

Full-Duplex Decode-and-Forward Relaying with Limited Self-Interference Cancellation

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Abstract—In this paper we address the transmit strategy design for a full-duplex (FD) one-way relaying system which operates in decode-and-forward (DF) mode. Our study starts with defining a system model which encompasses the limits of a FD system to overcome its own loopback self-interference. Afterwards we present a design strategy for the defined system focusing on the case with availability of the perfect channel state information (CSI) at all nodes. Furthermore, we generalize our solution to the case with erroneous CSI following the max-min design approach. In the end the proposed methods are evaluated via numerical simulations and the destructive effect of CSI error in the loopback channel is observed.

I. INTRODUCTION

The tempting idea of full-duplex (FD) communications, as the ability to establish two directions of communication at the same time and frequency, has been long considered to be infeasible due to the intrinsic self-interference. In theory, since each node is aware of its own transmitted signal, the interference from the loopback path can be estimated and suppressed. However, in practice this procedure is challenging due to the high strength of the self-interference channel, limited channel state information (CSI) precision, as well as the inaccuracies in the Rx and Tx chains (e.g., power amplifier nonlinearity, oscillator phase noise, limited analog to digital convertor (ADC) and digital to analog convertor (DAC) precision, ...). These sources of inaccuracy, while being ignorable in many classic Half-Duplex (HD) communication schemes, may render the transceiver disfunctional since, due to the proximity of Tx and Rx antennas on the same node, the aforementioned interference is passing through a much stronger channel compared to the deeply attenuated desired signal which originates from a distant source. Recently, via specialized designs, [1]–[8] have provided an adequate level of isolation between Tx and Rx directions to facilitate a FD communication. A common idea of these approaches is the accurate attenuation of main interference components in RF (prior to down-conversion), so that the remaining self-interference can be correctly processed in the effective dynamic range of the ADC and further attenuated in the digital domain. The reported result in [6] promises the suppression of self-interference down to the receiver noise floor for short distance scenarios throughout the bandwidth of 80 MHz. Hence investigating the possible gains by applying FD capability on the classic HD scenarios is becoming more promising. As an interesting use case, [9]–[13] have studied the FD gains and methodologies for scenarios of multi-hop wireless communication. As it has been shown, the majority of FD relaying scenarios while largely benefit from low-delay and efficient nature of FD relays, remain compatible with

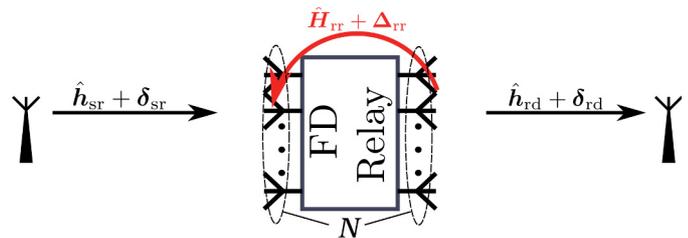


Fig. 1: Half-duplex end users are communicating via a full-duplex relay. Red arrow represents the self-interference signal which should be suppressed.

the HD operation of end users. In this work, we extend this trend by studying the robust transmit strategies for FD decode-and-forward (DF) relays and provide a convex optimization framework to tackle the available design complexities. Our paper is organized as follows: In Section II, inspired from the works in [9], [14] and [15], we define a system model that incorporates the channel knowledge error and hardware limits of the FD node to overcome its self-interference. In Section III we study the transmit strategies for a DF relaying system which benefits from the self interference cancellation mechanism (FD capability). While the design methodology for a FD-DF relaying system is addressed by [3], the presented solutions are neither optimal nor achievable in polynomial time. In Section IV we focus on methodology of dealing with channel knowledge inaccuracy and to enhance the respective worst case performance via appropriate polynomial time solutions. Our solution in Section V is utilizing the defined framework to present an energy efficient design with imperfect CSI and in the end, in Section VI, the performed numerical simulations investigate the effectiveness of the presented methods.

Notations: Vectors and matrices are denoted as bold-faced lower-case and upper-case letters, respectively. Complex conjugation, transpose, Hermitian transpose, vectorization of a matrix, trace operator, second norm of a vector and the Kronecker product are respectively denoted as $\{\cdot\}^*$, $\{\cdot\}^T$, $\{\cdot\}^H$, $\text{vec}(\cdot)$, $\text{Tr}\{\cdot\}$, $\|\cdot\|_2$ and \otimes .

II. SYSTEM MODEL

We investigate a scenario where a pair of single antenna HD users (source and destination) communicate via a FD relay with N transmit and N receive antennas (Fig. 1). The relay is operating in DF mode and the direct path between the end users is assumed to be ignorable. Channels are

following the uncorrelated Rayleigh flat-fading model where $\mathbf{h}_{sr} \in \mathbb{C}^N$ represents the channel between the source and the relay, $\mathbf{h}_{rd}^T \in \mathbb{C}^{1 \times N}$ is the channel between the relay and destination and $\mathbf{H}_{rr} \in \mathbb{C}^{N \times N}$ is the self-interference channel. The values $\rho_{sr}, \rho_{rr}, \rho_{rd} \in \mathbb{R}^+$ represent the variance of the respective channel coefficients. The estimated versions of the defined channels are presented as $\hat{\mathbf{h}}_{sr}, \hat{\mathbf{H}}_{rr}, \hat{\mathbf{h}}_{rd}$ and $\delta_{sr}, \Delta_{rr}, \delta_{rd}$ are the respective estimation errors. Similar to the model in [16] we follow the so-called *deterministic model* for the estimation error which defines a feasible error set as $\delta_{sr}^T \mathbf{T}_{sr} \delta_{sr}^* \leq \xi_{sr}$, $\delta_{rd}^T \mathbf{T}_{rd} \delta_{rd}^* \leq \xi_{rd}$, $\text{Tr}(\Delta_{rr} \mathbf{T}_{rr} \Delta_{rr}^H) \leq \xi_{rr}$ where $\mathbf{T}_{sr}, \mathbf{T}_{rr}, \mathbf{T}_{rd} \in \mathbb{C}^{N \times N}$ are known positive-definite matrices with Hermitian symmetry which shape the error's feasibility region for the corresponding channel. The real, positive values $\xi_{sr}, \xi_{rr}, \xi_{rd} \in \mathbb{R}^+$ are defining the radii of the error feasibility region which is dependent on the quality of our estimation process. The benefits of using this model for similar use cases is justified in [17] and [18]. The relay node continuously receives the transmitted signal from the source while dealing with the loopback interference signal from its own transmitter front-end:

$$\begin{aligned}
 \mathbf{r}_{in} &= \mathbf{h}_{sr} \sqrt{P_T} \cdot s + \mathbf{H}_{rr} \mathbf{r}_{out} + \mathbf{n}_r \\
 &= \underbrace{\mathbf{h}_{sr} \sqrt{P_T} \cdot s}_{\text{desired signal}} + \underbrace{\hat{\mathbf{H}}_{rr} \mathbf{r}_{out}}_{\text{suppressible interference}} \\
 &\quad + \underbrace{\mathbf{n}_r + \Delta_{rr} \mathbf{r}_{out}}_{\text{insuppressible interference}}
 \end{aligned} \tag{1}$$

where $\mathbf{r}_{in}, \mathbf{r}_{out} \in \mathbb{C}^N$ are respectively the received and transmitted signals from the relay node, $s \in \mathbb{C}$ is the transmitted signal from the source ($\mathbb{E}\{ss^*\} = 1$) and $\mathbf{n}_r \in \mathbb{C}^N$ is the zero-mean circularly symmetric complex Gaussian (ZMCSCG) noise with variance σ_{nr}^2 . It is clear that the major part of the interference is known to the receiver since it is basically generated by the same node. Following [9], [14] and [19] we assume that the known part of the interference can be estimated and canceled if its power does not exceed the functional range of the receiver, applying cancellation methods [2]–[6]. Hence we have

$$\mathbf{r}_{in, \text{supp}} = \mathbf{h}_{sr} \sqrt{P_T} \cdot s + \mathbf{n}_r + \Delta_{rr} \mathbf{r}_{out}, \tag{2}$$

$$\mathbb{E} \left\{ \left\| \hat{\mathbf{H}}_{rr} \mathbf{r}_{out} \right\|_2^2 \right\} \leq P_{int}, \tag{3}$$

where $\mathbf{r}_{in, \text{supp}}$ is the interference-suppressed version of the received signal, P_T represents the transmission power from the source and P_{int} defines the system interference power constraint which ensures accurate receiver operation in facing with loopback interference. Afterwards, the transmitted symbol from the source is decoded at the relay (applying a linear detection filter \mathbf{d} , where $\|\mathbf{d}\|_2 = 1$) and amplified to construct the relay output

$$\hat{s} = \mathcal{P}\{\mathbf{d}^H \cdot \mathbf{r}_{in, \text{supp}}\}, \quad \mathbf{r}_{out} = \mathbf{g} \cdot \hat{s}, \tag{4}$$

where $\mathcal{P}\{\cdot\}, \hat{s}$ represent the maximum likelihood detection and the respective detected symbol in the relay. The relay amplification vector is denoted as $\mathbf{g} \in \mathbb{C}^N$. Finally, the destination receives and decodes the signal from the relay

$$y = \mathbf{h}_{rd}^T \mathbf{r}_{out} + n_d, \tag{5}$$

TABLE I: Symbols which are needed for description of the defined communication cycle.

Notation	Description
N	number of Tx and Rx antennas on the relay
$\mathbf{h}_{sr}, \hat{\mathbf{h}}_{sr}$	accurate and estimated channels between source and relay
$\mathbf{h}_{rd}, \hat{\mathbf{h}}_{rd}$	accurate and estimated channels between relay and destination
$\mathbf{H}_{rr}, \hat{\mathbf{H}}_{rr}$	accurate and estimated self interference channel matrices
$\Delta_{rr}, \delta_{sr}, \delta_{rd}$	corresponding channel estimation error
\mathbf{g}	the relay amplification vector
\mathbf{d}	linear detection filter
$\mathbf{n}_r, \mathbf{n}_d$	AWGN noise on the relay and destination
$\sigma_{nr}^2, \sigma_{nd}^2$	noise variance in the relay and destination
P_T	transmit power from the source;
P_{\max}, P_{\max}^R	maximum allowed Tx power from the source and the relay
P_{int}	maximum allowed self interference power
$\mathbf{T}_{rr}, \mathbf{T}_{sr}, \mathbf{T}_{rd}$	feasible region shaping matrices for CSI error
$\xi_{rr}, \xi_{sr}, \xi_{rd}$	respective feasible CSI error radius
r_{in}, r_{out}	input and output signal from the relay node
$r_{in, \text{supp}}$	interference-suppressed relay input signal
s, \hat{s}	transmitted data symbol and the estimated version in the relay

where $y \in \mathbb{C}$ represents the received signal at the destination and $n_d \in \mathbb{C}$ is a ZMCSCG noise which is added to the received signal in destination with variance σ_{nd}^2 . In addition to the defined interference power constraint, both source and relay nodes are limited by their maximum allowed transmit power:

$$P_T \leq P_{\max}, \quad \mathbb{E} \left\{ \|\mathbf{r}_{out}\|_2^2 \right\} \leq P_{\max}^R, \tag{6}$$

where P_{\max} and P_{\max}^R respectively define the maximum allowed Tx power in the source and the relay. In the following parts of this paper, we investigate the optimal set of parameters $(\mathbf{g}, \mathbf{d}, P_T)$ which result in maximum system performance for different scenarios of CSI error, subject to the defined power constraints. In order to avoid confusion in our formulations, Table I presents a list of the used symbols which are involved in the defined communication cycle.

