An Improved Preamble-based SNR Estimation Algorithm for OFDM Systems

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***Abstract*—A crucial parameter required for adaptive transmission in orthogonal frequency division multiplexing (OFDM) systems is the signal-to-noise ratio (SNR). In this paper, certain modifications to previously proposed preamble-based SNR estimator for wireless OFDM systems are suggested in order to improve estimation performance. Proposed modifications, related to DFT interpolation, are based on the adaptive selection of significant channel impulse response (CIR) paths utilizing the average noise power estimate obtained in frequency domain. The modified estimator shows performance improvement of average SNR estimation in low SNR regime and considerably outperforms original algorithm for SNR per subcarrier estimation.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is a multicarrier modulation scheme that provides strong robustness against intersymbol interference (ISI) by dividing the broadband channel into many narrowband subchannels in such a way that attenuation across each subchannel stays flat. Orthogonalization of subchannels is performed with low complexity by using the fast Fourier transform (FFT). The serial high-rate data stream is converted into multiple parallel low-rate streams, each modulated on a different subcarrier.

An important task in the design of future OFDM system is to exploit frequency selective channels by adaptable transmission parameters (bandwidth, coding/data rate, power) to preserve power and bandwidth efficiency according to channel conditions at the receiver. In order to achieve such improvements, efficient and exact signal-to-noise ratio (SNR) estimation algorithm is requisite. The SNR is defined as the ratio of the desired signal power to the noise power and is widely used as a standard measure of signal quality for communication systems. SNR estimators derive estimate by averaging the observable properties of the received signal over a number of symbols. Prior to SNR per subcarrier estimation for adaptive transmission, the average SNR and channel frequency response have to be estimated.

Most of the SNR estimators proposed in the literature so far are related to single carrier transmission. In [1], a detailed comparison of various algorithms is presented, together with the derivation of the Cramer-Rao bound (CRB). Most of these algorithms can be directly applied to OFDM systems in additive white Gaussian noise (AWGN) [2], while the SNR estimation in frequency selective channels additionally requires efficient estimation of channel state information (CSI).

For packet based communications, block of information data is usually preceded by several training symbols (preambles) of known data used for synchronization and equalization purposes. Those preambles can be utilized for SNR estimation without additional throughput reduction.

In [3], we proposed an efficient and robust preamble-based algorithm, named periodic-sequence (PS) estimator, for SNR estimation in frequency selective time-invariant OFDM systems and compared its performance with several algorithms found in the literature. The PS estimator, based on second-order moments of received samples in frequency domain, utilizes preamble structure proposed by Morelli and Mengali in [4]. Compared to Schmidl and Cox synchronization method [5], it allows synchronization over a wider frequency offset range with only one preamble, hence reducing the training symbol overhead. The SNR per subcarrier is estimated using the average noise power estimate and channel estimates obtained by DFT interpolation, which is based on the fact that the channel power is concentrated on relatively small number of time domain samples [6]. However, it is shown in [3] that SNR per subcarrier estimates has bad performance at low SNR values which requires some more sophisticated mechanisms for channel estimation. In [7], authors proposed a method for adaptive selection of significant channel impulse response (CIR) paths. The rest of CIR paths, whose average power is below the threshold determined by noise power estimates, are nulled, thus improving the performance of channel estimation.

In this paper, we propose modifications to PS estimator, which utilize the method of significant CIR path selection proposed in [7]. Average noise power estimates from PS estimator are used to determine appropriate threshold for significant path selection. The modified PS estimator, named improved PS (IPS) estimator, offers the better performance of average SNR estimation in low SNR region compared to PS estimator and significantly improves the performance of SNR per subcarrier estimation.

The remainder of this paper is organized as follows. Section II provides the system model and specifies the SNR estimation

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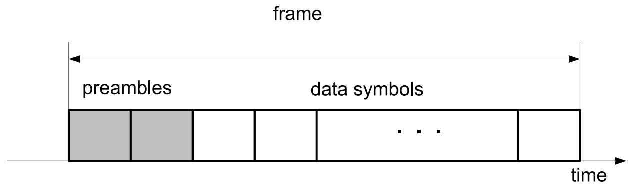

