

6. Übung

Aufgabe 1

$$P(N_s = i | N_t = j) \stackrel{\text{bedingte Wahrscheinlichkeit}}{=} \frac{P(N_s = i, N_t = j)}{P(N_t = j)} \\ \stackrel{\text{bedingte Wahrscheinlichkeit}}{=} \frac{P(N_t = j | N_s = i) P(N_s = i)}{P(N_t = j)}$$

Proposition 3.21
(Skript)

$$\stackrel{\text{Proposition 3.21}}{=} \frac{P(N_{(s,t]} = j-i) \cdot P(N_{(0,s]} = i)}{P(N_{(0,t]} = j)}$$

$$= \frac{e^{-\lambda(t-s)} \frac{(\lambda(t-s))^{j-i}}{(j-i)!} e^{-\lambda s} \frac{(\lambda s)^i}{i!}}{e^{-\lambda t} \frac{(\lambda t)^j}{j!}}$$

$$= \frac{j!}{i! (j-i)!} \underbrace{e^{-\lambda(t-s)} e^{\lambda t} e^{-\lambda s}}_{=1} \underbrace{\frac{\lambda^{j-i} \lambda^i}{\lambda^j}}_{=1} \frac{(t-s)^{j-i} s^i}{t^j}$$

$$= \binom{j}{i} t^{j-i} \left(1 - \frac{s}{t}\right)^{j-i} \frac{s^i}{t^i}$$

$$= \binom{j}{i} \left(\frac{s}{t}\right)^i \left(1 - \frac{s}{t}\right)^{j-i}$$
