(\frac{\partial \widetilde{\lambda}_j}{\partial \gamma_j} \right)^{-1} (\varrho). \tag{15}$$

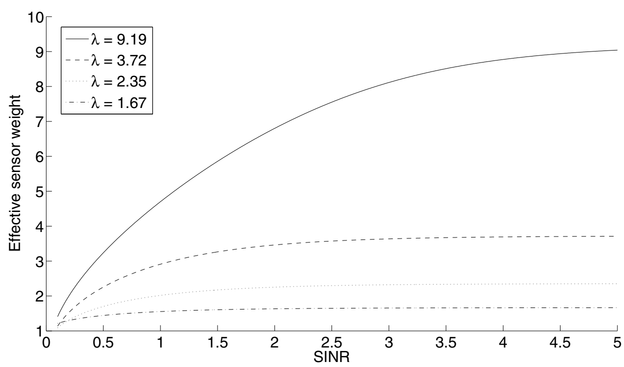

Fig. 1. Effective sensor weight $\widetilde{\lambda}$ as function of the SINR γ for different values of the initial sensor weight λ.

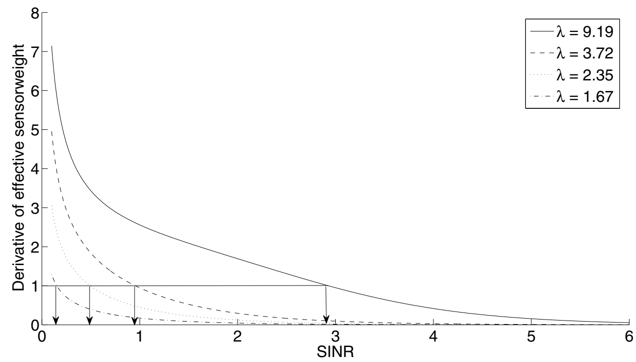

Fig. 2. Derivative $\partial\widetilde{\lambda}/\partial\gamma$ of the effective sensor weight $\widetilde{\lambda}$ with respect to the SINR γ. Here, threshold ϱ is chosen to be equal to 1.

Given γ_j, the bit-error rate ε_j of the jth channel C_j according to (8) can be computed by

$$\varepsilon_j = \frac{1}{2}\mathrm{erfc}(\sqrt{\gamma_j}). \tag{16}$$

The determined SINR levels $\gamma_1, \ldots, \gamma_N$ can be realized by appropriate power assignment as described in the following.

B. Achieving target SINRs by power assignment

The goal of the power assignment strategy is to find transmission power levels for all nodes such that the individual target SINRs γ_j according to (15) are met for all nodes.

For both steps, transmission from leaf nodes to cluster heads and transmission from cluster heads to the fusion center, a vector $\boldsymbol{p}$ with the optimal transmission power levels of the transmitting nodes as elements can be computed by

$$\boldsymbol{p} = [\boldsymbol{I} - \boldsymbol{\Gamma}\boldsymbol{N}^{-1}\boldsymbol{B}]^{-1}\boldsymbol{\tau}. \tag{17}$$

The diagonal matrices $\boldsymbol{\Gamma}$ and $\boldsymbol{N}$ contain the target SINRs γ_j and the number of pulse repetitions N_j for one data bit as entries. The entries b_{ij} of matrix $\boldsymbol{B}$ read as

$$b_{ij} = \begin{cases} \sigma^2 g_{jm_i}/g_{im_i}, & i \neq j \\ 0, & i = j \end{cases}.$$

The elements τ_j of the positive vector $\boldsymbol{\tau}$ are given by $\tau_j = (\eta_{m_j}\gamma_j)/(T_f N_j g_{jm_j})$. To decrease the computational effort of the power assignment to the leaf nodes an efficient reformulation of (17) can be used [3]. The power assignment to the cluster heads in the second step further simplifies to

$$p_j = \frac{\dfrac{\eta}{T_f\sigma^2}}{g_j\left(\dfrac{N_j}{\sigma^2\gamma_j}+1\right)\left(1 - \displaystyle\sum_{S_k\in\mathcal{M}}\frac{1}{\frac{N_k}{\sigma^2\gamma_k}+1}\right)}, \quad S_j \in \mathcal{M}. \tag{18}$$

VI. Numerical Results and Conclusions

In this section, we present simulation results for the proposed strategies. The scenario is generated by randomly deploying the sensor nodes uniformly in a rectangular area A. The fusion center is located in the middle of the scenario. As path loss model we assume signal attenuation according to $d^{-\beta}$. The involved parameters for the scenario and the IR-UWB transceivers are summarized in Table I. Fig. 3 illustrates the global probability of detection error P_e for the tree topology depending on the total transmission power p_{tot} and the total number of sensor nodes N. The contour lines on the ground plane indicate that there are several combinations of N and p_{tot} that result in the same probability of detection error P_e. The choice of the combination gives a degree of freedom to the network designer. A more detailed contour plot with different levels of probability of detection error is given in Fig. 4. It can be observed that the reasonable range of parameter combinations is limited. For a probability of detection error of $P_e = 10^{-3}$ and total transmission power of about $p_{\mathrm{tot}} = 0.5\,\mathrm{W}$, e.g., it is not reasonable to use a sensor number higher than about $N = 25$, because the power budget cannot be significantly reduced in return.