Fig. 1. Frame structure

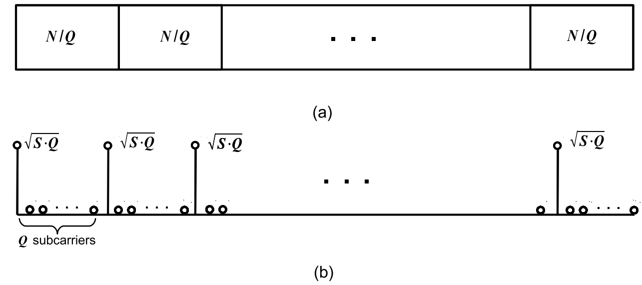

Fig. 2. Preamble structure in (a) time and (b) frequency domain

problem. In Section III, PS estimator is revised and appropriate SNR estimates are given. The proposed IPS estimator is introduced in Section IV. Its performance is analyzed by computer simulations in Section V. Finally, some concluding remarks are given in Section VI.

II. System Model

In many wireless OFDM systems, transmission is normally organized in frames. Typical frame structure is shown in Fig. 1 where sequence of data symbols is preceded by several preambles of known data used for the synchronization and/or channel estimation purposes. We consider general model of frame structure composed of I preambles where each preamble contains N modulated subcarriers. Let $C(i,n)$ denote the complex data symbol on nth subcarrier in ith preamble, where $i=0,\ldots,I-1$ and $n=0,\ldots,N-1$. It is assumed that modulated subcarrier has unit magnitude, i.e. $|C(i,n)|^2=1$, which is a regular assumption since present OFDM standards usually contain preambles composed of QPSK and/or BPSK modulated subcarriers. At the receiver, perfect synchronization is assumed, hence after FFT, received signal on nth subcarrier in ith preamble can be expressed as

$$Y(i,n)=\sqrt{S}C(i,n)H(i,n)+\sqrt{W}\eta(i,n), \tag{1}$$

where $\eta(i,n)$ is sampled complex zero-mean AWGN of unit variance, S and W are transmitted signal power and noise power on each subcarrier, respectively, and $H(i,n)$ is the channel frequency response given by

$$H(i,n)=\sum_{l=0}^{L-1}h(\tau_l+iT_s)\cdot e^{-j2\pi\frac{n\tau_l}{T_s}}, \tag{2}$$

where $h(\tau_l+iT_s)$ and τ_l denote the channel gain and delay of lth path during the ith preamble, respectively, T_s is the duration of the OFDM preamble and L is the length of the CIR. The channel path gains $h(\tau_l+iT_s)$ in each OFDM symbol independently experience Rayleigh fading, while $\sum_{l=0}^{L-1}E\left\{|h(\tau_l+T_s)|^2\right\}=1$ is satisfied. Our initial assumption is that channel is constant during the whole frame, since we consider SNR estimation algorithms for the purposes of adaptive transmission. Therefore, time index i is omitted during the estimation procedure, i.e. $h(\tau_l+iT_s)$ is replaced by $h(\tau_l)$ and $H(i,n)$ is replaced by $H(n)$. We can further assume that the channel is sample-spaced, i.e., CIR paths are integer multiples of the system sampling rate T_S/N giving $h(l)\equiv h(l\frac{T_S}{N})=h(\tau_l)$. It is also assumed that average SNR and SNR per subcarrier estimates are valid for all information data bearing OFDM symbols within the frame. As it is shown in [8], the average SNR of the ith received OFDM preamble can be expressed as

$$\begin{aligned}\rho_{av}&=\frac{E\{\sum_{n=0}^{N-1}|\sqrt{S}C(i,n)H(n)|^2\}}{E\{\sum_{n=0}^{N-1}|\sqrt{W}\eta(i,n)|^2\}}\\&=\frac{S}{W},\end{aligned} \tag{3}$$

where $\sum_{n=0}^{N-1}E\left\{|H(n)|^2\right\}=N$ is satisfied, while the SNR of the nth subcarrier is given by

$$\begin{aligned}\rho(n)&=\frac{E\{|\sqrt{S}C(i,n)H(n)|^2\}}{E\{|\sqrt{W}\eta(i,n)|^2\}}\\&=\frac{S|H(n)|^2}{W}=\rho_{av}\cdot|H(n)|^2.\end{aligned} \tag{4}$$

III. PS estimator

The key idea for PS estimator rests upon the time domain periodic preamble structure utilized for time and frequency synchronization in [5]. In order to cover a wider frequency range, in [4] a preamble of Q identical parts, each containing N/Q samples is proposed as depicted in Fig. 2a. The corresponding frequency domain representation is shown in Fig. 2b. In the sequel we assume that Q divides N, so that $N_p=N/Q$ is integer.