III. OPTIMAL DESIGN WITH PERFECT CSI

It is already established that for the defined multi-hop communication, the end to end mutual information is the minimum capacity of the available individual links [20]

$$C_{s,d} = \min \{C_{s,r}, C_{r,d}\}, \tag{7}$$

where $C_{s,r}, C_{r,d}, C_{s,d}$ represent the mutual information between source and the relay, the relay and the destination and the source and the destination, respectively. Due to the single antenna setup of the end users, we treat the rate maximization problem equivalently as an SINR maximization on the decision variables. We should note that while the aforementioned assumption does not hold in general, it provides a simplifying framework to formulate our problem. Hence, by assuming the zero-mean and Gaussian distribution for all signal, noise and interference components our equivalent problem can be written

as

$$\begin{aligned}
 & \max_{g, d, P_T} t \\
 & \text{s.t. } \text{SINR}_r \geq t, \text{ SINR}_d \geq t, \\
 & \mathbb{E} \left\{ \left\| \hat{\mathbf{H}}_{rr} \cdot \mathbf{r}_{\text{out}} \right\|_2^2 \right\} \leq P_{\text{int}}, \\
 & \mathbb{E} \left\{ \left\| \mathbf{r}_{\text{out}} \right\|_2^2 \right\} \leq P_{\text{max}}^R, P_T \leq P_{\text{max}}, \quad (8)
 \end{aligned}$$

where $\text{SINR}_r, \text{SINR}_d$ are the achieved SINR on the relay node (after applying the detection filter \mathbf{d}) and on the destination node, respectively. Furthermore, the transmit power constraint in the source is tight at least for one optimal solution (otherwise, it can be always scaled up to P_{max} and improve SINR_r while maintaining feasibility with no effect on SINR_d . This statement also holds for the general case with CSI error). For the scenario with availability of perfect CSI, due to the cancellation of the last term in (2), the source to relay and relay to destination paths become independent and hence (8) can be treated as two separate SINR maximization problems on the relay and destination. This readily results in $\mathbf{d} = \frac{\mathbf{h}_{sr}}{\|\mathbf{h}_{sr}\|_2}$ and maximum SINR in the relay to be $\text{SINR}_{r,\text{max}} = \frac{P_T \|\mathbf{h}_{sr}\|_2^2}{\sigma_{rr}^2}$. Optimal SINR in the destination can be respectively achieved via the following problem

$$\begin{aligned}
 & \max_g \frac{\mathbf{h}_{rd}^T \mathbf{g} \mathbf{g}^H \mathbf{h}_{rd}^*}{\sigma_{nd}^2} \\
 & \text{s.t. } \left\| \mathbf{H}_{rr} \cdot \mathbf{g} \right\|_2^2 \leq P_{\text{int}}, \left\| \mathbf{g} \right\|_2^2 \leq P_{\text{max}}^R, \quad (9)
 \end{aligned}$$

which by defining $\mathbf{G} = \mathbf{g} \mathbf{g}^H$ and ignoring the implicit rank-1 constraint on \mathbf{G} turns into

$$\begin{aligned}
 & \max_{\mathbf{G}} \text{Tr}(\mathbf{h}_{rd}^* \mathbf{h}_{rd}^T \mathbf{G}) \\
 & \text{s.t. } \text{Tr}(\mathbf{H}_{rr}^H \mathbf{H}_{rr} \mathbf{G}) \leq P_{\text{int}}, \\
 & \text{Tr}(\mathbf{G}) \leq P_{\text{max}}^R, \mathbf{G} \geq 0, \mathbf{G} = \mathbf{G}^H. \quad (10)
 \end{aligned}$$

Similar to [9] and due to the Corollary 3.4 in [21] we always obtain an optimal rank-1 \mathbf{G} for the above problem and hence the relaxed constraint is satisfied. The optimal \mathbf{g} can be consequently achieved via $\mathbf{g} = \mathbf{G}^{\frac{1}{2}}$. It is worth mentioning that while the presented solution offers an optimal rate maximizing strategy, it is not unique and is not taking into account the power consumption. The problem of energy efficient design will be further discussed in Section V.

IV. SEMI-DEFINITE RELAXATION FRAMEWORK FOR WORST-CASE MAXIMIZATION WITH ERRONEOUS CSI

In this section we investigate the effects of CSI imperfection on the performance of the defined system. We start with the effects of CSI error in the loopback channel which is of high importance due to the FD system architecture. Afterwards we extend our study to the general scenario of CSI error and provide respective robust designs in each step.

A. Imperfect CSI for Loopback Channel

As it is mentioned in our system model (Section II) the loopback channel estimation error directly acts as a residual interference component (2) and significantly degrades the achievable performance. This problem is considered as a

major bottleneck for cancellation techniques which aim at high suppression quality [6]. In this part we take into account the estimation error for the loopback channel and present a robust design based on the worst case optimization. Our equivalent SINR maximization problem in (8) turns into

$$\begin{aligned}
 & \max_{g, d, t} \min_{\Delta_{rr}} t \\
 & \text{s.t. } \frac{P_{\text{max}} \mathbf{d}^H \mathbf{h}_{sr} \mathbf{h}_{sr}^H \mathbf{d}}{\mathbf{d}^H [\sigma_{rr}^2 \mathbf{I}_N + \Delta_{rr} \mathbf{g} \mathbf{g}^H \Delta_{rr}^H] \mathbf{d}} \geq t, \\
 & \text{Tr}(\Delta_{rr} \mathbf{T}_{rr} \Delta_{rr}^H) \leq \xi_{rr}, \mathbf{h}_{rd}^T \mathbf{g} \mathbf{g}^H \mathbf{h}_{rd}^* \geq t \cdot \sigma_{nd}^2, \\
 & \text{Tr}(\hat{\mathbf{H}}_{rr}^H \hat{\mathbf{H}}_{rr} \mathbf{g} \mathbf{g}^H) \leq P_{\text{int}}, \|\mathbf{d}\|_2 = 1, \\
 & \text{Tr}(\mathbf{g} \mathbf{g}^H) \leq P_{\text{max}}^R. \quad (11)
 \end{aligned}$$

