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Exercise 11

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Problem 1. *(Shamir no-key protocol)* Alice and Bob are using Shamir's no-key protocol to exchange a secret message. They agree to use the prime $p = 31337$ for their communication. Alice chooses the random number $a = 9999$ while Bob chooses $b = 1011$. Alice's message is $m = 3567$.

a) Calculate all exchanged values c_1, c_2, and c_3 following the protocol.
Hint: You may use $6399^{1011} \equiv 29872 \pmod{31337}$.

Problem 2. *(Proof of 8.3)* Let $n = p \cdot q$, $p \neq q$ be prime and x a non-trivial solution of $x^2 \equiv 1 \ (mod\ n)$, i.e., $x \not\equiv \pm 1 (mod\ n)$.

Then

$$\gcd\ (x + 1, n) \in \{p, q\}$$

Problem 3. *(RSA encryption)* A uniformly distributed message $m \in \{1, \ldots, n-1\}$ with $n = p\,q$ with two primes $p \neq q$ is encrypted using the RSA-algorithm with public key (n, e).

a) Show that it is possible to compute the secret key d if m and n are not coprime, i.e., if $p \mid m$ or $q \mid m$.

b) Calculate the probability for m and n having common divisors.

c) How large is the probability of (b) roughly, if n has 1024 bits and the primes p and q are approximately of same size ($p, q \approx \sqrt{n}$).