Starting from the 0th, each Qth subcarrier is modulated with a QPSK signal $C_p(m)$, $m=0,1,\ldots,N_p-1$ with $|C_p(m)|=1$. The remainder of $N_z=N-N_p=\frac{(Q-1)}{Q}N$ subcarriers is not used (nulled). In order to maintain the total energy level over all symbols within the preamble, the power is scaled by factor Q yielding a total transmit power of SQ in the loaded subcarriers.

Write $n=mQ+q$, $m=0,\ldots,N_p-1$, $q=0,\ldots,Q-1$. The transmitted signal on the nth subcarrier is written as

$$C(n)=C(mQ+q)=\begin{cases}C_p(m), & q=0\\ 0, & q=1,\ldots,Q-1\end{cases}. \tag{5}$$

By (1) the nth received signal is given by

$$Y(n)=Y(mQ+q)=\begin{cases}Y_p(m), & q=0\\ Y_z(mQ+q), & q=1,\ldots,Q-1\end{cases},$$

where

$$Y_p(m)=\sqrt{SQ}\,C_p(m)H_p(m)+\sqrt{W}\,\eta(m) \tag{6}$$

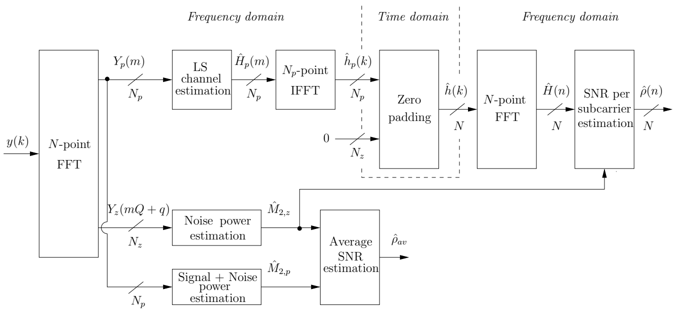

Fig. 3. Block diagram of the PS estimator

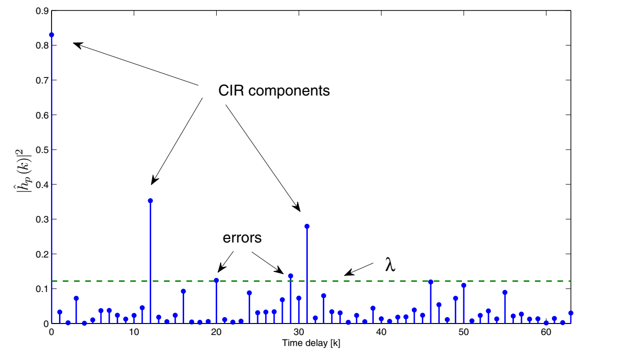

Fig. 4. Significant path selection for channel (c) with Q = 4 and SNR = -6 dB

denotes the received signal on loaded subcarriers, and

$$Y_z(mQ+q) = \sqrt{W}\,\eta(mQ+q) \tag{7}$$

is the received signal on nulled subcarriers containing only noise.

The empirical second-order moment of the received signal on loaded subcarriers is

$$\hat{M}_{2,p} = \frac{1}{N_p}\sum_{m=0}^{N_p-1}|Y_p(m)|^2 \tag{8}$$

with expected value $E\left\{\hat{M}_{2,p}\right\} = QS + W$ derived in [3].

Similarly, the empirical second moment of the received signal on nulled subcarriers,

$$\hat{M}_{2,z} = \frac{1}{N_p(Q-1)}\sum_{m=0}^{N_p-1}\sum_{q=1}^{Q-1}|Y_z(mQ+q)|^2, \tag{9}$$

has expectation $E\left\{\hat{M}_{2,z}\right\} = W$.

In summary, the average SNR ρ_{av} can be estimated by forming

$$\begin{aligned}\hat{\rho}_{av} &= \frac{1}{Q}\frac{\hat{M}_{2,p}-\hat{M}_{2,z}}{\hat{M}_{2,z}} \\ &= \frac{1}{Q}\left((Q-1)\frac{\sum_{m=0}^{N_p-1}|Y_p(m)|^2}{\sum_{m=0}^{N_p-1}\sum_{q=1}^{Q-1}|Y_z(mQ+q)|^2}-1\right),\end{aligned} \tag{10}$$

where, by the strong law of large numbers, $\hat{M}_{2,p}$ and $\hat{M}_{2,z}$ are strongly consistent unbiased estimators of $QS+W$ and average noise power W, respectively.