We can infer from (11) that the performance minimization over feasible estimation errors happens at the border of the error feasible area. In order to observe that, we note that for any error matrix (Δ_{rr}) for which $\text{Tr}(\Delta_{rr} \mathbf{T}_{rr} \Delta_{rr}^H) < \xi_{rr}$, we may scale up (with a positive scalar value) the error matrix so that the noted inequality becomes tight. Since Δ_{rr} is scaled up, the interference component $\mathbf{d}^H \Delta_{rr} \mathbf{g} \mathbf{g}^H \Delta_{rr}^H \mathbf{d}$ experiences an inevitable increase which results in smaller SINR value in the relay node with no effect on SINR_d . This justifies that at least one performance minimizing error matrix is located on the boundary of the respective feasible region. We may further investigate the noted condition as

$$\begin{aligned}
 & \max_{\Delta_{rr}} \mathbf{d}^H \Delta_{rr} \mathbf{g} \mathbf{g}^H \Delta_{rr}^H \mathbf{d} \\
 & \text{s.t. } \text{Tr}(\Delta_{rr} \mathbf{T}_{rr} \Delta_{rr}^H) = \xi_{rr}, \quad (12)
 \end{aligned}$$

which by defining $\Delta'_{rr} = \Delta_{rr} \mathbf{T}_{rr}^{\frac{1}{2}}$, $\mathbf{g}' = \mathbf{T}_{rr}^{-\frac{1}{2}} \mathbf{g}$ and $\delta'_{rr} = \text{vec}(\Delta'_{rr})$ turns into

$$\max_{\Delta'_{rr}} \frac{\delta'_{rr}{}^H \mathbf{A} \delta'_{rr}}{\delta'_{rr}{}^H \delta'_{rr}} \cdot \xi_{rr} = \lambda_{\text{max}}\{\mathbf{A}\} \cdot \xi_{rr}, \quad (13)$$

where $\mathbf{A} = (\mathbf{g}'^* \otimes \mathbf{d}) \cdot (\mathbf{g}'^* \otimes \mathbf{d})^H$ and $\lambda_{\text{max}}\{\cdot\}$ presents the maximum eigenvalue operator. Note that the above variable redefinition is always feasible due to the positive definite nature of error region shaping matrices (including \mathbf{T}_{rr}). Due to the rank-1 nature of \mathbf{A} and $\|\mathbf{d}\|_2 = 1$ we have

$$\begin{aligned}
 \lambda_{\text{max}}\{\mathbf{A}\} \cdot \xi_{rr} &= \left\| \mathbf{g}'^* \otimes \mathbf{d} \right\|_2^2 \cdot \xi_{rr} = \\
 & \text{Tr}(\mathbf{g}' \mathbf{g}'^H) \cdot \xi_{rr} = \text{Tr}(\mathbf{T}_{rr}^{-1} \cdot \mathbf{g} \mathbf{g}^H) \cdot \xi_{rr}. \quad (14)
 \end{aligned}$$

The above result shows that the worst-case insuppressible self-interference (corresponding to the worst-case Δ_{rr}) is not dependent on the design of \mathbf{d} and can be simplified as (14). This turns our problem in (11) as

$$\begin{aligned}
 & \max_{g, d, t} t \\
 & \text{s.t. } P_{\text{max}} \cdot \mathbf{d}^H \mathbf{h}_{sr} \mathbf{h}_{sr}^H \mathbf{d} - t \sigma_{rr}^2 - t \xi_{rr} \text{Tr}(\mathbf{T}_{rr}^{-1} \mathbf{g} \mathbf{g}^H) \geq 0, \\
 & \text{Tr}(\mathbf{g} \mathbf{g}^H) \leq P_{\text{max}}^R, \mathbf{h}_{rd}^T \mathbf{g} \mathbf{g}^H \mathbf{h}_{rd}^* - t \cdot \sigma_{nd}^2 \geq 0, \\
 & \text{Tr}(\hat{\mathbf{H}}_{rr}^H \hat{\mathbf{H}}_{rr} \mathbf{g} \mathbf{g}^H) \leq P_{\text{int}}, \|\mathbf{d}\|_2 = 1, \quad (15)
 \end{aligned}$$

It is observable that one optimal \mathbf{d} for the above problem is $\frac{\mathbf{h}_{sr}}{\|\mathbf{h}_{sr}\|_2}$. At this point, we apply a bisection search over values

of t which for each step turns into a feasibility problem. By repeating the same semi-definite relaxation procedure as defined in (10) our feasibility problem can be written as

$$\begin{aligned} & \max_{\mathbf{G}} 1 \\ & \text{s.t. } P_{\max} \|\mathbf{h}_{\text{sr}}\|_2^2 - t\sigma_{\text{nr}}^2 - t\xi_{\text{rr}} \cdot \text{Tr}(\mathbf{T}_{\text{rr}}^{-1}\mathbf{G}) \geq 0, \\ & \quad \text{Tr}(\mathbf{h}_{\text{rd}}^* \mathbf{h}_{\text{rd}}^{\text{T}} \mathbf{G}) - t\sigma_{\text{nd}}^2 \geq 0, \mathbf{G} = \mathbf{G}^{\text{H}}, \\ & \quad \text{Tr}(\hat{\mathbf{H}}_{\text{rr}}^{\text{H}} \hat{\mathbf{H}}_{\text{rr}} \mathbf{G}) \leq P_{\text{int}}, \mathbf{G} \geq 0, \\ & \quad \text{Tr}(\mathbf{G}) \leq P_{\max}^{\text{R}}, \end{aligned} \quad (16)$$

which should be examined for each value of t . The resulted \mathbf{G} is not a rank-1 matrix in general. To approach this problem we define an equivalent feasibility test to (16) by formulating our problem as

$$\begin{aligned} & \min_{\mathbf{G}} \text{Tr}(\mathbf{T}_{\text{rr}}^{-1}\mathbf{G}) \\ & \text{s.t. } \text{Tr}(\mathbf{h}_{\text{rd}}^* \mathbf{h}_{\text{rd}}^{\text{T}} \mathbf{G}) - t\sigma_{\text{nd}}^2 \geq 0, \mathbf{G} = \mathbf{G}^{\text{H}}, \\ & \quad \text{Tr}(\hat{\mathbf{H}}_{\text{rr}}^{\text{H}} \hat{\mathbf{H}}_{\text{rr}} \mathbf{G}) \leq P_{\text{int}}, \mathbf{G} \geq 0, \\ & \quad \text{Tr}(\mathbf{G}) \leq P_{\max}^{\text{R}}. \end{aligned} \quad (17)$$

