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Exercise 1
- Proposed Solution -

Friday, October 19, 2018

Solution of Problem 1

a) Straight-forward from (b) by setting k to 1.

b) *(Taken directly from the lecture's script)* Given two matrices $\mathbf{M} \in \mathbb{R}^{n \times m}$ and $\mathbf{N} \in \mathbb{R}^{m \times r}$, the goal is to compute $\mathbf{MN}$. Map function generates the following key-value pairs:

- For each element m_{ij} of $\mathbf{M}$ produce r key-value pairs $((i, k), (j, m_{ij}))$ for $k = 1, \ldots, r$.

- For each element n_{jk} of $\mathbf{N}$ produce n key-value pairs $((i, k), (j, n_{jk}))$ for $i = 1, \ldots, n$.

The Reduce function computes the multiplication as follows:

- For each key (i, k), find the values with the same j.

- Multiply m_{ij} and n_{jk} to get $m_{ij} n_{jk}$.

- Sum up all $m_{ij} n_{jk}$ over j to get $\sum_{j=1}^{m} m_{ij} n_{jk}$.

Solution of Problem 2

a) Given two vectors $\mathbf{v} \in \mathbb{R}^n$ and $\mathbf{w} \in \mathbb{R}^n$, the goal is to compute $\mathbf{v} + \mathbf{w}$. The map function generates the following key-value pairs:

- For each element v_i of $\mathbf{v}$ produce 1 key-value pair $((i), (v_i))$.
- For each element w_j of $\mathbf{w}$ produce 1 key-value pair $((j), (w_j))$.

The Reduce function computes the multiplication as follows:

- For all i, find all the key-value pairs with the same i.
- Sum v_i and w_i to get either $v_i + w_i$, v_i, or w_i.

b) Given one sparse vector $\mathbf{v} \in \mathbb{R}^n$, the goal is to compute $\frac{1}{n} \sum_{i=1}^{n} v_i$. The map function generates the following key-value pairs:

- For each element v_i of $\mathbf{v}$ produce 1 key-value pair $((i), (v_i))$.

The Reduce function computes the multiplication as follows:

- For all i, fetch v_i.
- Sum up all v_i and divide by n to get $\frac{1}{n} \sum_{i=1}^{n} v_i$.