Note that $\hat{\rho}_{av}$ does not need any knowledge of the transmitted symbols on loaded subcarriers. Only the arrangement of loaded and nulled subcarriers must be known to the receiver. However, channel estimates $\hat{H}(n)$ are requisite for the estimation of SNR per subcarrier (4). They are available only for the loaded subcarriers by the means of least square (LS) estimation as

$$\begin{aligned}\hat{H}_p(m) &= \frac{1}{\sqrt{Q}}C_p^*(m)Y_p(m) \\ &= \sqrt{S}H_p(m) + \sqrt{\frac{W}{Q}}C_p^*(m)\eta(m).\end{aligned} \tag{11}$$

As it is shown in Fig. 3 channel estimates for nulled subcarriers $\hat{H}(mQ+q)$, $m = 0,\dots,N_p-1$, $q = 1,\dots,Q-1$, are obtained by DFT interpolation. Therefore, the CIR estimates after IFFT can be written as

$$\begin{aligned}\hat{h}_p(k) &= \mathrm{IFFT}_{N_p}\left[\hat{H}_p(m)\right],\ 0 \le k \le N_p-1 \\ &= \sqrt{S}h(k) + \sqrt{\frac{W}{Q}}\widetilde{\eta}(k),\end{aligned} \tag{12}$$

where $\mathrm{IFFT}_{N_p}[\cdot]$ presents the N_p-point IFFT and $\widetilde{\eta}(k) = \mathrm{IFFT}_{N_p}\left[C_p^*(m)\eta(m)\right]$. In order to obtain channel estimates, the rest of $N_z = N - N_p$ samples are padded with zeros giving the CIR prior to N-point FFT as

$$\hat{h}(k) = \begin{cases}\hat{h}_p(k), & 0 \le k \le N_p-1 \\ 0, & N_p \le k \le N-1.\end{cases} \tag{13}$$

Channel estimates after N-point FFT are obtained as

$$\hat{H}(n) = \mathrm{FFT}_N\left[\hat{h}(k)\right],\ 0 \le n \le N-1. \tag{14}$$

It can be easily noticed that in order to preserve CIR information, the number of loaded subcarriers has to be larger or equal to the CIR length, i.e., $N_p \ge L$. Hence, $Q \le N/L$ must be satisfied, which puts a constraint to preamble design. Using (4) with noise power estimates obtained in (9), the SNR estimate on the nth subcarrier can be written as

$$\hat{\rho}(n) = \frac{|\hat{H}(n)|^2}{\hat{M}_{2,z}}. \tag{15}$$

IV. Improved PS Estimator

Proposed modifications to PS estimator are shown in Fig. 5. By comparing the average power estimates of individual CIR paths $|\hat{h}_p(k)|^2$ with the threshold λ determined by the average noise power estimate obtained in frequency domain $\hat{M}_{2,z}$, only the significant CIR paths are selected as inputs to N-point FFT. The rest of CIR paths, whose average power estimates are below the threshold, are nulled assuming that they present only noise samples. Fig. 4 shows one channel realization used in simulations and appropriate threshold value

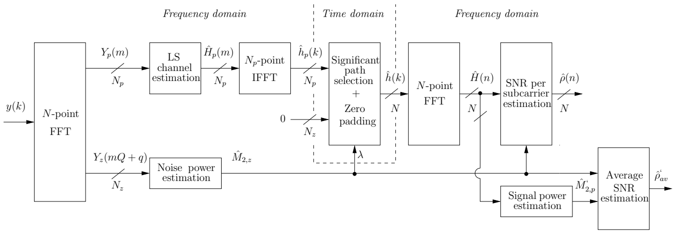

Fig. 5. Block diagram of the IPS estimator

used for significant path selection. Therefore, the CIR prior to N-point FFT can be written as

$$\hat{h}(k) = \begin{cases} \hat{h}_p(k), & |\hat{h}_p(k)|^2 > \lambda \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

