

Power-Aware Sensor Selection for Distributed Detection in Wireless Sensor Networks

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Abstract—The limited energy of sensor nodes in wireless sensor networks strongly recommends power-aware design methodologies. In this paper, a power-aware sensor selection strategy for wireless sensor networks is presented that is especially designed for distributed detection with soft decision fusion. The objective is to minimize the global probability of error at the fusion center under a total network power constraint. The cross-layer approach for the selection of a proper subset of sensors is based on a measure of individual sensor detection quality as well as location information. It corresponds to a low-complexity power allocation algorithm and enables significant performance gains in terms of reduction of the global probability of error compared to the inclusion of all sensors.

I. INTRODUCTION

Distributed detection of signal sources in a region of interest is one of the primary applications of wireless sensor networks [1]–[3]. In distributed detection, the sensor nodes process their observations locally and make preliminary decisions about the state of the monitored environment, e.g., absence or presence of a target. The local decisions are transmitted to a fusion center which combines the received decisions to obtain a final detection result which has high reliability. The main objective in the design of sensor networks for detection applications is the minimization of the global probability of error at the fusion center.

As the transmission channels of the battery-operated wireless sensors are subject to noise and interference, the resulting channel errors will affect the detection performance of the sensor network [4], [5]. On the other hand, wireless channel quality depends on the utilized transmission power. In wireless sensor networks, the available power budget should be allocated in a way that application-specific performance metrics are optimized, thereby exploiting dependencies between signal processing and wireless networking [6].

In this paper, we present a power-aware sensor selection strategy for distributed detection in wireless sensor networks in order to minimize the global probability of error at the fusion center under a total network power constraint. The approach for the selection of sensors is based on a measure of individual sensor detection quality as well as location information. Appropriate measures for sensor detection quality are derived from the asymptotic error exponents in hypothesis testing. The corresponding low-complexity power allocation algorithm is implemented by using a constant power level across a

properly chosen subset of sensors. The approach is similar to the concept of constant-power waterfilling as presented in [7]. The feasibility of the proposed sensor selection strategy is demonstrated for the general case of distributed detection with M -ary quantization and soft decision fusion where the number of quantization levels at the sensors is arbitrary.

To evaluate the performance of the proposed sensor selection strategy, we consider impulse radio ultra-wideband (IR-UWB) communication systems. IR-UWB transceivers are a promising candidate for wireless sensor nodes due to low power consumption, resilience against multipath fading, and low system complexity [8]. However, the sensor selection strategy can also be applied to other communication systems.

The remainder of this paper is organized as follows. In Section II, the problem of distributed detection with noisy channels and soft decision fusion is stated. The sensor selection strategy and the resulting low-complexity power allocation algorithm are presented in Section III. The considered IR-UWB system model is shortly described in Section IV. Finally, we present numerical results and conclusions in Section V.

II. DISTRIBUTED DETECTION

The problem of distributed detection in parallel fusion networks with M -ary quantization at the local sensors, noisy channels and soft decision fusion at the fusion center can be stated as follows (see Fig. 1). We consider a binary hypothesis testing problem with hypotheses H_0 and H_1 indicating the state of the monitored environment. The associated prior probabilities are $\pi_0 = P(H_0)$ and $\pi_1 = P(H_1)$. In order to detect the true state of nature, a network of N sensors S_1, \dots, S_N obtains random observations

$$(X_1, \dots, X_N)' \in \mathcal{X}_1 \times \dots \times \mathcal{X}_N \quad (1)$$

which are generated according to either H_0 or H_1 . The random observations X_1, \dots, X_N are assumed to be conditionally independent across sensors given the underlying hypothesis, i.e., the joint conditional probability density function of all the observations factorizes according to

$$f(x_1, \dots, x_N | H_k) = \prod_{j=1}^N f_j(x_j | H_k), \quad k = 0, 1. \quad (2)$$

According to the distributed nature of the problem, the sensors process their respective observations X_j independently by

