

A Semi-Stochastic Radio Propagation Model for Wireless MIMO Channels

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Abstract—Among current physical channel models, deterministic models, such as ray-launching, are accurate but site-specific. Geometry-based stochastic models are flexible, however, cannot benefit if site information is available. In this work, a semi-stochastic MIMO channel model is proposed. It integrates results from deterministic models into stochastic models, merging the advantages of both approaches.

Index Terms—MIMO, ray-launching, channel model, radio propagation

I. INTRODUCTION

Multiple-input multiple-output (MIMO) technique is well known for its enormous capacity gain [1]. In recent years, to serve the growing demand of high-speed data traffic in wireless communications, MIMO systems are brought from theory to practice. In the foreseeable future, MIMO technique will become essential in wireless telecommunication systems, especially in the 4G LTE (Long Term Evolution) systems [2]. Therefore, modeling the wireless MIMO radio channel becomes an important task in simulating wireless communication systems.

Initially, the MIMO channels are commonly modeled as flat fading channels without spatial correlation, and the channel impulse response is described analytically [3]. Due to the extensive deployment of wideband wireless devices, various methods are devoted to efficient modeling of multi-path channels [4] [5] [6]. It is also realized, that the spatial correlation has great influence on the channel capacity. Thus, correlation models, such as the Kronecker model [7] and the Weichselberger model [8], are developed to characterize the spatial correlation.

As an alternative of analytic models, physical channel models use a different methodology. Instead of describing the impulse response between antenna pairs in an analytical way, physical channel models focus on the aspect of electromagnetic wave propagation. Physical models can be further split into deterministic models and geometry-based stochastic models [9].

Deterministic models, such as ray-tracing and ray-launching, characterize the physical propagation parameters in a completely deterministic manner by following or launching

deflected rays from transmitters to receivers [10] [11]. In deterministic models, electromagnetic characteristics of radio links are explicitly calculated by means of a detailed description of the propagation environment. Deterministic models capture the nature of radio wave propagation, thus are intuitive and potentially accurate. However, they are site specific, namely, geometric information about the propagation environment must be given as prerequisite.

To model the typical behavior of different environments, geometry-based stochastic models choose the locations of scatterers in a random way following certain probability distributions. The channel impulse responses are then generated through ray-based calculations. The probability distributions of scatterers are categorized by different scenarios (urban, suburban, etc.). Therefore, geometry-based stochastic models are able to model certain propagation environment without detailed geometric data. The most commonly used geometry-based stochastic models include 3rd Generation Partnership Project (3GPP) spatial channel model (SCM) [12] and the WINNER (Wireless world INitiative NEw Radio) model [13].

Modern wireless networks demand for detailedly described channels which take the specific environment into account. In particular, for self-organizing networks (SON), or for network planning, radio channels for a given area should be simulated precisely.

In this work, a semi-stochastic model is proposed. In our approach, the environmental information is considered while generating the channel matrix by means of statistical and stochastic components.

The remainder of this paper is divided into four parts. In Section II, existing deterministic models and geometry-based stochastic channel models are introduced. The proposed semi-stochastic model is elaborated in Section III. Implementation issues of the new channel model are addressed in Section IV. Finally, conclusions are given in Section V. For notation, $(\cdot)^T$ is used to denote matrix transpose.

II. MIMO SYSTEM MODEL

Consider a MIMO system with U antennas on the receiver (Rx) and S antennas on the transmitter (Tx) side, as depicted in Fig. 1. With t denoting time index, assuming signal vector $\mathbf{x}(t) = [x_1(t), x_2(t), \dots, x_S(t)]^T$ is transmitted, the received signal vector $\mathbf{y}(t) = [y_1(t), y_2(t), \dots, y_U(t)]^T$ can be written

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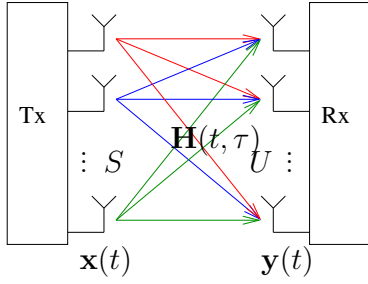


Fig. 1. MIMO system model.