The selection of the threshold λ is based on the reduction of mean square error (MSE) of the individual channel estimate. It is shown in [7] that the MSE is reduced when

$$\sigma_h^2(k) > \frac{1}{\rho_{av}},\ k = 0, \ldots, N-1 \tag{17}$$

holds, where $\sigma_h^2(k) = E\left\{|h(k)|^2\right\}$ denote the average power of the kth CIR path. Since, from (12), only CIR estimates $\hat{h}_p(k)$ are available, $\sigma_{\hat{h}_p}^2(k)$ can be written as

$$\sigma_{\hat{h}_p}^2(k) = S\sigma_h^2(k) + \frac{W}{Q}. \tag{18}$$

Replacing (18) in (17) it can be derived that MSE is reduced when

$$\sigma_{\hat{h}_p}^2(k) > (1 + \frac{1}{Q})W. \tag{19}$$

The average power of kth path $\sigma_{\hat{h}_p}^2(k)$ and average noise power W in (19) can be replaced with available unbiased estimates, $|\hat{h}_p(k)|^2$ and $\hat{M}_{2,z}$, respectively. Therefore, appropriate threshold can be derived as

$$|\hat{h}_p(k)|^2 > (1 + \frac{1}{Q})\hat{M}_{2,z} = \lambda. \tag{20}$$

After significant path selection and FFT, channel estimates $\hat{H}(n)$ are obtained using (14), while SNR per subcarrier estimates $\hat{\rho}(n)$ are derived from (15). Since performed CIR filtering significantly reduces the amount of noise present in channel estimates, average power estimate can be written as

$$\hat{M'}_{2,p} = \frac{1}{N}\sum_{n=0}^{N-1}|\hat{H}(n)|^2, \tag{21}$$

giving the average SNR estimate as

$$\hat{\rho'}_{av} = \frac{\hat{M'}_{2,p}}{\hat{M}_{2,z}}. \tag{22}$$

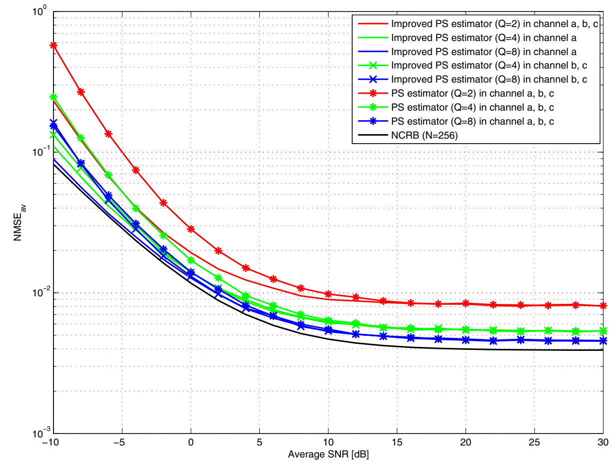

Fig. 6. NMSE of the average SNR

V. Simulation Results

The performance of IPS estimator is evaluated and compared with the performance of PS estimator using Monte-Carlo simulation. OFDM system parameters used in the simulation are taken from WiMAX specifications giving $N = 256$ subcarriers and cyclic prefix length of 32 samples [9]. Performance is evaluated for three different channels: (a) AWGN channel, (b) a 3-tap time-invariant fading channel with a root mean square delay spread $\tau_{rms} = 2$ samples and (c) a 3-tap time-invariant fading channel with a $\tau_{rms} = 10$ samples. Parameters for considered channels are taken from [10]. The number of independent trials is set to $N_t = 100000$ assuring the high confidence interval of the estimates. The evaluation of the performance is done in terms of normalized MSE (NMSE) of the estimated average SNR values following

$$\text{NMSE}_{\text{av}} = \frac{1}{N_t}\sum_{i=i}^{N_t}\left(\frac{\hat{\rho}_{av,i} - \rho_{av}}{\rho_{av}}\right)^2, \tag{23}$$

where $\hat{\rho}_{av,i}$ is the estimate of the average SNR in the ith trial, and ρ_{av} is the true value. Second considered performance measure is the NMSE of the estimated SNR per subcarrier given by

$$\text{NMSE}_{\text{sc}} = \frac{1}{NN_t}\sum_{i=i}^{N_t}\sum_{n=0}^{N}\left(\frac{\hat{\rho}(n)_i - \rho(n)}{\rho(n)}\right)^2, \tag{24}$$

where $\hat{\rho}(n)_i$ is the estimate of the $\rho(n)$ in the ith trial.