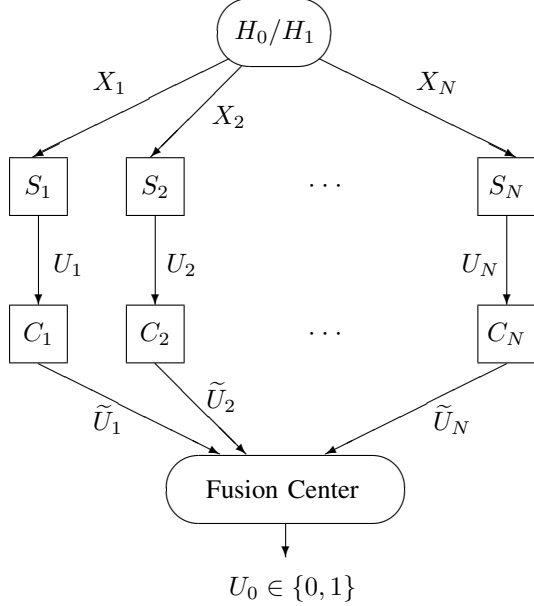


Fig. 1. Parallel fusion network with noisy channels.

forming local decisions

$$U_j = \delta_j(X_j), \quad j = 1, \dots, N. \quad (3)$$

Thus, the local decision U_j of sensor S_j does only depend on its own observation X_j and not on the observations of the other sensors.

A. Local sensor decision rules

In the general case of M -ary quantization at the local sensors, the local sensor decision rules δ_j are mappings

$$\delta_j: \mathcal{X}_j \rightarrow \{1, \dots, M\}, \quad j = 1, \dots, N. \quad (4)$$

Warren and Willett have shown that the sensor decision rules leading to jointly optimal configurations under the minimum probability of error criterion are monotone likelihood ratio partitions of the sensor observation spaces $\mathcal{X}_1, \dots, \mathcal{X}_N$, provided that the observations are conditionally independent across sensors [9]. Hence, it is only necessary to consider sensor decision rules δ_j that can be parameterized by a set of real quantization thresholds $\tau_{j_1}, \dots, \tau_{j_{M-1}}$, where $\tau_{j_0} = -\infty$, $\tau_{j_M} = \infty$, and $\tau_{j_k} \leq \tau_{j_{k+1}}$. In this way, each sensor S_j is characterized by the conditional probabilities

$$\alpha_{jk} = P(U_j = k|H_0) = P(\tau_{j_{k-1}} < L_j \leq \tau_{j_k}|H_0), \quad (5)$$

$$\beta_{jk} = P(U_j = k|H_1) = P(\tau_{j_{k-1}} < L_j \leq \tau_{j_k}|H_1), \quad (6)$$

where $L_j = \log(f_j(X_j|H_1)/f_j(X_j|H_0))$ is the local log-likelihood ratio of observation X_j . The probability vectors $\alpha_j = (\alpha_{j_1}, \dots, \alpha_{j_M})'$ and $\beta_j = (\beta_{j_1}, \dots, \beta_{j_M})'$ are computable given the local observation statistics $f_j(\cdot|H_k)$ and the quantization thresholds $\tau_{j_1}, \dots, \tau_{j_{M-1}}$ for each $j = 1, \dots, N$.

B. Transmission of local decisions

Upon local decision-making, the sensor nodes transmit their local decisions

$$(U_1, \dots, U_N)' \in \{1, \dots, M\}^N \quad (7)$$

to the fusion center in order to perform decision combining. We model the communication link C_j between sensor S_j and the fusion center by a discrete noisy channel with transition matrix T_j . The channel transition matrix $T_j = (T_{kl}^{(j)})_{1 \leq k, l \leq M}$ is an $M \times M$ matrix with the kl th entry defined as

$$T_{kl}^{(j)} = P(\tilde{U}_j = k|U_j = l), \quad k, l \in \{1, \dots, M\}, \quad (8)$$

where $\sum_{k=1}^M T_{kl}^{(j)} = 1$ for any $l \in \{1, \dots, M\}$. Because of the noisy channels, the fusion center receives a vector of potentially corrupted decisions

$$(\tilde{U}_1, \dots, \tilde{U}_N)' \in \{1, \dots, M\}^N. \quad (9)$$

The distribution of the corrupted decisions \tilde{U}_j is determined by the conditional probabilities

$$\tilde{\alpha}_{jk} = P(\tilde{U}_j = k|H_0) = \sum_{l=1}^M T_{kl}^{(j)} \alpha_{jl}, \quad (10)$$

$$\tilde{\beta}_{jk} = P(\tilde{U}_j = k|H_1) = \sum_{l=1}^M T_{kl}^{(j)} \beta_{jl}. \quad (11)$$

Assuming knowledge of the channel transition matrices T_j , the probability vectors $\tilde{\alpha}_j = T_j \alpha_j$ and $\tilde{\beta}_j = T_j \beta_j$ characterizing the distribution of the received local decisions $\tilde{U}_1, \dots, \tilde{U}_N$ under each of the two hypotheses can be calculated.