as

$$\mathbf{y}(t) = \int_{\tau} \mathbf{H}(t; \tau) \mathbf{x}(t - \tau) d\tau + \mathbf{w}(t), \quad (1)$$

where τ is excess delay, $\mathbf{w}(t) = [w_1(t), w_2(t), \dots, w_U(t)]^T$ is the additive white Gaussian noise (AWGN) term. The time variant channel impulse response is denoted by $\mathbf{H}(t; \tau)$, which is a $U \times S$ matrix

$$\mathbf{H}(t; \tau) = \begin{pmatrix} h_{1,1}(t; \tau) & h_{1,2}(t; \tau) & \dots & h_{1,S}(t; \tau) \\ h_{2,1}(t; \tau) & h_{2,2}(t; \tau) & \dots & h_{2,S}(t; \tau) \\ \vdots & \vdots & \ddots & \vdots \\ h_{U,1}(t; \tau) & h_{U,2}(t; \tau) & \dots & h_{U,S}(t; \tau) \end{pmatrix} \quad (2)$$

with the entry $h_{u,s}(t; \tau)$ representing the channel impulse response between Tx antenna s and Rx antenna u .

Considering the double-directional radio channel, $h_{u,s}(t; \tau)$ can be written as [14]

$$h_{u,s}(t; \tau) = \int_{\phi} \int_{\psi} h_{u,s}(t, \tau, \phi, \psi) \cdot \sqrt{G_{\text{Tx},s}(\phi)} \sqrt{G_{\text{Rx},u}(\psi)} d\phi d\psi, \quad (3)$$

where G_{Tx} and G_{Rx} are antenna pattern of Tx and Rx side, respectively. Angle of departure (AoD) and angle of arrival (AoA) are denoted by ϕ and ψ . The double-directional channel impulse response $h_{u,s}(t, \tau, \phi, \psi)$ is a composition of several multipath components (MPC).

A. Deterministic models

In deterministic models, all possible paths from the Tx to the Rx are determined by considering propagation phenomena like reflections at walls and diffractions at wedges. Usually, the propagation environment is described by polyhedrons. A visibility tree is build to capture the radio propagation paths. The visibility tree consists of nodes and branches, represent objects (walls, wedges, Rx, etc.) and line of sight (LoS) connections between objects, respectively. The layered structure of the visibility tree shows the depth of interactions.

For ray-tracing, the images of Tx relative to the reflecting planes are computed, as depicted in Fig. 2-(a). Each reflected or diffracted ray from the Tx to the Rx is exactly determined. This leads to a very high accuracy, because all the relevant objects are always considered for the selection of interactions. However, as the number of interactions increase, the computational complexity grows exponentially.

With ray-launching, the rays are launched from the transmitter homogeneously with a discrete angle increment [10]. After each interaction, the reflected or diffracted rays will be computed and traced further as illustrated in Fig. 2-(b). The tracing can be terminated, when the power of a ray drops under a predetermined threshold. The disadvantage is, the constant increment between two rays leads to the problem that it is hard to determine whether a wedge is hit or not.

Despite the high accuracy, the computational burden makes ray-tracing incapable of handling large wireless communication scenarios [15]. Thus, it is mostly used for indoor or microcell environment. The ray-launching method has many advantages in predicting field strength for a large area [16]. Especially, in urban scenarios, a cube oriented 3D ray launching algorithm (CORLA) proposed in [17] offers both fast and accurate field strength prediction.

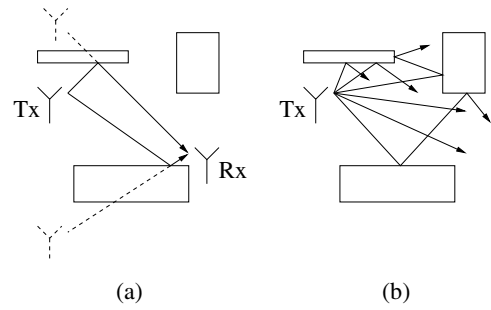


Fig. 2. Compare ray-tracing and ray-launching methods: (a) Ray-tracing; (b) ray-launching.

B. Geometry-based stochastic models

Unlike deterministic models, which use site-specific geometric information to calculate the propagating radio wave, stochastic models benefit from statistical properties of the radio wave propagation. A large number of measured data is analyzed to extract statistic distributions. The physical channels are then randomly generated based on the statistical distributions.

Usually, rays with similar delay and angles are grouped as clusters. Furthermore, it is assumed that each cluster has M distinct rays and a total number of N clusters are generated. Each ray (m th ray within n th cluster) is specified by its

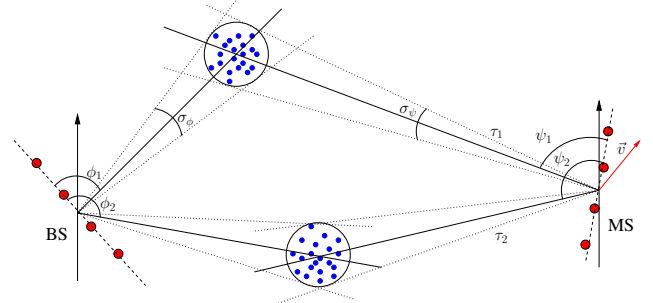


Fig. 3. Geometry-based stochastic MIMO channel model.