It is clear that the feasibility of (16) readily results in the feasibility of (17). On the other hand, (16) is feasible if (17) is feasible and the resulted \mathbf{G} satisfies: $P_{\max} \|\mathbf{h}_{\text{sr}}\|_2^2 - t\sigma_{\text{nr}}^2 - t\xi_{\text{rr}} \cdot \text{Tr}(\mathbf{T}_{\text{rr}}^{-1}\mathbf{G}) \geq 0$. The benefit of applying the formulation in (17) is the fact that if it is feasible for a given t the corresponding rank-1 \mathbf{G} always exists and can be obtained following the Corollary 3.4 in [21]. The defined feasibility test will be repeated for different values of t following the bi-section search procedure until the desired solution precision is achieved.

B. Generalized Design with Imperfect CSI

In order to address the generalized case, where the CSI corresponding to all paths are suffered due to the channel estimation inaccuracy, we formulate our robust SINR maximization problem as

$$\begin{aligned} & \max_{\mathbf{g}, \mathbf{d}, t} \min_{\Delta_{\text{rr}}, \delta_{\text{sr}}, \delta_{\text{rd}}} t \\ & \text{s.t. } \frac{P_{\max} \mathbf{d}^{\text{H}} (\mathbf{h}_{\text{sr}} + \delta_{\text{sr}}) (\mathbf{h}_{\text{sr}} + \delta_{\text{sr}})^{\text{H}} \mathbf{d}}{\mathbf{d}^{\text{H}} [\sigma_{\text{nr}}^2 \mathbf{I}_N + \Delta_{\text{rr}} \mathbf{g} \mathbf{g}^{\text{H}} \Delta_{\text{rr}}^{\text{H}}] \mathbf{d}} \geq t, \\ & \quad (\mathbf{h}_{\text{rd}} + \delta_{\text{rd}})^{\text{T}} \mathbf{g} \mathbf{g}^{\text{H}} (\mathbf{h}_{\text{rd}} + \delta_{\text{rd}})^* \geq t \cdot \sigma_{\text{nd}}^2, \\ & \quad \text{Tr}(\hat{\mathbf{H}}_{\text{rr}}^{\text{H}} \hat{\mathbf{H}}_{\text{rr}} \mathbf{g} \mathbf{g}^{\text{H}}) \leq P_{\text{int}}, \text{Tr}(\mathbf{g} \mathbf{g}^{\text{H}}) \leq P_{\max}^{\text{R}}, \\ & \quad \text{Tr}(\delta_{\text{sr}}^{\text{T}} \mathbf{T}_{\text{sr}} \delta_{\text{sr}}^*) \leq \xi_{\text{sr}}, \text{Tr}(\delta_{\text{rd}}^{\text{T}} \mathbf{T}_{\text{rd}} \delta_{\text{rd}}^*) \leq \xi_{\text{rd}}, \\ & \quad \text{Tr}(\Delta_{\text{rr}} \mathbf{T}_{\text{rr}} \Delta_{\text{rr}}^{\text{H}}) \leq \xi_{\text{rr}}, \|\mathbf{d}\|_2 = 1, \end{aligned} \quad (18)$$

which by recalling (14) and applying a bi-section search over values of t turns into the following feasibility problem

$$\begin{aligned} & \max_{\mathbf{g}, \mathbf{d}} \min_{\Delta_{\text{rr}}, \delta_{\text{sr}}, \delta_{\text{rd}}} 1 \\ & \text{s.t. } \text{Tr} \left(P_{\max} (\hat{\mathbf{h}}_{\text{sr}} + \delta_{\text{sr}})^{\text{T}} \mathbf{d}^* \mathbf{d}^{\text{T}} (\hat{\mathbf{h}}_{\text{sr}} + \delta_{\text{sr}})^* \right. \\ & \quad \left. - t \cdot \sigma_{\text{nr}}^2 - t \cdot \xi_{\text{rr}} \cdot \text{Tr}(\mathbf{T}_{\text{rr}}^{-1} \mathbf{g} \mathbf{g}^{\text{H}}) \right) \geq 0, \\ & \quad \text{Tr} \left((\hat{\mathbf{h}}_{\text{rd}} + \delta_{\text{rd}})^{\text{T}} \mathbf{g} \mathbf{g}^{\text{H}} (\hat{\mathbf{h}}_{\text{rd}} + \delta_{\text{rd}})^* \right) \\ & \quad \quad \quad - t \cdot \sigma_{\text{nd}}^2 \geq 0, \\ & \quad \text{Tr}(\hat{\mathbf{H}}_{\text{rr}}^{\text{H}} \hat{\mathbf{H}}_{\text{rr}} \mathbf{g} \mathbf{g}^{\text{H}}) \leq P_{\text{int}}, \text{Tr}(\mathbf{g} \mathbf{g}^{\text{H}}) \leq P_{\max}^{\text{R}}, \\ & \quad \text{Tr}(\delta_{\text{sr}}^{\text{T}} \mathbf{T}_{\text{sr}} \delta_{\text{sr}}^*) \leq \xi_{\text{sr}}, \|\mathbf{d}\|_2 = 1, \\ & \quad \text{Tr}(\delta_{\text{rd}}^{\text{T}} \mathbf{T}_{\text{rd}} \delta_{\text{rd}}^*) \leq \xi_{\text{rd}}. \end{aligned} \quad (19)$$

It is observable that each of the constraints is affected by no more than one of the channel estimation errors. Clearly the feasibility of (19) is achieved iff all of the included constraints remain feasible for all of the feasible channel estimation errors. In order to proceed we apply the following theorem which can be inferred from the derivations in [9], [16]