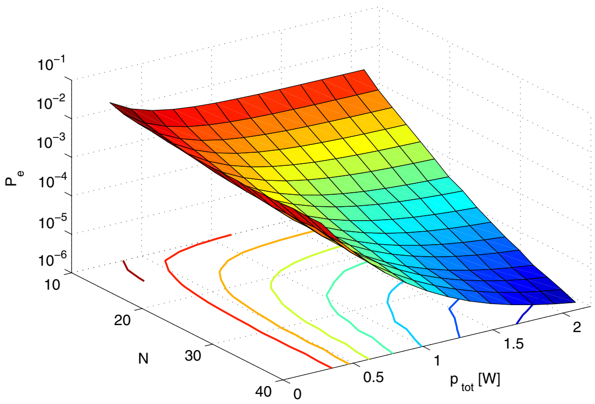

Fig. 3. Global probability of detection error P_e at the fusion center depending on the total number of nodes N in the network and the total power p_{tot}. The contour lines show combinations of N and p_{tot} that result in the same P_e.

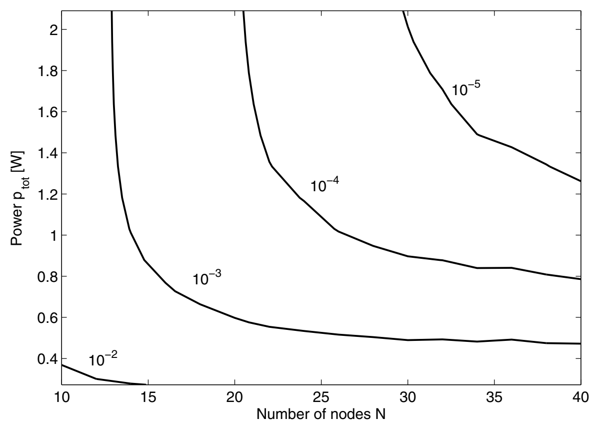

Fig. 4. Each combination of N and p_{tot} on a contour line results in the same probability of detection error P_e at the fusion center.

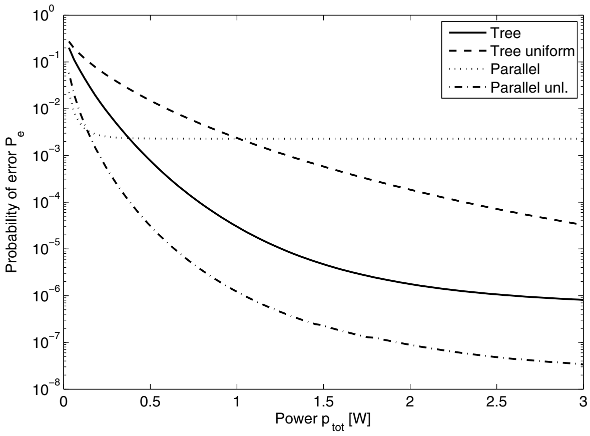

Fig. 5. Probability of detection error P_e depending on the total transmission power p_{tot} for the tree network and the parallel network with limited transmission range. While the dashed line shows the performance of the tree using the clustering algorithm and uniform power allocation, the solid line states the performance with additional opportunistic resource allocation after clustering.

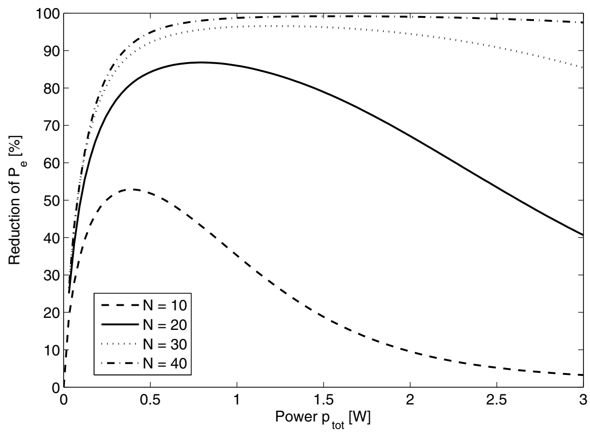

Fig. 6. Relative performance gain of the opportunistic resource allocation approach in terms of a reduction of P_e compared to uniform power assignment to all nodes.

Fig. 5 shows the global probability of detection error P_e depending on the total transmission power for the tree network and the parallel topology. In the parallel topology, all nodes transmit directly to the fusion center with power levels determined as described in Section V. It can be observed, that for an unlimited transmission range the parallel topology is always advantageous. In case of a realistic limitation of the transmission range d_{tr} however, the situation is completely different. Due to the limitation, in the parallel topology some nodes cannot connect to the fusion center and the detection performance P_e is deteriorated. Using the the described clustering algorithm with simple uniform power allocation, there exists a point of intersection from which on the tree topology is advantageous even without opportunistic resource allocation. The probability of error P_e and the power p_{tot} at the point of intersection can be further decreased significantly by additionally employing the described opportunistic resource allocation strategy. The additional gain compared to uniform power assignment for different numbers of sensor nodes N is illustrated in Fig. 6. For all considered N, there exists a point with a maximum gain and for high p_{tot} the gain decreases due to a relative decrease of the influence of channel errors on P_e in both strategies. Note, that the maximum is located at power levels, where the tree topology outperforms the parallel one, justifying the additional use of the opportunistic resource allocation strategy after clustering.

TABLE I

PARAMETERS USED IN THE SIMULATION.

parameter	value
N	40
A	$100\,\text{m} \times 100\,\text{m}$
d_{tr}	35 m
β	2
ς^2	$1.9966 \cdot 10^{-3}$
N_j	10
T_c	2 ns
T_f	100 ns
η	10^{-11} J

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