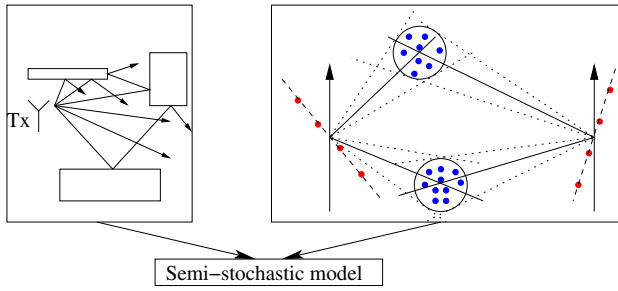


Fig. 4. Semi-stochastic model combines ray-launching and geometry-based stochastic channel models.

AoD $\phi_{n,m}$, AoA $\psi_{n,m}$, delay $\tau_{n,m}$, initial phase $\Phi_{n,m}$ and the Doppler shift $v_{n,m}$ as illustrated in Fig. 3. Assuming all the rays within the same cluster have identical power, the wideband MIMO channel response can be generated as

$$\begin{aligned}
 h_{u,s,n}(t, \tau) = & \sqrt{\frac{P_n}{M}} \sum_{m=1}^M \sqrt{G_{\text{Tx},s}(\phi_{n,m})} \sqrt{G_{\text{Rx},u}(\psi_{n,m})} \\
 & \exp\left(j \left[\frac{2\pi}{\lambda} \vec{k}_{\text{Tx},n,m} \cdot \vec{d}_{\text{Tx},s} + \Phi_{n,m} \right]\right) \\
 & \exp\left(j \frac{2\pi}{\lambda} \vec{k}_{\text{Rx},n,m} \cdot \vec{d}_{\text{Rx},u}\right) \\
 & \exp\left(j \frac{2\pi}{\lambda} v_{n,m} t\right) \delta(\tau - \tau_{n,m}), \quad (4)
 \end{aligned}$$

where P_n is the cluster power. $\vec{k}_{\text{Tx},n,m}$ is the directional vector at the Tx with the direction of the AoD of the m th ray in n th cluster. On the two-dimensional plane, $\vec{k}_{\text{Tx},n,m}$ is $[\cos \phi_{n,m}, \sin \phi_{n,m}]$. On the Rx side, the directional vector is represented by $\vec{k}_{\text{Rx},n,m}$. The wave length is denoted by λ . The relative position vector of s th Tx antenna and u th Rx antenna are denoted by $\vec{d}_{\text{Tx},s}$ and $\vec{d}_{\text{Rx},u}$, respectively [18]. The Doppler frequency component $v_{n,m}$ due to movement of the mobile station is defined by

$$v_{n,m} = \|v\| \cos(\psi_{n,m} - \theta), \quad (5)$$

where $\|v\|$ and θ are the scalar velocity and direction of the mobile station, respectively.

In the stochastic models, large scale parameters (LSP), such as distributions of AoD, AoA, delay, etc., are decided according to the chosen propagation environment (urban, suburban, etc.). Afterwards, small scale parameters (AoD, AoA, delay, etc.) are generated in a random manner following the large scale parameters. For instance, to generate AoD, a cluster angle ϕ_n is first randomly generated according to a pre-defined power-azimuth spectrum (PAS). After that, offset angles α_m are applied for each ray using

$$\phi_{n,m} = \phi_n + c_{\text{AoD}} \alpha_m \quad (6)$$

where c_{AoD} is the cluster-wise root mean square azimuth spread of departure angles [13]. The generation of AoA is analogous.

Parameters	Description
n, N	Path index, total number of paths
l_n	Per path attenuation
d_n	Propagation distance
r_n	Number of reflections
φ_n	Additional phase shift due to propagation and reflections
ϕ_n, ψ_n	Path AoD and AoA

TABLE I
PATH INFORMATION GENERATED BY CORLA

III. SEMI-STOCHASTIC MODEL

In this section, the modeling procedure of the proposed semi-stochastic model is explained in detail. As depicted in Fig. 4, the semi-stochastic model is a reasonable combination of ray-launching and geometric-based stochastic models. We take the paths of the deterministic step to create the clusters of the WINNER model. Apart from that, the WINNER model is applied. Enhancements of the semi-stochastic model compared to deterministic and stochastic models are addressed. Finally, some possible extensions are discussed.

