Problem 1. (Optimality conditions) Consider the optimization problem

$$
\begin{aligned}
\operatorname{minimize} & x_{1}^{2}+x_{2}^{2} \\
\text { subject to } & \left(x_{1}-1\right)^{2}+\left(x_{2}-1\right)^{2} \leq 1, \\
& \left(x_{1}-1\right)^{2}+\left(x_{2}+1\right)^{2} \leq 1
\end{aligned}
$$

with variable $\boldsymbol{x} \in \mathbb{R}^{2}$.
a) Sketch the feasible set and level sets of the objective. Find the optimal point $\boldsymbol{x}^{*}$ and the optimal value $p^{*}$.
b) Give the expression of the associated Langrangian and state the KKT conditions. Do there exist Lagrange multipliers $\lambda_{1}^{*}$ and $\lambda_{2}^{*}$ that prove that $\boldsymbol{x}^{*}$ is optimal?
c) Derive and solve the Lagrange dual problem. Does strong duality hold?