C. Optimal channel-aware fusion rule

At the fusion center, the received soft decisions $\tilde{U}_1, \dots, \tilde{U}_N$ are fused to the final detection result $U_0 = \delta_0(\tilde{U}_1, \dots, \tilde{U}_N)$, where the fusion rule δ_0 is a binary-valued mapping

$$\delta_0: \{1, \dots, M\}^N \rightarrow \{0, 1\}. \quad (12)$$

The sensor network detection performance is measured in terms of the global probability of error

$$P_e = \pi_0 P_f + \pi_1 P_m, \quad (13)$$

which can be written as a weighted sum of the global probability of false alarm $P_f = P(U_0 = 1|H_0)$ and the corresponding global probability of miss $P_m = P(U_0 = 0|H_1)$.

The optimal fusion rule under the minimum probability of error criterion can be performed by evaluating a log-likelihood ratio test of the form

$$\sum_{j=1}^N \mathcal{L}_j \begin{matrix} U_0 = 1 \\ \geq \\ U_0 = 0 \end{matrix} \log\left(\frac{\pi_0}{\pi_1}\right) = \vartheta, \quad (14)$$

where $\mathcal{L}_j = \log(P(\tilde{U}_j|H_1)/P(\tilde{U}_j|H_0))$ is the log-likelihood ratio of the corrupted decision \tilde{U}_j and ϑ is the fusion threshold.

D. Global error probabilities

When using the optimal fusion rule according to (14), the global probability of false alarm P_f and the global probability of miss P_m are determined by the conditional tail probabilities

$$P_f = P\left(\sum_{j=1}^N \mathcal{L}_j \geq \vartheta | H_0\right) \quad (15)$$

and

$$P_m = P\left(\sum_{j=1}^N \mathcal{L}_j < \vartheta | H_1\right). \quad (16)$$

In order to efficiently evaluate the sensor network detection performance in terms of the global probability of error P_e , we employ an approach introduced in [10] which provides tight upper bounds on the global probability of false alarm (15) and the global probability of miss (16).

III. SENSOR SELECTION STRATEGY

In the following, we propose a power-aware sensor selection strategy based on a measure for the detection quality or discrimination power of the individual sensors. Furthermore, the relative path gain of the channel between the individual sensors and the fusion center is used as a weighting factor. The sensor selection strategy corresponds to a low-complexity power allocation algorithm that distributes a total power budget evenly among all sensor nodes for which the weighted sensor detection quality exceeds a specified cut-off parameter.

A. Measure for sensor detection quality

A measure for the detection quality of each sensor is supposed to assess the discrimination power of the corresponding sensor with respect to the underlying binary hypothesis testing problem. Initially, every sensor S_j is characterized by the probability vectors α_j and β_j which determine the conditional distribution of the local decision U_j transmitted by sensor S_j under hypothesis H_0 or H_1 , respectively. A meaningful measure $Q(\alpha_j, \beta_j)$ for the detection quality of sensor S_j should therefore be based on some kind of distance between the probability vectors α_j and β_j . Examples of quality measures $Q(\alpha_j, \beta_j)$ based on the distance between probability vectors are given in Table I.

TABLE I
QUALITY MEASURES

Kullback-Leibler distance	$Q(\alpha_j, \beta_j) = \sum_{k=1}^M \alpha_{j_k} \log\left(\frac{\alpha_{j_k}}{\beta_{j_k}}\right)$
Chernoff distance	$Q(\alpha_j, \beta_j) = -\min_{0 \leq t \leq 1} \log \sum_{k=1}^M \alpha_{j_k}^t \beta_{j_k}^{1-t}$
Variational distance	$Q(\alpha_j, \beta_j) = \sum_{k=1}^M \alpha_{j_k} - \beta_{j_k} $

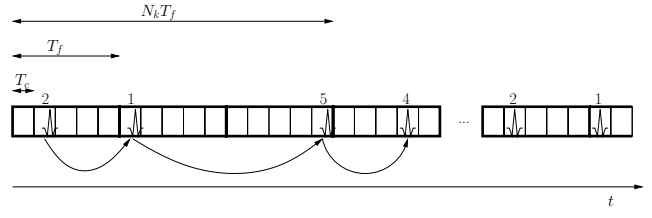


Fig. 2. Illustration of parameters used in the system model. In the example $c^{(k)} = (2, 1, 5, 4)$, $d_1^{(k)} = 1$, $d_2^{(k)} = 0$, and $N_k = 3$.