Theorem 4.1: Given a vector $\mathbf{h} \in \mathbb{C}^N$ and values $\xi, t \in \mathbb{R}^+$, the feasible set on \mathbf{Q} , $\mathbf{Q} = \mathbf{Q}^{\text{H}}$ which fits in the following constraint:

$$\begin{cases} \forall \delta, \delta^{\text{T}} \mathbf{T} \delta^* \leq \xi \Rightarrow \\ (\mathbf{h} + \delta)^{\text{T}} \mathbf{Q} (\mathbf{h} + \delta)^* \geq t, \end{cases} \quad (20)$$

is equivalent to the feasible set over \mathbf{Q} defined by

$$\begin{aligned} & \max_{\mathbf{Q}, \mathbf{Z}, \mu \geq 0} 1 \\ & \text{s.t. } \text{Tr}[(\mathbf{Q} - \mathbf{Z}) \mathbf{h}^* \mathbf{h}^{\text{T}}] - \xi \cdot \mu \geq t, \\ & \quad \begin{bmatrix} \mathbf{Z} & \mathbf{Q} \\ \mathbf{Q} & \mathbf{Q} + \mu \mathbf{T} \end{bmatrix} \geq 0, \end{aligned} \quad (21)$$

where $\mathbf{T} > 0$, $\mathbf{T} = \mathbf{T}^{\text{H}}$ and the matrix $\mathbf{Z} \in \mathbb{C}^{N \times N}$ and $\mu \in \mathbb{R}^+$ are auxiliary variables.

Proof: The proof is similar to the derivations in [16] (equations 13-25). The main components of this reasoning is the so-called S-procedure [22] as a powerful tool to deal with robust quadratic problems as well as the famous Schur's complement [23]. The same derivation steps has been also discussed in details in [9]. ■

By defining $\mathbf{D} = \mathbf{d}^* \mathbf{d}^{\text{T}}$, $\mathbf{G} = \mathbf{g} \mathbf{g}^{\text{H}}$ and applying the same semi-definite relaxation procedure on (19) we reformulate our

problem as

$$\begin{aligned}
 & \max_{\mathbf{G}, \mathbf{D}, \mathbf{Z}_\ell, \mu_\ell \geq 0} 1 \\
 \text{s.t.} \quad & \begin{bmatrix} \mathbf{Z}_1 & \mathbf{D} \\ \mathbf{D} & \mathbf{D} + \mu_1 \mathbf{T}_{\text{sr}} \end{bmatrix} \geq 0, \\
 & \begin{bmatrix} \mathbf{Z}_2 & \mathbf{G} \\ \mathbf{G} & \mathbf{G} + \mu_2 \mathbf{T}_{\text{rd}} \end{bmatrix} \geq 0, \\
 & \text{Tr} \left[(\mathbf{D} - \mathbf{Z}_1) \hat{\mathbf{h}}_{\text{sr}}^* \hat{\mathbf{h}}_{\text{sr}}^{\text{T}} \right] - \xi_{\text{sr}} \cdot \mu_1 \geq t \cdot \left[\frac{\sigma_{\text{nr}}^2}{P_{\text{max}}} \right. \\
 & \quad \left. + \frac{\xi_{\text{rr}} \text{Tr}(\mathbf{T}_{\text{rr}}^{-1} \mathbf{G})}{P_{\text{max}}} \right], \\
 & \text{Tr} \left[(\mathbf{G} - \mathbf{Z}_2) \hat{\mathbf{h}}_{\text{rd}}^* \hat{\mathbf{h}}_{\text{rd}}^{\text{T}} \right] - \xi_{\text{rd}} \cdot \mu_2 \geq t \cdot \sigma_{\text{nd}}^2, \\
 & \text{Tr}(\hat{\mathbf{H}}_{\text{rr}}^{\text{H}} \hat{\mathbf{H}}_{\text{rr}} \mathbf{G}) \leq P_{\text{int}}, \quad \text{Tr}(\mathbf{G}) \leq P_{\text{max}}^{\text{R}}, \\
 & \text{Tr}(\mathbf{D}) = 1, \quad \mathbf{D} = \mathbf{D}^{\text{H}}, \mathbf{D} \geq 0, \\
 & \mathbf{G} = \mathbf{G}^{\text{H}}, \mathbf{G} \geq 0, \ell \in \{1, 2\}. \tag{22}
 \end{aligned}$$

where t is treated as a constant in each step of the feasibility check. Despite the additional complication imposed by introducing the new variables $(\mathbf{Z}_\ell, \mu_\ell)$, it is clear that (22) follows the semi-definite programming structure and can be efficiently solved within a polynomial time. We have to note that the achieved \mathbf{G}, \mathbf{D} are in general not rank-1 matrices. The respective rank-1 matrices can be achieved by applying randomization [24] or via direct singular decomposition. Having the rank-1 approximations we calculate

$$\mathbf{d}^* = \left(\mathbf{D}^{\frac{1}{2}} \right)^*, \mathbf{g}^* = \mathbf{G}^{\frac{1}{2}}, \tag{23}$$

where $\mathbf{g}^*, \mathbf{d}^*$ are the desired solutions to \mathbf{g}, \mathbf{d} in each iteration. The defined process is continued for different values of t , following the bi-section search steps, until the required precision is achieved.