Proposed method is evaluated for 3 different cases of preamble's repeated parts, i.e. $Q = 2, 4$ and 8. Fig. 6 shows the NMSE_{av} of considered estimators. In order to assess the absolute performances of the estimators, they are compared with the Cramer-Rao bound (CRB) which is the lower bound for the variance of any unbiased estimator, see [11]. Normalized CRB (NCRB) for OFDM signal with N QPSK modulated subcarriers in AWGN channel can be expressed as

$$\text{NCRB} = \frac{1}{N}\left(\frac{2}{\rho_{av}} + 1\right). \tag{25}$$

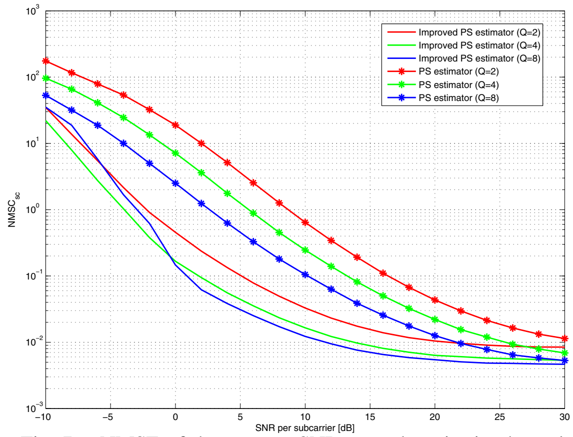

Fig. 7. NMSE of the average SNR per subcarrier in channel (b)

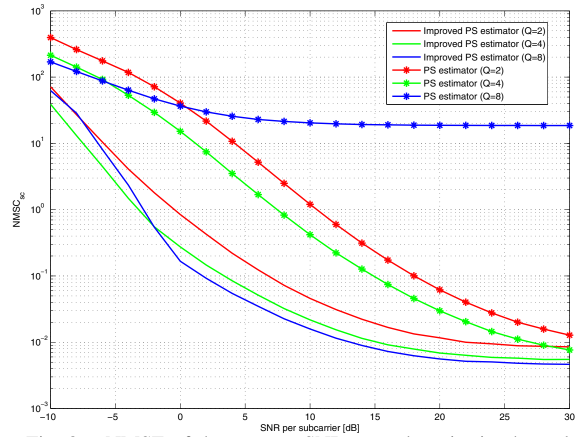

Fig. 8. NMSE of the average SNR per subcarrier in channel (c)

Fig. 6 shows that the PS estimator has the same performance in all considered channels and that the increase of the number of identical parts Q in the preamble brings its performance closer to the NCRB. It can be explained with the notion that more subcarriers are used for the average noise power estimation (9) while at the same time transmitted signals on loaded subcarriers are getting more power due to the scaling by Q, giving the more accurate estimate in (8). It can be also noticed that the IPS estimator outperforms PS estimator in low SNR regime. However, the performance improvement for the $Q = 4$ is slightly worse in frequency selective channels compared to AWGN channel. For $Q = 8$, IPS estimator reaches the NCRB at low SNR values, while there is no improvement in frequency selective channels compared with PS estimator and performance is slightly worse compared to $Q = 4$ at SNR values less than -7 dB.

Fig. 7 compares the NMSE_{sc} of considered estimators in time-invariant frequency selective channel (b) which corresponds to moderate selectivity. It is shown that IPS outperforms PS estimator for all values of Q. The IPS estimator for $Q = 8$ shows worse performance compared to $Q = 4$ case for SNR values less than 0 dB and worse performance compared to $Q = 2$ case for SNR values less than -6 dB.

The performance of considered estimators in time-invariant frequency selective channel (c) which corresponds to strong selectivity is shown in Fig. 8. In contrast to PS estimator which stops to benefit from the increase of Q and become biased for $Q = 8$, IPS estimator shows good performance and similar tendency as in the channel (b). From Fig. 7 and Fig. 8 it can be noticed that in the region of high values of SNR, channel estimates stop to act as deteriorating factor and NMSE_{sc} approaches the NMSE_{av}.

VI. Conclusion

In order to improve estimation performance of previously proposed preamble-based SNR estimator for wireless OFDM systems, we suggested a modifications to DFT interpolation which exploits the adaptive selection of significant CIR paths utilizing the estimate of average noise power obtained in frequency domain. It is shown that proposed modifications improve the performance of average SNR estimation in low SNR regime and considerably outperform original algorithm for SNR per subcarrier estimation.

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