It is well known from information theory, that the Kullback-Leibler and Chernoff distances occur as asymptotic error exponents in Neyman-Pearson and Bayesian hypothesis testing, respectively [11]. Accordingly, for the minimum probability of error criterion, it might be reasonable to measure the detection quality of the sensors in terms of the Chernoff distance.

B. Low-complexity power allocation

Our approach to power allocation is essentially based on the idea to allocate transmission power only to those sensors S_j whose detection quality $Q(\alpha_j, \beta_j)$ exceeds some specified threshold value. Furthermore, wireless sensors with high path gain should be favored due to high channel quality. Therefore, we use a weighting factor given by the path gain g_j of the channel C_j between sensor S_j and the fusion center normalized by the maximum path gain g_{\max} . In effect, we want to select those sensors S_j that reliably add a large contribution of sensor detection quality $Q(\alpha_j, \beta_j)$ to the fusion center. Eventually, we determine the allocated transmission power p_j of sensor S_j according to

$$p_j = \begin{cases} p_{\text{tot}}/N_\kappa, & \text{if } (g_j/g_{\max}) \cdot Q(\alpha_j, \beta_j) \geq \kappa \\ 0, & \text{otherwise} \end{cases} \quad (17)$$

where p_{tot} is the total transmission power and N_κ is the number of sensors with positive transmission power when using the cut-off parameter κ . Note, that for $\kappa = 0$ we obtain uniform power assignment among all N nodes of the sensor network, i.e., in this case we obtain $p_j = p_{\text{tot}}/N$ for every sensor S_j .

IV. IR-UWB SYSTEM MODEL

Due to low power consumption and low transceiver complexity, IR-UWB is a promising candidate as an air interface for wireless sensor nodes. Therefore, we assume each sensor node to be equipped with an IR-UWB transceiver unit. In particular, we consider IR-UWB with pulse position modulation with modulation index α and pseudo random time hopping codes as multiple access scheme as described in [12]. The transmitted signal from sensor S_j can then be written as

$$s_j(t) = A_j \sum_{i=-\infty}^{\infty} w(t - iT_f - c_i^{(j)}T_c - \alpha d_{[i/N_j]}^{(j)}), \quad (18)$$

where T_f denotes the length of a time frame in which one impulse of form $w(t)$ is transmitted. In the frame, the impulse is delayed by an integer multiple of the chip length T_c

according to the time hopping code $c_i^{(j)}$. Each data bit $d^{(j)}$ belonging to the local decision U_j of sensor S_j is transmitted by a number of N_j equally modulated pulses with amplitude A_j . Some exemplary parameters for one user are illustrated in Fig. 2.

A. Signal-to-interference-and-noise ratio

According to [13], in a multi-user scenario the signal-to-interference-and-noise ratio (SINR) γ_j of the communication link between sensor S_j and the fusion center can be written as

$$\gamma_j = N_j \frac{g_j p_j}{\zeta^2 \sum_{k \neq j} g_k p_k + \frac{1}{T_f} \eta}, \quad (19)$$

with p_j denoting the transmission power of sensor node S_j . The parameter ζ^2 depends on the correlation properties of the employed pulse form $w(t)$. The path gain between sensor S_j and the fusion center is denoted by g_j . The transmitted signal is subject to additive white Gaussian noise with energy η .