V. OPTIMAL DESIGN FRAMEWORK WITH MINIMUM POWER CONSUMPTION

While the solution in (22) offers a polynomial time procedure to obtain maximum communication rate, the corresponding optimal solution is not unique. Since the end to end communication rate is usually limited by only one of the source to relay or the relay to destination links, it will be inefficient to spend unnecessary power on a link while it is not the communication bottleneck. Furthermore, there is no consideration of system power consumption in the offered design in (22). This encourages us to investigate the energy efficient transmit strategy as a separated design, given a pre-defined requirement of end to end communication rate. For this purpose, we assume that the system power consumption can be approximated by addition of the transmit signal power from the source and from the relay node

$$P_{\text{tot}} = P_{\text{T}} + \|\mathbf{g}\|_2^2, \tag{24}$$

where P_{tot} is the total system transmit power. Following the same semi-definite relaxation framework as in (22) and defining

$$\bar{\mathbf{D}} := P_{\text{T}} \mathbf{d}^* \mathbf{d}^{\text{T}}, \tag{25}$$

we are able to formulate our optimization problem as

$$\begin{aligned}
 & \min_{\mathbf{G}, \bar{\mathbf{D}}, \mathbf{Z}_\ell, \mu_\ell \geq 0} \text{Tr}(\bar{\mathbf{D}} + \mathbf{G}) \\
 \text{s.t.} \quad & \begin{bmatrix} \mathbf{Z}_1 & \bar{\mathbf{D}} \\ \bar{\mathbf{D}} & \bar{\mathbf{D}} + \mu_1 \mathbf{T}_{\text{sr}} \end{bmatrix} \geq 0, \\
 & \begin{bmatrix} \mathbf{Z}_2 & \mathbf{G} \\ \mathbf{G} & \mathbf{G} + \mu_2 \mathbf{T}_{\text{rd}} \end{bmatrix} \geq 0, \\
 & \text{Tr} \left[(\bar{\mathbf{D}} - \mathbf{Z}_1) \hat{\mathbf{h}}_{\text{sr}}^* \hat{\mathbf{h}}_{\text{sr}}^{\text{T}} \right] - \xi_{\text{sr}} \cdot \mu_1 \geq t_0 \cdot \left[\sigma_{\text{nr}}^2 \right. \\
 & \quad \left. + \xi_{\text{rr}} \text{Tr}(\mathbf{T}_{\text{rr}}^{-1} \mathbf{G}) \right], \\
 & \text{Tr} \left[(\mathbf{G} - \mathbf{Z}_2) \hat{\mathbf{h}}_{\text{rd}}^* \hat{\mathbf{h}}_{\text{rd}}^{\text{T}} \right] - \xi_{\text{rd}} \cdot \mu_2 \geq t_0 \cdot \sigma_{\text{nd}}^2, \\
 & \text{Tr}(\hat{\mathbf{H}}_{\text{rr}}^{\text{H}} \hat{\mathbf{H}}_{\text{rr}} \mathbf{G}) \leq P_{\text{int}}, \quad \text{Tr}(\mathbf{G}) \leq P_{\text{max}}^{\text{R}}, \\
 & \bar{\mathbf{D}} = \bar{\mathbf{D}}^{\text{H}}, \bar{\mathbf{D}} \geq 0, \text{Tr}(\bar{\mathbf{D}}) \leq P_{\text{max}}, \\
 & \mathbf{G} = \mathbf{G}^{\text{H}}, \mathbf{G} \geq 0, \ell \in \{1, 2\}. \tag{26}
 \end{aligned}$$

where the value t_0 represents the pre-defined requirement on the end to end communication rate and the term $\text{Tr}(\bar{\mathbf{D}})$ is equal to the source transmit power due to the definition (24). It is worth mentioning that due to the pre-defined rate requirement (t_0), there is no need to apply bi-section search steps and the optimal general rank solution is achievable via single iteration. Similar to (22), the resulted $\mathbf{G}, \bar{\mathbf{D}}$ are in general not rank-1 matrices. The corresponding rank-1 solutions are achieved via randomization technique [24] or direct singular decomposition of the general rank matrices. Having the rank-1 approximations we calculate

$$\bar{\mathbf{d}}^* = \left(\bar{\mathbf{D}}^{\frac{1}{2}} \right)^*, \mathbf{g}^* = \mathbf{G}^{\frac{1}{2}}, \tag{27}$$

and the respective desired \mathbf{d} and the transmit power of the source are achieved as

$$P_{\text{T}}^* = \|\bar{\mathbf{d}}^*\|_2^2, \mathbf{d}^* = \frac{\bar{\mathbf{d}}^*}{\|\bar{\mathbf{d}}^*\|_2}, \tag{28}$$

where $\mathbf{g}^*, \mathbf{d}^*, P_{\text{T}}^*$ are the desired energy efficient solutions to $\mathbf{g}, \mathbf{d}, P_{\text{T}}$ corresponding to t_0 as the pre-defined link quality in (26).

VI. SIMULATION RESULTS

In this section we evaluate the performance of the introduced methods via Monte Carlo simulations. We follow the defined system setup and channel model as described in Section II and average our results over 100 realizations. The comparison is made between the performance of the equivalent HD system (with equal number of Tx and Rx chains as well as the same CSI error condition as the FD setup), the derived robust design with FD setup in Section IV, and the non-robust FD design (assuming no CSI error). As the comparison metric we have chosen the worst-case communication rate which maintains for all feasible CSI error conditions. For the aforementioned system and design strategies, the resulted performance is been observed for various system settings. In Fig.2 the effect of self-interference suppression capability ($\beta := \frac{P_{\text{int}}}{P_{\text{max}}}$) is been studied. As it can be observed for small values of CSI error and high suppression capability,

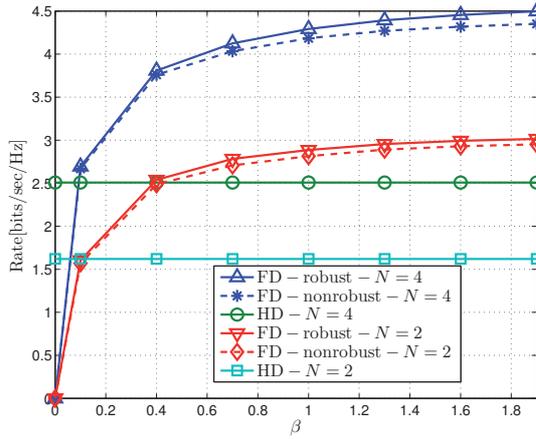


Fig. 2: Achievable rate (worst-case) vs. Interference suppression quality (β). $\xi_{sr}^n = \xi_{rd}^n = \xi_{rr}^n = 10^{-4}$. The FD two-fold gain is observable for high suppression quality and small CSI error.

