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## Tutorial 9

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Problem 1. (One-dimensional trust region problem) Consider the one-dimensional, real-valued trust region problem.

$$
\begin{array}{cc}
\operatorname{minimize} & a x^{2}+2 b x \\
\text { subject to } & x^{2} \leq 1
\end{array}
$$

a) Determine all pairs $(a, b)$ for which the problem is non-convex.

In the following the problem shall be non-convex.
b) Calculate the dual function $L_{D}(\lambda)$
c) Give the optimal parameter $\lambda^{*}$ which maximizes $L_{D}$ and the corresponding value $d^{*}$.
d) Show that the optimal value of the primal problem $p^{*}$ equals $d^{*}$.

Problem 2. (Dual problem bounds) For the following optimization problems with optimization variable $\boldsymbol{x} \in \mathbb{R}^{2}$, compute the dual problem and the maximum lower bound $d^{*}$ for the optimal value $p^{*}$.
a)

$$
\begin{array}{rc}
\text { minimize } & 2 x_{1}^{2}+8 x_{2}^{2} \\
\text { subject to } & 3 x_{1}+6 x_{2}=10
\end{array}
$$

b)

$$
\begin{array}{rc}
\operatorname{maximize} & 2 x_{1} x_{2} \\
\text { subject to } & x_{1}^{2}+x_{2}^{2}=1
\end{array}
$$

Remark: Convert problem (b) into a minimization problem first.