A. Fusion of ray-launching and stochastic models

As mentioned in last section, the radio propagation environments in geometry-based stochastic models are abstract, while various scenarios are categorized and different geometries are translated into tabulated LSP. In the proposed semi-stochastic model, the geometric information of the radio propagation environment is represented by 3D maps. For a given 3D map, the propagation paths (clusters) are explicitly determined by ray-launching. Afterwards, similar procedures as in geometry-based stochastic models are adopted to reproduce the random channel characteristics. In this paper, CORLA is used as the ray-launching algorithm and the WINNER model is used as the geometry-based stochastic model. In principle, other ray-launching algorithms and stochastic models can also be used for the semi-stochastic model with minor modifications [19].

The first step of the proposed model is the deterministic step, namely, to perform ray-launching for every (Rx) point on the map. Being limited by the computational power, ray-launching algorithms are usually performed on maps with relatively low resolution, where a few square meters are mapped into one pixel. The output parameters of each pixel should be stored in look-up tables to speed up the subsequent modeling process. For CORLA, the available output parameters are listed in Table I. The per path attenuation l_n is calculated by considering the free space loss, reflection, diffraction and transmission. The overall attenuation can be calculated by

$$l = \frac{1}{\sum_{n=1}^N \frac{1}{l_n}}, \quad (7)$$

whereas the normalized path power P_n can be written as

$$P_n = \frac{l}{l_n}. \quad (8)$$

The additional phase shift φ_n is due to propagation and reflections

$$\varphi_n = \frac{2\pi}{\lambda} d_n + \pi r_n, \quad (9)$$

as in each reflection, the phase of the ray is altered by 180 degrees. The cluster delay can also be calculated by

$$\tau_n = d_n/c, \quad (10)$$

where c is the speed of light. Normally, the propagation environment does not change during the simulations. Therefore, the deterministic step should be performed only once at the initialization stage.

After the deterministic step, the stochastic steps are applied. For all receivers and for all time instances t between t_0 and t_{\max} , the following procedure is performed to calculate the channel matrices $\mathbf{H}(t)$. The location of the mobile station is calculated according to its speed and direction of movement. The cluster power, delay, AoD and AoA of the current location are obtained from the look-up table built in the deterministic step. Assuming each ray has the same amount of power, the individual delay, AoD and AoA are generated following the methodology in the WINNER model. In the two strongest clusters, rays are spread into three sub-clusters with different intra-cluster delay offset as shown in Table II. The ray AoD and AoA are calculated using (6), the offset angles are listed in Table III. The random coupling of AoD and AoA is then performed for rays within a cluster or a sub-cluster in case of two strongest clusters. Moreover, the initial phase $\Phi_{n,m}$ is also drawn randomly following a uniform distribution $\text{Uni}(0, 2\pi)$. Finally, with given antenna pattern, the channel coefficient can be generated using (4). The whole procedure is summarized in Algorithm 1, where a receive point is denoted by p . The sets of cluster, ray, antenna, and channel parameters are denoted by \mathcal{C} , \mathcal{R} , \mathcal{G} , and \mathcal{H} , respectively.

B. Enhancements

In geometry-based stochastic models, while choosing propagation scenarios, the existence of LoS component must be specified and so does the Ricean K factor, if applicable. In contrast, in the semi-stochastic model, the existence and strength of the LoS component are both known.

Due to the non-ideal surface of the scatterers, signal power may be reflected to other than the specular directions. This diffused scattering phenomena leads to an increasing number of rays as well as number of interactions [20]. Diffused scattering is normally not covered by the deterministic approaches because of the high computational burden. In the semi-stochastic model, the diffused scattering is implied by the stochastic step with small computational effort.

C. Extensions

Polarized antennas are popular in MIMO systems for higher diversity [21]. When different polarizations are used, the

sub-clusters #	mapping to rays	power	delay offset
1	1,2,3,4,5,6,7,8,19,20	10/20	0 ns
2	9,10,11,12,17,18	6/20	5 ns
3	13,14,15,16	4/20	10 ns

TABLE II
INTRA-CLUSTER DELAY SPREAD FOR TWO STRONGEST PATHS [13].

Ray number m	Basis vector of offset angle α_m
1,2	± 0.0447
3,4	± 0.1413
5,6	± 0.2492
7,8	± 0.3715
9,10	± 0.5129
11,12	± 0.6797
13,14	± 0.8844
15,16	± 1.1481
17,18	± 1.5195
19,20	± 2.1551

TABLE III
RAY OFFSET ANGLES WITHIN CLUSTERS, GIVEN FOR 1° RMS ANGLE SPREAD [13].