B. Bit error rate and channel transition matrix

Using the standard Gaussian approximation for multiple access interference as discussed in [14], the bit error rate ε_j of sensor node S_j can be expressed as

$$\varepsilon_j = \frac{1}{2} \operatorname{erfc}(\sqrt{\gamma_j}). \quad (20)$$

If we assume $M = 2^b$ possible values for the local decision U_j , we have to transmit b bits for each local decision. Assuming consecutive and independent transmission of the bits, this results for, e.g., $b = 2$ in the channel transition matrix

$$\mathbf{T}_j = \begin{pmatrix} (1 - \varepsilon_j)^2 & \varepsilon_j(1 - \varepsilon_j) & \varepsilon_j(1 - \varepsilon_j) & \varepsilon_j^2 \\ \varepsilon_j(1 - \varepsilon_j) & (1 - \varepsilon_j)^2 & \varepsilon_j^2 & \varepsilon_j(1 - \varepsilon_j) \\ \varepsilon_j(1 - \varepsilon_j) & \varepsilon_j^2 & (1 - \varepsilon_j)^2 & \varepsilon_j(1 - \varepsilon_j) \\ \varepsilon_j^2 & \varepsilon_j(1 - \varepsilon_j) & \varepsilon_j(1 - \varepsilon_j) & (1 - \varepsilon_j)^2 \end{pmatrix}$$

for the discrete noisy channel C_j . Via equations (19) and (20), the channel transition matrix \mathbf{T}_j becomes a function of the allocated transmission power levels p_1, \dots, p_N . The bottom line is, that for any fixed power allocation the global probability of error P_e of the parallel fusion network with noisy channels can be evaluated explicitly.

V. NUMERICAL RESULTS AND CONCLUSIONS

In this section, we investigate the performance of the power-aware sensor selection strategy from Section III compared to the inclusion of all sensors by simulations. The scenario is generated by randomly deploying sensor nodes uniformly in a rectangular area A . The fusion center is supposed to be located in the middle of the scenario. The path gain g_j is determined by the path loss model $g_j = d_j^{-\beta}$, where d_j is the distance between sensor S_j and the fusion center and β is the path loss exponent. We consider the case of quaternary sensors, i.e., the local sensors act as 2-bit quantizers. The involved parameters are summarized in Table II.

TABLE II
PARAMETERS USED IN THE SIMULATION

parameter	value
N	50
A	100 m \times 100 m
β	2
ζ^2	$1.9966 \cdot 10^{-3}$
N_j	10
T_c	2 ns
T_f	100 ns
η	10^{-11} J

A. Joint distribution of sensor observations

As an illustrative example, we consider the problem of detecting the presence or absence of a deterministic signal in Gaussian noise, i.e., we assume that the observations X_1, \dots, X_N at the local sensors are conditionally independent distributed according to

$$\begin{aligned} H_0: X_j &\sim \mathcal{N}(0, \sigma_j^2), \\ H_1: X_j &\sim \mathcal{N}(\mu_j, \sigma_j^2). \end{aligned} \quad (21)$$

The variance σ_j^2 describes the Gaussian background noise and the mean μ_j indicates the deterministic signal component under hypothesis H_1 at sensor S_j , $j = 1, \dots, N$. The local observation signal-to-noise ratio (SNR) at sensor S_j is given by

$$\text{SNR}_j = 10 \log_{10} \left(\frac{\mu_j^2}{\sigma_j^2} \right), \quad (22)$$

where the SNR is measured in dB. In the simulation, we assume the local observation signal-to-noise ratios $\text{SNR}_1, \dots, \text{SNR}_N$ to be independent and identically uniformly distributed between -5 and 5 dB.

B. Distribution and quantization of log-likelihood ratios

The log-likelihood ratio L_j of the observation X_j is again a Gaussian random variable and is conditionally distributed according to

$$\begin{aligned} H_0: L_j &\sim \mathcal{N} \left(-\frac{\mu_j^2}{2\sigma_j^2}, \frac{\mu_j^2}{\sigma_j^2} \right), \\ H_1: L_j &\sim \mathcal{N} \left(\frac{\mu_j^2}{2\sigma_j^2}, \frac{\mu_j^2}{\sigma_j^2} \right). \end{aligned} \quad (23)$$

For simplicity, the quantization thresholds of the log-likelihood ratio L_j are uniformly chosen to be $\tau_{j1} = -1$, $\tau_{j2} = 0$, and $\tau_{j3} = 1$ for all sensors S_1, \dots, S_N . However, optimal selection of the quantization thresholds with respect to the employed detection quality measure $Q(\alpha_j, \beta_j)$ is also possible.

