Prof. Dr. Rudolf Mathar, Dr. Arash Behboodi, Qinwei He

Exercise 10

Friday, July 7, 2017

Problem 1. Let $n \in \mathbb{N}$. If there exists a primitive element modulo n, then there exist $\varphi(\varphi(n))$ many.

Problem 2. *(properties of the discrete logarithm)* We examine the properties of the discrete logarithm.

a) Compute the discrete logarithm of 18 and 1 in the group $\mathbb{Z}_{79}^*$ with generator 3 (by trial and error if necessary).

b) How many tryings would be necessary to determine the discrete logarithm in the worst case?

Problem 3. *(prove Proposition 7.5)* Prove Proposition 7.5 from the lecture, which gives a possibility to generate a primitive element modulo n:

Let $p > 3$ be prime, $p - 1 = \prod_{i=1}^{k} p_i^{t_i}$ the prime factorization of $p - 1$. Then,

$$a \in \mathbb{Z}_p^* \text{ is a primitive element modulo } p \Leftrightarrow a^{\frac{p-1}{p_i}} \not\equiv 1 (\operatorname{mod} p) \text{ for all } i \in \{1, \ldots, k\}.$$