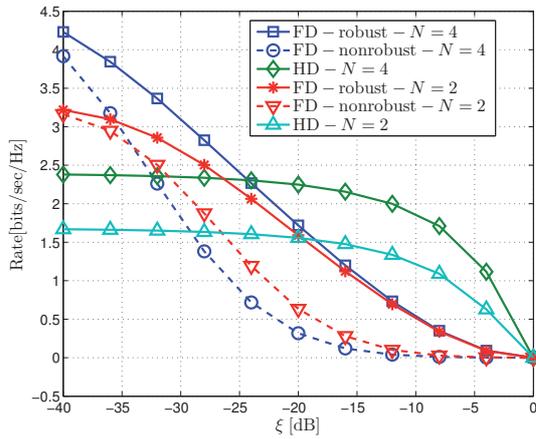


Fig. 3: Achievable rate (worst-case) vs. CSI error intensity. $\xi = \xi_{sr}^n = \xi_{rd}^n = \xi_{rr}^n$. FD operation is highly sensitive to the CSI error in the loopback path.

the FD system is capable of achieving nearly twice of rate compared to an equivalent HD system. Fig.3 illustrates the extremely destructive effect of the CSI error ($\xi_{sr} = \xi_{sr}^n \cdot \|\mathbf{h}_{sr}\|_2^2$, $\xi_{rd} = \xi_{rd}^n \cdot \|\mathbf{h}_{rd}\|_2^2$ and $\xi_{rr} = \xi_{rr}^n \cdot \|\mathbf{H}_{rr}\|_{fro}^2$) in the FD operation. The aforementioned definition of the values ξ_{sr}^n , ξ_{rd}^n , ξ_{rr}^n adapts the defined error radii for different channel realizations. Fig.3 verifies our expectation from Section II since any inaccuracy of the self-interference channel directly results in the residual interference in the receiver which is not suppressible, since it is not known. That follows due to the fact that the perfect estimation of the received interference becomes impossible due to the defined CSI error. Fig.4 illustrates the effect of N on the system performance and on the effectiveness of our robust design. As it was expected, while the overall system performance experiences an observable increase as the number of antennas increases, this enhancement is only achieved via employing the robust transmit strategy. In general, in a setting with erroneous CSI, more antennas generate more sources of

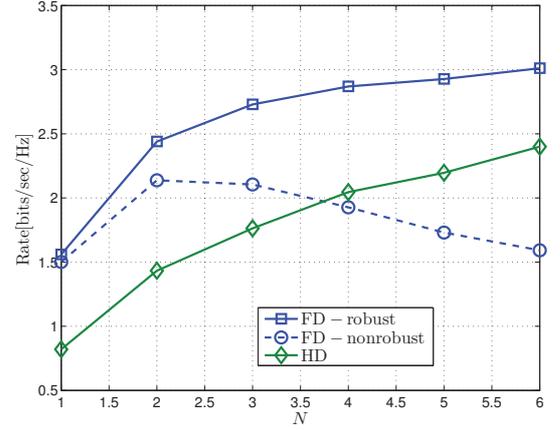


Fig. 4: Achievable rate (worst-case) vs. Number of antennas (N). The necessity of the robust approach increases for higher number of antennas.

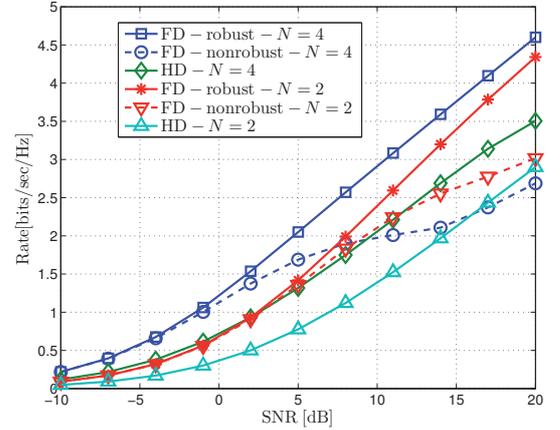


Fig. 5: Achievable rate (worst-case) vs. SNR [dB]. The robust design achieves higher gains for high SNR regions.

error and consequently more components of insuppressible interference in the loopback path which can be fatal for a FD operation. On the other hand, higher N promises a higher spatial degree of freedom to adapt our precoding design with the system conditions. Following the aforementioned arguments, it can be observed from Fig.4 that while higher N reduces the performance of the error-prone system, our robust transmit strategy provides a significant level of resilience against this degradation. As the other observation, Fig.5 depicts the effect of noise level on the resulted performance ($\text{SNR} := \frac{P_{\max} \cdot \rho_{sr}}{\sigma_{nr}^2 + \sigma_{rd}^2}$). As it is observed, as the SNR increases, the sensitivity of the system to the error components increases as well. Hence the robust design shows more enhancements with the higher SNR region. Unless stated otherwise, the values of Table II has been chosen for different system parameters in our simulations.

VII. CONCLUSION

In this paper we have presented a semi-definite relaxation framework to address the transmit strategy design in a full-duplex decode-and-forward relaying system. Our study

TABLE II: Unless stated otherwise the following values are set in our simulations.

Parameters	Value
$T_{sr} = T_{rd} = T_{rd}$	I_N
ξ_{sr}^n	5×10^{-2}
ξ_{rd}^n	5×10^{-2}
ξ_{rr}^n	5×10^{-4}
$\rho_{sr} = \rho_{rd}$	-20dB
ρ_{rr}	0dB
SNR	10dB
$P_{\max} = P_{\max}^R$	1W
β	4

encompasses different scenarios of channel knowledge inaccuracy and provides a robust and computationally efficient design framework to tackle the resulted degradation. As it has been shown, while benefits of the full-duplex operation is degraded due to the effects of inaccurate channel estimation, our proposed design provides a level of robustness for the resulted system performance. In the end we should once again note the sensitivity of the FD system to the loopback CSI error which requires dedicated estimation intervals and specialized solutions.

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