C. Results and conclusions

Fig. 3 depicts the simulation results for the Chernoff distance. The suggested sensor selection strategy reduces the global probability of error P_e up to nearly 40 % compared to the inclusion of all sensors given a fixed total transmission power of $p_{\text{tot}} = 0.1400$ W. The maximal

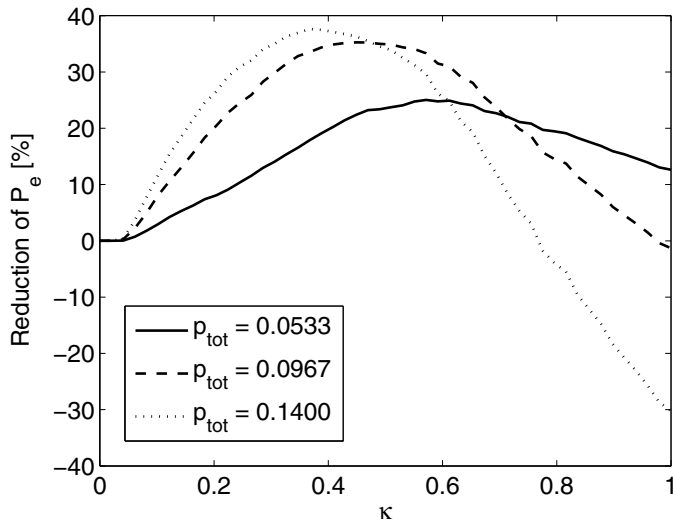


Fig. 3. Performance gain of the sensor selection strategy in terms of reduction of the global probability of error P_e compared to the inclusion of all sensors for several values of the total transmission power p_{tot} .

achievable performance gain decreases slightly for a lower total transmission power of $p_{\text{tot}} = 0.0967$ W. For a very low total transmission power of $p_{\text{tot}} = 0.0533$ W, the maximal achievable performance gain is about 25 %, where the robustness of the gain with respect to the selected cut-off parameter κ is significantly improved. In all cases, very high values of the cut-off parameter κ result in the exclusion of too many sensors from the distributed detection system and thus in a deterioration of the global detection performance.

The fraction of selected sensors as a function of the cut-off parameter κ is depicted in Fig. 4. E.g., for a total transmission power of $p_{\text{tot}} = 0.1400$ W, a performance gain of 40 % can be achieved by selecting approximately 50 % of the sensors. For a total transmission power of $p_{\text{tot}} = 0.0533$ W, the maximal performance gain of 25 % can be achieved by selecting only 30 % of the sensors. Since the sensor selection strategy based on the weighted quality measure $(g_j/g_{\text{max}}) \cdot Q(\alpha_j, \beta_j)$ is independent of the total transmission power p_{tot} , the fraction of selected sensors as a function of the cut-off parameter κ is also independent of p_{tot} .

For all considered values of total transmission power p_{tot} , there exists an optimal value of the cut-off parameter κ for which the performance gain of the sensor selection strategy is maximal. In our future work, we plan to address the issue of optimally determining the value of κ with respect to given wireless sensor network parameters.

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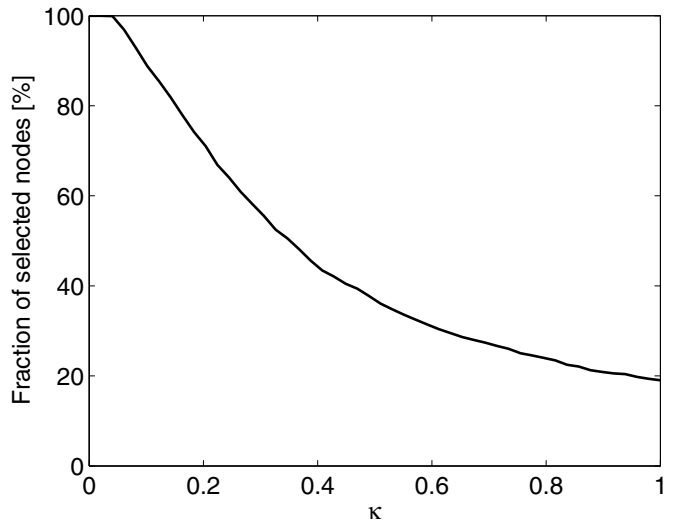


Fig. 4. Fraction of selected sensors for distributed detection in percent as a function of the cut-off parameter κ . Since the selection is based on the quality measure $Q(\alpha_j, \beta_j)$ of the initial probability vectors, the fraction is independent of the total transmission power p_{tot} .

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