Algorithm 1 SemiStochasticChannelModel()

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initLookUpTable( $\mathcal{P}$ ,  $\mathcal{G}$ , CORLA())
for all Rx do
  for  $t = t_0 \rightarrow t_{\max}$  step  $\Delta t$  do
     $p \leftarrow \text{getRxPos}(t)$ 
     $N \leftarrow \text{getNumberOfClusters}(p)$ 
    for  $u = 1 \rightarrow U$  do
      for  $s = 1 \rightarrow S$  do
        for  $n = 1 \rightarrow N$  do
           $\mathcal{C}_n \leftarrow \text{getClusterInformation}(p,n)$ 
          for  $m = 1 \rightarrow M$  do
             $\mathcal{R}_{n,m} \leftarrow \text{getRayParameters}(\mathcal{C}_n, m)$ 
            randomlyCoupleRays()
             $\mathcal{G}_{u,s,n,m} \leftarrow \text{getAntennGain}(\phi_{n,m}, \psi_{n,m})$ 
          end for
           $\mathcal{H}_{u,s,n}(t) \leftarrow \text{getChCoeff}(\mathcal{C}_n, \mathcal{R}_{n,m}, \mathcal{G}_{u,s,n,m})$ 
        end for
      end for
    end for
  end for

```

channel matrix \mathbf{H} can be decomposed into four submatrices

$$\mathbf{H} = \begin{pmatrix} \mathbf{H}_{v,v} & \sqrt{\kappa}\mathbf{H}_{v,h} \\ \sqrt{\kappa}\mathbf{H}_{h,v} & \mathbf{H}_{h,h} \end{pmatrix} \quad (11)$$

where κ is the cross polarization ratio, $\mathbf{H}_{v(h),v(h)}$ is the channel matrix from vertical (horizontal) polarized Tx antenna to vertical (horizontal) polarized Rx antenna. Therefore, additionally, initial phase for different polarization combination must be generated. Also the polarized antenna pattern should be calculated.

In the WINNER model, elevation angles are only defined for few scenarios, although in CORLA the 3D angles are adopted. It is possible in the future to extend the semi-stochastic model to a full 3D model. Nevertheless, the methodology behind is quite straight forward.

IV. IMPLEMENTATION

Although the deterministic part of the semi-stochastic model is time consuming, it only needs to be run once for fixed BS locations. It is feasible to implement CORLA on a general

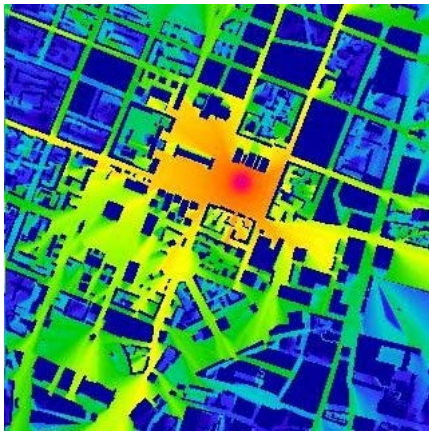


Fig. 5. Field strength prediction in Munich city using CORLA.

purpose computer [17], as shown in Fig. 5. However, due to the massive parallel structure of the ray-launching algorithm, multi-processor hardware, such as graphic processing units (GPU), are proved to be more advantageous [22].

The future implementation of the semi-stochastic channel model involves the hardware channel simulator from our industrial partner Qosmotec GmbH. The Qosmotec Propagation Effects Replicator (QPER) [23] is dedicated to produce replicable simulation of channel effects. Using Field-Programmable Gate Array (FPGA) as calculation unit, QPER is capable of simulating systems with 40MHz bandwidth and 4×4 MIMO channels.

V. CONCLUSION

In this paper, a semi-stochastic model is proposed for wide-band wireless MIMO channels. The semi-stochastic model combines deterministic channel models and geometry-based stochastic channel models. By utilizing the precision of ray-launching algorithms and randomness of the stochastic models, the semi-stochastic model has a great potential of simulating wireless systems in environments with detailed geometric description.

In the planning stage of the next generation cellular networks, it is preferable to have the site specific information with stochastic properties, such that the planning can be thoroughly tested. In the operating stage, to simulate the SON functionality, dynamically generated channels depending on the geometric data are also required. Therefore, the proposed semi-stochastic model is a good candidate for simulating the next generation cellular networks.

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